ROBUST APPROACH FOR SOLVING THE FULLY FUZZY LPP WITH TRAPEZOIDAL FUZZY NUMBERS

S. Ramathilagam[1], M. Mohamed Salih Mukthar[2]

[1]Assistant Professor, Department of Mathematics, Periyar Arts College, (Affiliated to Thiruvalluvar University, Serkkadu, Vellore-632115) Cuddalore-607001, Email: thilagaiitm@mail.com
[2]Assistant Professor, PG and Research Department of Mathematics, C. Abdul Hakeem College (Autonomous), (Affiliated to Thiruvalluvar University, Serkkadu, Vellore-632115) Melvisharam-632509
Email: salih.mat@cahc.edu.in

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Abstract: *In this research paper, we prove the following theorem for the general case of trapezoidal fuzzy numbers: A fully fuzzy linear programming problem of equality constraints containing all the parameters are trapezoidal fuzzy numbers has fuzzy basic feasible solution if it has fuzzy feasible solution. Next, we develop a fuzzy algebraic method for obtaining the solutions of fully fuzzy linear programming problems, which contains trapezoidal fuzzy numbers, without converting them into crisp linear programming problems. Particularly in this method, we use the modified operations of subtraction and division of trapezoidal fuzzy numbers. Finally, we illustrate the algorithm by solving some problems.*

Keywords: *Fully Fuzzy linear programming problem, Trapezoidal fuzzy number, Fuzzy algebraic method, Fuzzy arithmetic operations*

1. INTRODUCTION

In the history of fuzzy theory, Prof. Lotfi Zadeh [1] was being the key initiator for developing many concepts. Especially, Zadeh [1] developed one interesting concept that is a fuzzy number and it does not like a usual crisp number. Because of its development, several types of fuzzy numbers were developed by some authors. Das, Mandal and Edalatpanah [2] have described about the standard operations of trapezoidal fuzzy numbers. S.Ramathilagam and M.Mohamed Salih Mukthar [3] have modified the two operations that is subtraction and division of trapezoidal fuzzy numbers for the purpose of obtaining inverse operations of addition and multiplication of trapezoidal fuzzy numbers.
Nasseri [4], Towfik [5] and Ghoushchi. S.J,[6] have proposed some methods for obtaining the solutions of fuzzy linear programming problems in their articles. Akram, M [7], D. S. Shelar [8] have given modified approach for solving the fuzzy linear programming problems. Ebrahimnejad [9] has discussed some results on the solutions of fuzzy linear programming problems and has given a method for solving them. Most of the authors have restricted all the parameters only to symmetric trapezoidal fuzzy numbers. Through this paper, we prove the main theorem in the next section for the general case of trapezoidal fuzzy numbers. Hence, the theorem will be generalized. Later, we propose a fuzzy algebraic method for solving fuzzy linear programming problems without converting them into crisp problems.
First, we recall the following terminologies:

Definition 1.1

A fuzzy number $\widetilde{A_T} = (p, q, m, n)$ is called a trapezoidal fuzzy number [10] if its membership function is given by: $\mu_{\widetilde{A_T}}(x) = \begin{cases} \frac{x-p}{q-p}, & p \le x \le q \\ 1, & q \le x \le m \\ \frac{x-n}{m-n}, & m \le x \le n \\ 0, & elne \end{cases}$

Definition 1.2

A trapezoidal fuzzy number $\widetilde{A_T} = (p, q, m, n)$ is said to be non-positive (non-negative) [10] if $p \le 0$ (p $\ge$ 0).

Definition 1.3

Let $\widetilde{A_{T1}} = (p, q, m, n)$ and $\widetilde{A_{T2}} = (w, x, y, z)$ be two arbitrary trapezoidal fuzzy numbers. Then [10] $\widetilde{A_{T1}} < \widetilde{A_{T2}}$ if and only if (q - p) < (x - w) or q < x and q - p = x - w or q = x, q - p = x - w and $\frac{q+m}{2} < \frac{x+y}{2}$ or q = x, q - p = x - w, $\frac{q+m}{2} = \frac{x+y}{2}$ and s - r < z - y.

Definition 1.4

Let $\widetilde{A_{T1}} = (p, q, m, n)$ and $\widetilde{A_{T2}} = (w, x, y, z)$ be two arbitrary trapezoidal fuzzy numbers.
Then the addition of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [10] is defined as

$$\widetilde{A_{T1}} + \widetilde{A_{T2}} = (p, q, m, n) + (w, x, y, z) = (p + w, q + x, m + y, n + z)$$

Definition 1.5

Let $\widetilde{A_{T1}} = (p, q, m, n)$ and $\widetilde{A_{T2}} = (w, x, y, z)$ be two arbitrary trapezoidal fuzzy numbers.
Then the multiplication of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [10] is defined as
$\widetilde{A_{T1}} X\, \widetilde{A_{T2}} = (p, q, m, n)$ X $(w, x, y, z) = (a, b, c, d)$,
where $a = \min(pw, px, qw, qx)$, $b = \max(pw, px, qw, qx)$,
$c = \min(my, mz, ny, nz)$, $d = \max(my, mz, ny, nz)$

Definition 1.6

Let $\widetilde{A_{T1}} = (p, q, m, n)$ and $\widetilde{A_{T2}} = (w, x, y, z)$ be two arbitrary trapezoidal fuzzy numbers, where $p \le q \le m \le n$ and $w \le x \le y \le z$. Then the modified subtraction of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [3] is defined as

$$\widetilde{A_{T1}} - \widetilde{A_{T2}} = (p, q, m, n) - (w, x, y, z) = (p - w, q - x, m - y, n - z)$$

Definition 1.7

Let $\widetilde{A_{T1}} = (p, q, m, n)$ and $\widetilde{A_{T2}} = (w, x, y, z) \ne (0,0,0,0)$ be two arbitrary trapezoidal fuzzy numbers, where $p \le q \le m \le n$ and $w \le x \le y \le z$. Then the modified division of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [3] is defined as

$$\widetilde{A_{T1}} / \widetilde{A_{T2}} = (p, q, m, n)/(w, x, y, z) = (p/w, q/x, m/y, n/z).$$

3. Robustic Result On Fully Fuzzy Linear Programming Problem

Theorem 3.1

If the fuzzy system of linear equality constraints in nonnegative variables of the fuzzy linear programming problem has a fuzzy feasible solution, then it also has a fuzzy basic feasible solution.

Remark 3.1

- This theorem proved by A. Ebrahimnejad [9] only for the case of Fuzzy linear programming problems with Symmetric Trapezoidal fuzzy numbers.

Now, we verify the similar result for the case of Fully Fuzzy linear programming problems with any Trapezoidal fuzzy numbers. The following verification of the theorem strengthen the originality of the result.

Theorem 3.2
If the fully fuzzy linear programming problem of linear equality constraints with any trapezoidal fuzzy numbers has a fuzzy feasible solution, then it also has a fuzzy basic feasible solution.

Proof
Let us consider a fully fuzzy linear programming problem (FFLPP) as given below:
Maximize (or Minimize) $\tilde{z} = \sum_{j=1}^{n} \tilde{c_j}\tilde{x_j}$
Subject to the constraints
$\sum_{j=1}^{n} \widetilde{a_{ij}}\tilde{x_j} = \tilde{b_i}$, i = 1, 2, 3, …, m ----------(i)
$\tilde{x_j} \geq \tilde{0}$, for all j =1,2,…,n, where $\widetilde{a_{ij}}$, $\tilde{x_j}$, $\tilde{c_j}$, $\tilde{b_i}$ are trapezoidal fuzzy numbers.

Case 1
$\tilde{b_i} = \tilde{0}$
(i)$\Longrightarrow$ $\tilde{x_j} = \tilde{0}$ for all i = 1, 2, 3, …, m
i.e., it is the fuzzy basic feasible solution.

Case 2
$\tilde{b_i} \neq \tilde{0}$
Then $\tilde{x_j} = \tilde{0}$ is not a fuzzy feasible solution to the given FFLPP.
So, let us assume that $\tilde{x_j} = (\tilde{x}_{J_1}, \tilde{x}_{J_2}, \dots, \tilde{x}_{J_k})$ is a fuzzy feasible solution to the given FFLPP, where $\{\widetilde{a_p}: \tilde{x}_{J_p} > \tilde{0}\}=\{\widetilde{a_1}, \widetilde{a_2},\dots, \widetilde{a_k}\}$
Then $\tilde{x}_{J_p} = \tilde{0}$, for p$\notin$\{1,2,…,k\}
Now, $\sum_{p=1}^{k} \widetilde{a_p}\tilde{x}_{J_p} = \widetilde{a_1}\tilde{x}_{J_1} + \widetilde{a_2}\tilde{x}_{J_2} + \dots + \widetilde{a_k}\tilde{x}_{J_k} = \tilde{b_i}$ ----------------------(ii)
If $\{\widetilde{a_1}, \widetilde{a_2},\dots, \widetilde{a_k}\}$ is linearly independent vectors, then
(ii)$\Longrightarrow$ $\widetilde{x_{Jp}} = \tilde{0}$ for all p = 1, 2, 3, …, k
Hence this is the fuzzy basic feasible solution.

If the vectors $\{\widetilde{a_1}, \widetilde{a_2},\dots, \widetilde{a_k}\}$ are linearly dependent, then there exist some trapezoidal fuzzy numbers $\widetilde{y_{J1}}, \widetilde{y_{J2}},\dots, \widetilde{y_{Jk}}$ not all zeros such that $\widetilde{a_1}\tilde{y}_{J_1} + \widetilde{a_2}\tilde{y}_{J_2} + \dots + \widetilde{a_k}\tilde{y}_{J_k} = \tilde{0}$
Now, we can find some t such that $\tilde{y}_{J_t} \neq \tilde{0}$

$\Longrightarrow \tilde{a}_t\tilde{y}_{J_t} = - \sum_{p\neq t}^{k} \widetilde{a_p}\tilde{y}_{J_p}$, p = 1, 2, 3, …, k

$\Longrightarrow \tilde{a}_t = - \sum_{p\neq t}^{k} \widetilde{a_p}\frac{\widetilde{y_{J_p}}}{\widetilde{y_{J_t}}}$, p = 1, 2, 3, …, k -----------(iii)

Combining (2) and (3), we get

$\sum_{p\neq t}^{k}\left(\tilde{x}_{J_p} - \tilde{x}_{J_t}\frac{\widetilde{y_{J_p}}}{\widetilde{y_{J_t}}}\right)\widetilde{a_p} = \tilde{b_i}$, p = 1, 2, 3, …, k -----------(iv)

In this manner, a fuzzy feasible solution with at most (k-1) positive variables obtained.
To do this, we choose $\tilde{a}_t$ in such a way that

$\tilde{x}_{J_p} - \tilde{x}_{J_t}\frac{\widetilde{y_{J_p}}}{\widetilde{y_{J_t}}} \geq \tilde{0}$, p = 1, 2, 3, …, k and p≠t ----------(v)

If suppose assume that $\tilde{y}_{J_p} = \tilde{0}$, then $\tilde{x}_{J_p} \geq \tilde{0}$ for any p.
Hence the above condition is satisfied.

If $\widetilde{y_{J_p}} \neq \tilde{0}$, then we obtain

(v) $\Rightarrow \frac{\widetilde{x_{J_p}}}{\widetilde{y_{J_p}}} - \frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}} \geq \tilde{0}, \ \widetilde{y_{J_p}} > \tilde{0}$ ------------------(vi)

and $\frac{\widetilde{x_{J_p}}}{\widetilde{y_{J_p}}} - \frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}} \leq \tilde{0}, \ \widetilde{y_{J_p}} < \tilde{0}$ ------------------(vii)

(vi) and (vii) give the method of selecting $\widetilde{\alpha_t}$ such that (k-1) variables are positive.

To calculate the maximum value of $\frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}}$ in both (vi) and (vii),

we must choose $\frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}} = \min \left\{ \frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}}, \ \widetilde{y_{J_p}} > \tilde{0} \right\}$

For getting the positive value of $\frac{\widetilde{x_{J_t}}}{\widetilde{y_{J_t}}}$, each variable in (iv) should be non-negative. So, a fuzzy feasible solution might be obtained that has at most (k-1) positive variables.

If the vectors corresponding to the above positive variables are linearly independent, then the current solution is a fuzzy basic feasible solution.

Otherwise, the process of removing the positive variables one by one is carried out till a fuzzy feasible solution is obtained. Then by following the same procedure, we get a fuzzy basic feasible solution.

4. Effective Solution Of Fully Fuzzy Linear Programming Problem

The authors Amit Kumar & Jagdeep Kaur [11] have developed some methods for solving fuzzy linear programming and they restricted only to symmetric trapezoidal fuzzy numbers. Some authors have suggested that convert the fully fuzzy linear programming into crisp linear programming and then solve it. In the next section, we construct a new algorithm of fuzzy algebraic method, for solving the fully fuzzy linear programming problem in which all the parameters are trapezoidal fuzzy numbers by using the modified operations (subtraction and division) of trapezoidal fuzzy numbers.

4.1 Fuzzy Algebraic Method - Algorithm

Step 1: In the given fully fuzzy linear programming problem with trapezoidal fuzzy numbers, rewrite the given constraints into the equations form by adding (or subtracting) slack (or surplus) variables for $\leq$ (or $\geq$) constraints.

Step 2: Assume that all the non-basic variables are zeros.

Step 3: Calculate the values of the basic (remaining) variables.

Step 4: Check the feasibility conditions to the basic variables.

Step 5: If the solutions are feasible, then compute the corresponding values of $\tilde{z}$.

If the solutions are not feasible, then left the $\tilde{z}$ column.

Step 6: Similarly, assume that the other possible combinations of the decision variables are zeros. Then follow step 3.

Step 7: If all the possible combinations of the decision variables are over, then find the maximum (or minimum) of all the values of $\tilde{z}$. This maximum (or minimum) $\tilde{z}$ value will be the optimum solution to the given problem.

Next, we demonstrate the algorithm of fuzzy algebraic method by solving the following example problems.

Example 4.1.1

Consider the following maximization fully fuzzy linear programming problem:

Maximize $\tilde{z} = (1,6,9,12)\,\widetilde{x_1} + (2,3,8,9)\,\widetilde{x_2}$

Subject to $(2,3,4,5)\,\widetilde{x_1} + (1,2,3,4)\,\widetilde{x_2} \leq (6,16,30,48)$

$(-1,1,2,3)\widetilde{x_1} + (1,3,4,6)\,\widetilde{x_2} \leq (0,17,30,54)$

$\widetilde{x_1}, \widetilde{x_2} \geq 0$, where $\widetilde{x_1}$ and $\widetilde{x_2}$ are trapezoidal fuzzy numbers

Solution
To solve this problem, we use Fuzzy Algebraic method as follows:
Adding slack variables $\widetilde{x_3}, \widetilde{x_4}$ (trapezoidal fuzzy numbers) for ≤ constraints, the given problem becomes
Maximize $\tilde{z} = (1,6,9,12)\, \widetilde{x_1} + (2,3,8,9)\, \widetilde{x_2} + (0,0,0,0)\widetilde{x_3} + (0,0,0,0)\widetilde{x_4}$
Subject to $(2,3,4,5)\widetilde{x_1} + (1,2,3,4)\widetilde{x_2} + (1,1,1,1)\widetilde{x_3} = (6,16,30,48)$
$(-1,1,2,3)\widetilde{x_1} + (1,3,4,6)\widetilde{x_2} + (1,1,1,1)\widetilde{x_4} = (0,17,30,54)$
$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4} \geq 0$

Basic Variables	**Non-basic Variables**	**Solution**	**Nature of Solution**	**Objective Function (Value of $\tilde{z}$)**
$\widetilde{x_3}$ and $\widetilde{x_4}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_2} = (0,0,0,0)$	$\widetilde{x_3} = (6,16,30,48)$ and $\widetilde{x_4} = (0,17,30,54)$	Feasible	(0,0,0,0)
$\widetilde{x_1}$ and $\widetilde{x_3}$	$\widetilde{x_2} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_1} = (0,15,17,18)$ and $\widetilde{x_3} = (-42,-38,-29,6)$	Infeasible	---
$\widetilde{x_1}$ and $\widetilde{x_4}$	$\widetilde{x_2} = (0,0,0,0)$ and $\widetilde{x_3} = (0,0,0,0)$	$\widetilde{x_1} = (3,5.33,7.5,9.6)$ and $\widetilde{x_4} = (5.33,11.67,15,25.2)$	Feasible	(3,31.98,67.5,302.4)
$\widetilde{x_2}$ and $\widetilde{x_3}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_2} = (0,5.67,7.5,9)$ and $\widetilde{x_3} = (4.66,6,7.5,12)$	Feasible	(0,17.01,60,81)
$\widetilde{x_2}$ and $\widetilde{x_4}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_3} = (0,0,0,0)$	$\widetilde{x_2} = (6,8,10,12)$ and $\widetilde{x_4} = (6,24,40,72)$	Feasible	(12,72,80,108)
$\widetilde{x_1}$ and $\widetilde{x_2}$	$\widetilde{x_3} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_1} = (2.38,2.76,4.54,4.61)$ and $\widetilde{x_2} = (-2.4,4.4,6.32,6.33)$	Infeasible	---

Using the definition of maximum of trapezoidal fuzzy numbers, from the above table, we obtain the Optimum solution (maximum value) that is $\tilde{z} = (12,72,80,108)$, when $\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_2} = (6,8,10,12)$.

Example 4.1.2
Let us consider the following minimization fully fuzzy linear programming problem:
Minimize $\tilde{z} = (-5,-4,-3,-2)\, \widetilde{x_1} + (1,2,3,4)\, \widetilde{x_2}$
Subject to $(-8,-7,-6,-5)\, \widetilde{x_1} + (4,5,6,7)\, \widetilde{x_2} \geq (-4,-3,-2,-1)$
$(6,7,8,9)\widetilde{x_1} + (-10,-9,-8,-7)\, \widetilde{x_2} \geq (-6,-5,-4,-3)$
$\widetilde{x_1}, \widetilde{x_2} \geq 0$, where $\widetilde{x_1}$ and $\widetilde{x_2}$ are trapezoidal fuzzy numbers
Solution
Now, we use the algorithm of Fuzzy Algebraic method for obtaining the optimum solution.
First, subtracting surplus variables $\widetilde{x_3}, \widetilde{x_4}$ (trapezoidal fuzzy numbers) for ≥ constraints, the given problem becomes
Minimize $\tilde{z} = (-5,-4,-3,-2)\, \widetilde{x_1} + (1,2,3,4)\, \widetilde{x_2} + (0,0,0,0)\widetilde{x_3} + (0,0,0,0)\widetilde{x_4}$
Subject to $(-8,-7,-6,-5)\, \widetilde{x_1} + (4,5,6,7)\, \widetilde{x_2} - (1,1,1,1)\widetilde{x_3} = (-4,-3,-2,-1)$
$(6,7,8,9)\widetilde{x_1} + (-10,-9,-8,-7)\, \widetilde{x_2} - (1,1,1,1)\widetilde{x_4} = (-6,-5,-4,-3)$
$\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4} \geq 0$

Basic Variables	**Non-basic Variables**	**Solution**	**Nature of Solution**	**Objective Function (Value of $\tilde{z}$)**
$\widetilde{x_3}$ and $\widetilde{x_4}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_2} = (0,0,0,0)$	$\widetilde{x_3} = (1,2,3,4)$ and $\widetilde{x_4} = (3,4,5,6)$	Feasible	(-7,3,4,18)
$\widetilde{x_1}$ and $\widetilde{x_3}$	$\widetilde{x_2} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_1} = \left(-1,\frac{-5}{7},\frac{-1}{2},\frac{-1}{3}\right)$ and $\widetilde{x_3} = \left(\frac{11}{3},4,9,11\right)$	Infeasible	---
$\widetilde{x_1}$ and $\widetilde{x_4}$	$\widetilde{x_2} = (0,0,0,0)$ and $\widetilde{x_3} = (0,0,0,0)$	$\widetilde{x_1} = \left(\frac{1}{5},\frac{1}{3},\frac{3}{7},\frac{1}{2}\right)$ and $\widetilde{x_4} = \left(\frac{22}{5},\frac{36}{5},\frac{52}{7},\frac{15}{2}\right)$	Feasible	$\left(\frac{41}{15},\frac{68}{5},\frac{291}{14},\frac{204}{7}\right)$
$\widetilde{x_2}$ and $\widetilde{x_3}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_2} = \left(\frac{3}{7},\frac{1}{2},\frac{5}{9},\frac{3}{5}\right)$ and $\widetilde{x_3} = \left(\frac{26}{5},\frac{48}{9},\frac{11}{2},\frac{40}{7}\right)$	Feasible	$\left(\frac{27}{10},\frac{564}{63},\frac{147}{10},\frac{1370}{63}\right)$
$\widetilde{x_2}$ and $\widetilde{x_4}$	$\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_3} = (0,0,0,0)$	$\widetilde{x_2} = \left(-1,\frac{-3}{5},\frac{-1}{3},\frac{-1}{7}\right)$ and $\widetilde{x_4} = \left(5,\frac{17}{3},\frac{57}{5},15\right)$	Infeasible	---
$\widetilde{x_1}$ and $\widetilde{x_2}$	$\widetilde{x_3} = (0,0,0,0)$ and $\widetilde{x_4} = (0,0,0,0)$	$\widetilde{x_1} = \left(-\infty,\frac{-29}{28},\frac{66}{49},\frac{7}{3}\right)$ and $\widetilde{x_2} = \left(\frac{-6}{7},\frac{9}{7},2,\infty\right)$	Infeasible	---

Using the definition of minimum of trapezoidal fuzzy numbers, from the above table, we get the **Optimum solution** (minimum value) that is $\tilde{z} = \left(\frac{27}{10},\frac{564}{63},\frac{147}{10},\frac{1370}{63}\right)$, when $\widetilde{x_1} = (0,0,0,0)$ and $\widetilde{x_2} = \left(\frac{3}{7},\frac{1}{2},\frac{5}{9},\frac{3}{5}\right)$.

5. CONCLUSION

At the beginning of this paper, we reviewed the developments of trapezoidal fuzzy numbers and some methods of fuzzy linear programming problems. Next, we proved one main theorem stating that fully fuzzy linear programming problems of equality constraints, which contain the coefficients and decision variables are trapezoidal fuzzy numbers, have fuzzy basic feasible solutions, while they are having fuzzy feasible solutions. Hence, the theorem is generalized and it is applicable to any trapezoidal fuzzy numbers. To solve the fully fuzzy linear programming problems, many authors have converted them into crisp linear programming problems and then solved. Through this paper, we constructed an algorithm to obtain the solution of the fully fuzzy linear programming problems without converting them into crisp linear programming problems. Hence, we obtain the optimum solutions in terms of trapezoidal fuzzy numbers. Finally, we have solved the maximization and minimization fully fuzzy linear programming problems for applicability of this algorithm in both cases. Further, we suggest this method for the researchers to those who needs the solutions in terms of fuzziness.

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