STRONGLY EQUAL ODD EVEN MULTIPLICATIVE LABELING OF GRAPHS

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Abstract
A graph $G = (V, E)$ with p vertices and q edges is said to be strongly equal odd even multiplicative if the vertices of G can be labeled with distinct integers from $\{1,2,3, \dots, p\}$ such that labels induced on the edges by the product of the labels of end vertices are all distinct and $Of^*(uv) = Ef^*(uv)$,where $Of^*(uv)$ =number of edges with odd labeling under f^* and $Ef^*(uv)$= number of edges with even labeling under f^*. We discuss strongly equal odd even multiplicative labeling of star graph, double star graph bistar graph and lilly graph.

Keywords: Strongly multiplicative, Star graph, Double star, Bistar graph, lilly graph.

AMS Subject Classifications:05C78.

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Introduction

The assignments of values typically represented by integers using several appropriate mathematical rules to the vertices and / or edges of the given graph is termed as graph labeling. Formally, in 1967 Alex Rosa[1] introduced the concept of graph labeling. There is an enormous literature regarding labeling of many familiar classes of graphs. In this paper we deal only finite, simple, connected and undirected graphs. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$. Hence $|V(G)|$ and $|E(G)|$ are the number of vertices and number of edges of G respectively.

In 2001, Beineke and Hegde introduced multiplicative labeling and proved that every graph admits a multiplicative labeling. The Strongly multiplicative labeling was introduced by Beineke and Hegde. Now strongly multiplicative labeled graphs often serve as models in a wide range of applications. Such applications including coding theory and communication network. Joice Punitha and Josephine Lissie[13] have introduced the notion of even and odd strongly multiplicative graphs. In this sequel we introduced strongly equal odd even multiplicative labeling. A graph $G=(V,E)$ with p vertices and q edges is said to be strongly equal odd even multiplicative if the vertices of G can be labeled with distinct integers from $\{1,2,3,\dots,p\}$ such that labels induced on the edges by the product of the labels of end vertices are all distinct and $Of^*(uv)=Ef^*(uv)$,where $Of^*(uv)$ =number of edges with odd labeling under f^* and $Ef^*(uv)$= number of edges with even labeling under f^*. The following are the basic definitions and results needed for the main section.

1 Preliminaries

Definition:1.1[**2**] The star graph S_n is a complete bipartite graph $K_{1,n}$ where n represents the number of vertices and S_n has n-1 edges.

Definition 1.2.[**6**] The bistar $B_{(m,n)}$ is a graph obtained from K_2 by joining m pendant edges to each end of K_2.The edge K_2 is called the central edge $B_{(m,n)}$ and the vertices of K_2 are called the central vertices of $B_{(m,n)}$.

Definition:1.3[15] The double star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge of the existing n pendant vertices. It has $2n+1$ vertices and $2n$ edges.

Definition:1.4[8] The lilly graph $I_n,\ n\geq 2$ can be constructed by two star graphs $2K_{1,n},\ n\geq 2$ joining two path graphs $2P_n,\ n\geq 2$ with sharing a common vertex. i.e) $I_n=2K_{1,n}+2P_n$.

2. Main Results.

Theorem 2.1 The star $K_{1,n}$ is a strongly equal odd even multiplicative graph for all n is even.

Proof. Let $K_{1,n}$ be the star graph with vertex set $V(K_{1,n})=\{u,u_i\colon 1\leq i\leq n\}$ and the edge set $E(K_{1,n})=\{uu_i\colon 1\leq i\leq n\}$.

We note that $|V(K_{1,n})|=n+1$ and $|E(K_{1,n})|=n$

We define a vertex labeling $f\colon V(K_{1,n})\to\{1,2,3,\dots,n+1\}$ as follows.

$$f(u)=1 \qquad f(u_i)=i+1\,, \qquad 1\leq i\leq n$$

The induced edge function $f^*\colon E(K_{1,n})\to$N defined by

$$f^*(uu_{2i-1})=2i \qquad 1\leq i\leq\frac{n}{2} \qquad \to 1$$

$$f^*(uu_{2i})=2i+1 \qquad 1\leq i\leq\frac{n}{2} \qquad \to 2$$

Clearly the vertex and edge labels are distinct.

From equations 1 and 2, we get $Of^*(uv)=\frac{n}{2}$ and $Ef^*(uv)=\frac{n}{2}$

Thus f^* admits a strongly equal odd even multiplicative labeling.

Hence the star graph $K_{1,n}$ is a strongly equal odd even multiplicative graph.

Illustration: The strongly equal odd even multiplicative labeling of $K_{1,4}$

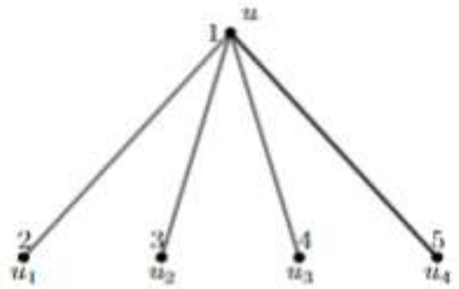

Figure :1

Theorem 2.2 The double star $K_{1,n,n}$ is a strongly equal odd even multiplicative graph for all $n \geq 2$.

Proof. Let $K_{1,n,n}$ be the double star graph with vertex set $V(K_{1,n,n}) = \{w, u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(K_{1,n,n}) = \{wu_i : 1 \leq i \leq n\} \cup \{wv_i : 1 \leq i \leq n\}$.

We note that $|V(K_{1,n,n})| = 2n + 1$ and $|E(K_{1,n,n})| = 2n$

We define a vertex labeling $f: V(K_{1,n,n}) \to \{1,2,3, \ldots, 2n + 1\}$ as follows.

$$f(w) = 1 \qquad f(u_i) = 2i + 1, \qquad 1 \leq i \leq n$$
$$f(v_i) = 2i, \qquad 1 \leq i \leq n$$

The induced edge function $f^*: E(K_{1,n,n}) \to$N defined by

$$f^*(wu_i) = 2i + 1 \qquad 1 \leq i \leq n \qquad \to 1$$
$$f^*(u_i v_i) = 2i(2i + 1) \qquad 1 \leq i \leq n \qquad \to 2$$

Clearly the vertex and edge labels are distinct.

From equations 1 and 2, we get $Of^*(uv) = n$ and $Ef^*(uv) = n$

Thus f^* admits a strongly equal odd even multiplicative labeling.

Hence the double star graph $K_{1,n,n}$ is a strongly equal odd even multiplicative graph.

Illustration: The strongly equal odd even multiplicative labeling of $K_{1,8,8}$

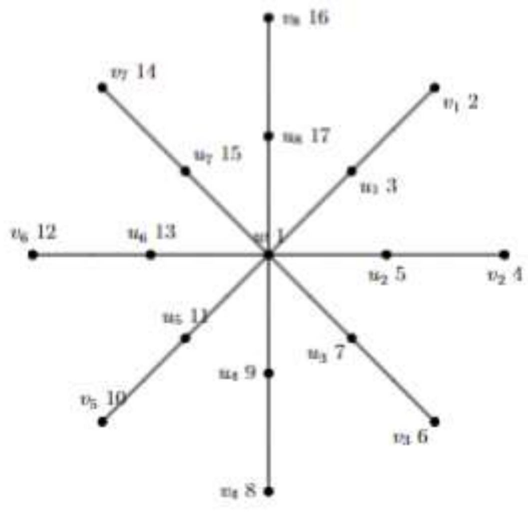

Figure 2

Theorem 2.3 The Bistar star $B_{m,n}$ is a strongly equal odd even multiplicative graph for all $m \geq 2$ and $m = n + 1$.

Proof. Let $B_{m,n}$ be the bistar star graph with vertex set $V(B_{m,n}) = \{u, u_i : 1 \leq i \leq m\} \cup \{v, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{m,n}) = \{uu_i : 1 \leq i \leq m\} \cup \{v, v_i : 1 \leq i \leq n\} \cup \{uv\}$.

We note that $|V(B_{m,n})| = 2m + 1$ and $|E(B_{m,n})| = 2m$

We define a vertex labeling $f: V(B_{m,n}) \rightarrow$ f $\{1,2,3,\ldots,2m+1\}$ as follows.

$$(u) = 1 \qquad f(v) = 2 \qquad f(u_i) = 2i + 1, \qquad 1 \leq i \leq m$$
$$f(v_i) = 2i + 2, \qquad 1 \leq i \leq n$$

The induced edge function $f^*: E(B_{m,n}) \rightarrow$N defined by

$$f(uv) = 2 \qquad \rightarrow 1$$
$$f(uu_i) = 2i + 1 \qquad 1 \leq i \leq m \qquad \rightarrow 2$$
$$f(vv_i) = 2(2i + 2) \qquad 1 \leq i \leq n \qquad \rightarrow 3$$

Clearly the vertex and edge labels are distinct.
From equation 2, we get $Of^*(uv) = m$ and from equations 1 and 3 we get, $Ef^*(uv) = n + 1$
Thus f^* admits a strongly equal odd even multiplicative labeling.

Hence the bistar star graph $B_{m,n}$ is a strongly equal odd even multiplicative graph.

Illustration: The strongly equal odd even multiplicative labeling of $B_{4,3}$

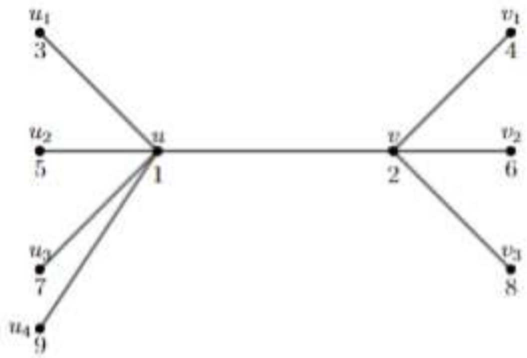

Figure 3

Theorem 2.4 Let G be a graph obtained by joining a pendent vertex with a vertex of degree two on one end of a comb graph. Then G is a strongly equal odd even multiplicative graph for all $n \geq 2$.

Proof. Let G be a graph obtained by joining a pendent vertex with a vertex of degree two on one end of a comb graph. Let G be a graph obtained by joining a pendent vertex w to v_n. Let $V(G) = \{w, v_i, u_i: 1 \leq i \leq n\}$ and the edge set $E(G) = \{v_i v_{i+1}: 1 \leq i \leq n-1\} \cup \{u_i v_i: 1 \leq i \leq n\} \cup \{v_n w\}$.

We note that $|V(G)| = 2n + 1$ and $|E(G)| = 2n$

We define a vertex labeling $f: V(G) \rightarrow \{1,2,3,\ldots,2n+1\}$ as follows.

$$f(w) = 2n + 1$$
$$f(u_i) = 2i, \qquad 1 \leq i \leq n$$
$$f(v_i) = 2i - 1, \qquad 1 \leq i \leq n$$

The induced edge function $f^*: E(G) \rightarrow$N defined by

$$f^*(v_n w) = (2n + 1)(2n - 1) \qquad \rightarrow 1$$
$$f^*(v_i v_{i+1}) = (2i - 1)(2i + 1) \qquad 1 \leq i \leq n - 1 \qquad \rightarrow 2$$
$$f^*(v_i u_i) = (2i - 1)(2i) \qquad 1 \leq i \leq n \qquad \rightarrow 3$$

Clearly the vertex and edge labels are distinct.
From equations 1 and 2, we get $Of^*(uv) = n$ and from equation 3 we get, $Ef^*(uv) = n$
Thus f^* admits a strongly equal odd even multiplicative labeling.

Hence the graph G is a strongly equal odd even multiplicative graph.

Illustration: The strongly equal odd even multiplicative labeling of G with $n = 4$

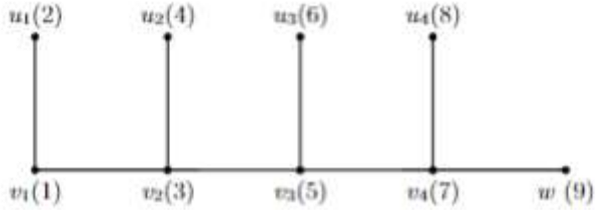

Figure 4

Theorem 2.5 The lilly graph I_n is a strongly equal odd even multiplicative graph for all $n \geq 2$.
Proof. Let I_n be the lilly graph with vertex set $V(I_n) = \{w, u_i, v_i: 1 \leq i \leq n\} \cup \{v_i', u_i': 1 \leq i \leq n-1\}$ and the edge set $E(I_n) = \{wv_i, wu_i: 1 \leq i \leq n\} \cup \{u_i'u_{i+1}',, v_i'v_{i+1}': 1 \leq i \leq n-2\} \cup \{wu_1', wv_1'\}$.
We note that $|V(I_n)| = 4n-1$ and $|E(I_n)| = 4n-2$

We define a vertex labeling $f: V(I_n) \rightarrow \{1,2,3, \dots ,4n-1\}$ as follows.

$$f(w) = 1$$
$$f(v_i) = 2i\,, \qquad 1 \leq i \leq n$$
$$f(u_i) = 2i+1\,, \qquad 1 \leq i \leq n$$
$$f(v_i') = 2n+2i\,, \qquad 1 \leq i \leq n-1$$
$$f(u_i') = 2n+2i+1\,, \qquad 1 \leq i \leq n-1$$

The induced edge function $f^*: E(I_n) \rightarrow$N defined by

$$f^*(wv_i) = 2i \qquad 1 \leq i \leq n \qquad \rightarrow 1$$
$$f^*(wu_i) = (2i+1) \qquad 1 \leq i \leq n \qquad \rightarrow 2$$
$$f^*(v_i'v_{i+1}') = (2n+2i)(2n+2i+2) \qquad 1 \leq i \leq n-2 \qquad \rightarrow 3$$
$$f^*(u_i'u_{i+1}') = (2n+2i+1)(2n+2i+3) \qquad 1 \leq i \leq n-2 \qquad \rightarrow 4$$

Clearly the vertex and edge labels are distinct.
From equations 2 and 4, we get $Of^*(uv) = 2n-1$ and from equations 1 and 3 we get, $Ef^*(uv) = \frac{n}{2}$
Thus f^* admits a strongly equal odd even multiplicative labeling.

Hence the bistar star graph I_n is a strongly equal odd even multiplicative graph.

Illustration: The strongly equal odd even multiplicative labeling of I_4

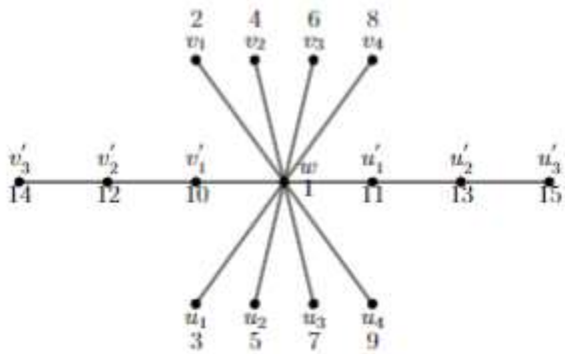

Figure 5

Conclusion

In this paper, we discussed the strongly equal odd even multiplicative labeling. The strongly equal odd even multiplicative labeling conditions are satisfied the some classes of graphs. There may be many interesting strongly equal odd even multiplicative graphs can be constructed in future.

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