Acquisitive Knowledge Optimization with Layered Color Set for Discrete Optimization based Decision problems in Group theory

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Article History: Received: 28.05.2023 Revised: 19.06.2023 Accepted: 17.07.2023

ABSTRACT This research proposes the most effective Discrete Optimization (DO) based decision making problems such as Subset Sum Problem (SSP), Knapsack problem (KP), and Submonoid Memebership Problem (SMP) along with calculating their generic case complexity. Also, to solve these DO based decision making problems a novel Discrete Gain Acquisitive Knowledge Optimization with Layer Splitting Color Set (DGAKO-LSCS) which utilized to solve the knapsack problem, Subset sum and Submonoid membership problem by splitting the elements in group H into $logm$. Furthermore, the generic case complexity of these techniques are computed and showed that the solutions are effective with low time complexity and standard deviation.

Keywords : Group theory, Rough set, Optimization, Generic case complexity, Subset sum problem, Knapsack problem, Submonoid memebership problem.

DOI:10.48047/ecb/2023.12.10.456

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1. Introduction

The group that is algebraic structure in group theory [1,2] is referred to as the most important component in computer science, mathematics, science, and statistics. In group theory, discrete optimization (DO) problems occurs in arbitrary groups and cause problems in binary operation [3-7] and decision making by increasing the computational complexity. The most crucial DO problems are subset sum problem, knapsack problem, and submonoid memebership problem [8-12]. In general form, the subset sum problem asks whether any subset of the integers sums to exactly T in which a target-sum is T and a multiset of integers is S. The issue is recognized as being Non deterministic Polynomial time (NP) and has some of its restricted variants are NP - complete as well. The idea of Interval-valued Intuitionistic Fuzzy sets (IVIFSs) [13 -17], which is a generalization of fuzzy sets and Intuitionistic Fuzzy sets (IFSs, was therefore been developed and generalized, but it is still limited in its ability to cover all potential uncertainties in various physical problems. To determine worst case complexity of arbitrary group decision making algorithms, a variety of methods has been used but they all entail calculating the expected value of the algorithm's running time with respect to some metric on the input set [18-20]. Generic case complexity [21,22] solves the problem in other complexity methods in which decision problem under consideration must be decidable and require a complete algorithm to resolve. A Group Theory - Based Optimization Algorithm (GTOA) [23-25] that incorporates algebraic group operations into the evolution procedure in order to solve knapsack problems. The main contribution of this paper are Definitions and Theorems for various DO-based decision making problems such as subset sum problem, knapsack problem, and submonoid memebership problem has been provided. To solve DO based decision making problems, [18-20] a novel Discrete Gain Acquisitive Knowledge Optimization with Layer Splitting Color Set (DGAKO – LSCS) has been presented that select optimistic elements from group without decision problem and reduce complexity by splitting the elements in the group. To determine time complexity of proposed approach, generic case complexity [23-28] has been calculated with theoretical proof.

2. DEFINITIONS

2.1 : Subset sum problem (SSP)

Given $h_1, \ldots . h_j, h \in H$ decide if $h = h_1^{\mu_1}, \ldots . h_j^{\mu_j}$ ------------ (1)

for some coefficients $\mu_1, \dots, \mu_j \in \{0,1\}$ when subjected to optimization condition

$\sum_t \mu_t$ is minimal.

Otherwise output has no solutions. If $Y = \{1\}$ then SSP occurs in unary form so it is in P-case. If $Y = \{2^m | m \in M\}$ then SSP is binary form so it is in NP-complete.

Two ways are provided to state the search variation of the problem. The first one only takes into account instances of the problem that are a solution to the instance exists; the second one is more demanding because it calls for solving the decision problem as well as discovering a solution (if one exists) for a specific instance.

SSP 1 for H: Given $h_1, \dots. h_j, h \in H$ find $\mu_1, \dots, \mu_j \in M \cup \{0\}$ with least possible distance between h and $h = h_1^{\mu_1}, \dots. h_j^{\mu_j}$ in Cayley graph $Cay(H, Y)$.

SSP 2 for H: Given $h_1, \dots. h_j, h \in H$ find $\mu_1, \dots, \mu_j \in M \cup \{0\}$ such that $h = h_1^{\mu_1}, \dots. h_j^{\mu_j}$ belongs to segment $[1, h]$ and the distance between h and $h = h_1^{\mu_1}, \dots. h_j^{\mu_j}$ in Cayley graph $Cay(H, Y)$ is least possible.

The aforementioned optimization issues relate to the situation where the weight function k is a constant function on H with $k = 1$.

2.2 : Knapsack problem (KP)

Given $h_1, \dots. h_j, h \in H$ decide if $h = Hh_1^{\mu_1}, \dots. h_j^{\mu_j}$ ----------- (2)

for some non-negative integers $\mu_1, \dots, \mu_j$. Integer knapsack problem (IKP), where the coefficients $\mu_1, \dots, \mu_j$ are arbitrary integers, is another variant of this issue. However, it is simple to see that for any group H, IKP is P-time reducible to KP. The third problem is identical to KP in the classical (abelian) case, but it is fundamentally different and of great interest to algebra in general. The search variation of the problem can be stated in one of two ways. The first one only considers instances of the problem for which a solution already exists; the second one is more difficult because it requires determining both the decision problem and a solution (if one exists) for a given instance.

KP 1 for H: Given $h_1, \ldots . h_j, h \in H$ find $\mu_1, \ldots ., \mu_j \in M \cup \{0\}$ with least possible distance between h and $h = h_1^{\mu_1}, \ldots . h_j^{\mu_j}$ in Cayley graph $Cay(H, Y)$.

This formula allows for solutions whose "total weight" is greater than the knapsack's carrying capacity. Also, require the following in order to specify with precision when a given solution fits in the knapsack geometrically. If there is a geodesic path in $Cay(H, Y)$ from h to i that contains v, then the element v belongs to the segment $[h, i]$ for elements $h, i, v \in H$ thereby the problem is defined.

KP 2 for H: Given $h_1, \ldots . h_j, h \in H$ find $\mu_1, \ldots ., \mu_j \in M \cup \{0\}$ such that $h = h_1^{\mu_1}, \ldots . h_j^{\mu_j}$ belongs to segment $[1, h]$ and the distance between h and $h = h_1^{\mu_1}, \ldots . h_j^{\mu_j}$ in Cayley graph $Cay(H, Y)$ is least possible.

2.3 : Submonoid Membership problem (SMP)

Given $h_1, \ldots . h_j, h \in H$ decide if h belongs to the submonoid generated by $h_1, \ldots . h_j$ in H i.e, if the following inequality holds for some

$h_{i1}, \ldots . h_{iS} \in \{h_1, \ldots . h_j\}\ S \in M\ h = h_{i1}, \ldots . h_{iS}$ ----------- (3)

A well-known problem in group theory known as the Generalized Word Problem (GWP) or the uniform subgroup membership problem in H is the restriction of SMP to the case when the set of generators $\{h_1, \ldots . h_j\}$ is closed under inversion so the submonoid is actually a subgroup of H. The bounded versions of KP and SMP make sense to take into account, as is customary in complexity theory, at least because they are always decidable in groups where the word problem is decidable. Given that the number of factors in these equalities is constrained by a natural number m that is given in the unary form, i.e., as the word 1^m, the problem in this instance is to determine whether the corresponding equalities (2) and (3) hold for a given h.

Bounded SMP for H: Given $h_1, \ldots . h_j, h \in H$ and $1^m \epsilon M$ decide if h is equal in H to product of a form $h = h_{i1}, \ldots . h_{iS}$ where $h_{i1}, \ldots . h_{iS} \in \{h_1, \ldots . h_j\}$ and $S \leq m$.

Most of the time, simultaneously solved the decision and search versions of the aforementioned issues while determining the algorithms' maximum allowable levels of time complexity. The search problems' optimization versions, just like in the classic case, may be the most intriguing variants and is NP-complete. Given $h_1, \ldots . h_j, h \in H$ express h as a product with minimum number of factors m that is $h = Hh_{i1}, \ldots . h_{im}$

------------ (4)

2.4 : Generic case complexity

Let C be a complexity class an $d \subseteq (Y^*)^j$ be a decision problem. Assume that ω is a correct partial algorithm for d, which means that whenever it makes a decision about whether a tuple in $(Y^*)^j$ belongs to d or not, that decision is accurate. If there is a generic subset $S \subseteq (Y^*)^j$ such that the algorithm ω terminates on the input T for every tuple S within the complexity bound C, then solves d with the generic-case complexity C. If the set S is also strongly generic, then the partial algorithm ω solves the problem d with strongly C generic-case complexity. Reiterate that performance on tuples outside of S is completely disregarded, and the definition thus applies in cases where d has arbitrarily high worst-case complexity or is in fact undecidable. Decision problem "generic" complexity classes can now be defined in a straightforward manner. As a result, while most group-theoretic decision problems do not depend on the selection of a finite generating set for a group's worst-case complexity, it is not entirely clear that the generic-case complexity is unrelated to the set of generators chosen.

3. Theoretical proof

The Theoretical proofs for DO based decision problems such as subset sum problem, knapsack problems and Submonoid memebership problem and generic case complexity has been provided in this subsection.

Theorem 3.1: $\mathbf{SSP(BSW(1,2))}$ is NP complete

Consider the popular Baumslag-Solitar metabelian group as

$$BSM(a,b) = \langle \beta, t | t^{-1}\beta^a t = \beta^b \rangle$$ ----------- (5)

Proof : $SSP(Z,Y)$ where Z is embedded subgroup is NP-complete for generating set $Y = \{y_m = 2^m | m \in M \cup \{0\}\}$. The map is $y_m \to t^{-m}\beta t^m$ ------------ (6)

is obviously P-time computable and defines an embedding $\gamma: Z \to BSM(1,2)$ because $t^{-m}\beta t^m = \beta^{2m}$ hence $SSP(Z,Y)$ is P-time reduced to $SSP(BSW(1,2))$. Thus $SSP(BSW(1,2))$ is NP-complete.

Moreover, it is easy to prove that $SSP\big(BSM(a,b)\big)$ is NP-complete whenever $|a| \neq |b|$ and $a, b \neq 0$. It is less obvious that $SSP\big(BSM(b, \pm b)\big)$ is in P. Hence, it is proved that $SSP\big(BSW(1,2)\big)$ is NP-complete.

Theorem 3.2 : Let H be a hyperbolic group given by a finite presentation $\langle X|K\rangle$. There exist a polynomial time algorithm that given $h_1, \ldots. h_j, h \in H$ and a unary $M \in M \cup \{0\}$ finds $\mu_1, \ldots., \mu_j \in \{0,1\}$ such that distance between h and $h = h_1^{\mu_1}, \ldots. h_j^{\mu_j}$in Cayley graph $Cay(H, Y)$ does not exceed M or outputs "No solutions" if no such $\mu_1, \ldots., \mu_j$ exists

Proof : By using a standard argument based on graph in figure 1, the corresponding decision problem is sufficient to determine: given $h_1, \ldots. h_j, h \in H$ *and a unary* $M \in M \cup \{0\}$, decide whether there exist $\mu_1, \ldots., \mu_j \in \{0,1\}$ such that distance between h and $h = h_1^{\mu_1}, \ldots. h_j^{\mu_j}$in Cayley graph $Cay(H, Y)$ does not exceed M.

Consider graph $\alpha = \alpha\big(u_1, \ldots. u_j, u, M\big)$ accepting a ball of radius M centered at u as shown in figure A.

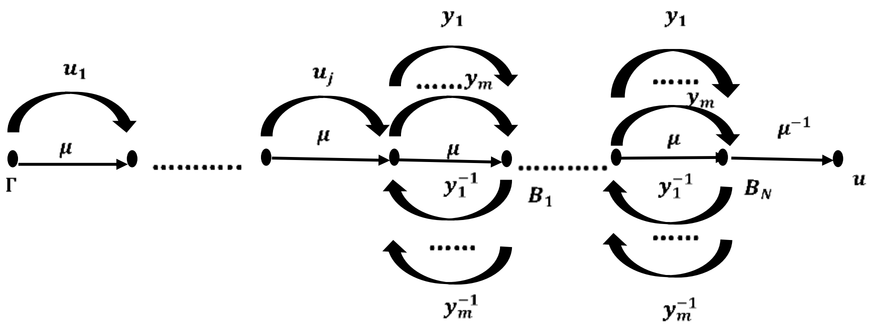

Figure A : Graph $\alpha\big(u_1, \ldots. u_j, u, M\big)$

It is clear that the problem has a positive aspect if and only if $1 \in L(\alpha)$ and it suffices to check whether the graph $\Delta = \mathcal{F}\big(C^{O(\log|w| + \Sigma_u |v_i| + mM)}(\alpha)\big)$ contains edge $\Gamma \to^{\mu} u$.

Hence, it is proved that if there exist $\mu_1, \ldots., \mu_j \in \{0,1\}$ such that distance between h and $h = h_1^{\mu_1}, \ldots. h_j^{\mu_j}$in Cayley graph $Cay(H, Y)$ does not exceed M otherwise no solutions were exists.

Theorem 3.3 : There is a finitely generated subgroup $G = \langle g_1, \dots, g_j \rangle$ in $F_2 \times F_2$ such that $BGWP(F_2 \times F_2, g_1, \dots, g_j)$ is NP complete then BSMP(H) is NP-complete.

Consider $(h, 1^m)$ is a positive instance of $BGWP(H, g_1, \dots, g_j)$ if and only if $\left(g_1, \dots, g_j, g_1^{-1}, \dots., g_j^{-1}, h, 1^m\right)$ is recognized as a positive integer of $BSMP(H)$.

Proof : There exist a finitely presented group H with NP-complete word problem and polynomial Dehn function. Assume that the subset of F(Y) defined by $d_H = g_1, \dots, g_j$ is $BGWP(F(Y) \times F(Y); d_H)$ is NP-hard. The fact that $F_2 \times F_2$ contains a subgroup that is isomorphic to $F(Y) \times F(Y)$ makes $BGWP(F_2 \times F_2; d_H)$ is NP-hard as well. There is only one thing left to mention that is $BGWP(F_2 \times F_2; d_H)$ is NP-complete because the word problem in $F_2 \times F_2$ is P-time decidable. The argument is based on Mikhailova's creation of an undecidable membership $F_2 \times F_2$ subgroup. Hence it is proved that $BSMP(H)$ is also NP-complete since it is identified as a non-negative integer.

Theorem 3.4 : Let H be a finitely generated group and let $G \leq H$ be a finitely generated subgroup of infinite index then the generic-case complexity of the decision problem for G in H is strongly in C.

Proof: Let H_1 be a finite index subgroup of H such that G≤H, and let $\gamma: H_1 \rightarrow H$ be an epimorphism. Assume that the subgroup J of infinite index in H contains the group $G = \gamma\,(G)$ and that the membership problem for J in H belongs to the complexity class C. Therefore, the generic-case complexity in C of the membership problem for G in H. Moreover, the generic-case complexity of the membership problem for G in H is strongly in C if the graph $\alpha(H, J, B)$ is non-amenable for some and thus any finite generating set B of H, then the membership problem G in H has a strong C generic-case complexity. For instance, Theorem B's "strong" conclusion is valid if H is a non-elementary hyperbolic group and J is a quasi-convex subgroup of H. The graph $\alpha(H, J, B)$ is in fact not amenable in this situation. Theorem B suggests that the decision membership problem for G in H is strongly generically in linear time because it is solvable in linear time for a quasi-convex subgroup of a hyperbolic group. Hence this section provides the definitions and theoretical proofs of DO based decision problems such as subset sum problem, knapsack problems and Submonoid memebership problem along with generic case complexity. Furthermore, there is a need to provide the solutions for the DO based decision problems with calculating the generic case complexity therefore the solution strategy is derived and which is explained in the next subsection.

4. Discrete Gain Acquisitive Knowledge Optimization with Layer Splitting Color Set

Discrete Gain Acquisitive Knowledge Optimization with Layer Splitting Color Set (DGAKO-LSCS) has been presented to solve the DO based decision problems and also calculates the generic case complexity. In which Discrete gain acquisitive knowledge optimization solves the knapsack problem by making the elements in the group divisible through knowledge sharing phase and also reduce the population size of optimization technique to attain high quality solutions without computational complexity thereby solves Knapsack optimization problems. Then, Layer splitting color set has been utilized to solve subset sum and subnomoid membership problem in which initially the layer splitting is performed to split the elements in group H into $logm$ that reduce the decision making problem but these problem are NP-complete that is they have problem in both decision making and computation hence color setting is incorporated in layer splitting that reduces the computational complexity. Furthermore, the generic case complexity of these techniques are computed to show the effectiveness of solutions in decision making and computation.

5. Discrete gain acquisitive knowledge optimization

In Discrete gain acquisitive knowledge optimization, knapsack problem has been solved by sharing the knowledge factor and reducing the dimension size of the group. Initially the number of elements (population size) in the group is assumed as *j*. Let $H(h_1, \dots . h_j)$ be the elements of the group and $h_j^{\mu_j} = h_1^{\mu_1}, \dots . h_D^{\mu_D}$ where D is the knowledge factor provided to an individual and $F_J(h_1, \dots . h_j)$ is the objective function values. To obtain a solution for knapsack problem, the initial number of elements has been obtained. The initial population is randomly created with some boundary constraints is given in the equation

$$h_j^{\mu_j}(0) = l_j + randj * (U_j - l_j) \quad \text{-----------} (7)$$

where, l_j and U_j are lower bound and upper bound, $randj$ stands for a 0–1 uniformly distributed random number. Then, in the knowledge acquisitive phase, the knowledge has been shared to the group as early-middle and middle-later stage. In order to improve their skills, group impart their knowledge or expertise to the most qualified elements. The knowledge factor is taken into consideration when calculating the early-middle and middle-later stage dimensions. The dimension of early-middle and middle-later stage has been provided in the equation

$$D_{early} = D \times \left(\frac{gener^{max} - gn}{gener^{max}}\right)^{J} \quad \text{------------ (8)}$$

$$D_{middle} = D - D_{early} \quad \text{------------ (9)}$$

where the knowledge rate $J > 0$, controls the experience rate. The dimensions for the early-middle and middle-later stage are represented by D_{early}, D_{middle} respectively. gn stands for the generation number, and $gener^{max}$ is the maximum number of generations. In the early-middle stage, the elements are updated by arranging them in ascending order as per the objective function which is given in the equation

$$h_{best}, \dots, h_{j-1}, h_j, h_{j+1} \dots, h_{worst} \quad \text{----------- (10)}$$

Choose the closest best h_j and worst h_{j+1} for each $H(h_1, \dots . h_j)$, as well as a random choice $randj$, to learn the information. Therefore, the the knowledge rate, $J > 0$ is used to update the elements in group. The impact and effect of other elements (whether positive or negative) on the individual is covered in middle-later stage. The element's update is ascertained in the manner described below:

- Following the ascending order of the individuals, the three categories of "best," "middle," and "worst" are determined based on the objective function values. The equation (11) provides the determination of best, middle and worst element as:

$$\left.\begin{aligned} bestelement &= 100_t\%(h_{tbest}) \\ middleelement &= D - 2(100_t\%(h_{tmiddle})) \\ worstelement &= 100_t\%(h_{tworst}) \end{aligned}\right\} \quad \text{----------- (11)}$$

- Choose two random vectors representing the top and bottom $100_t\%$ of each element $h_j^{\mu_j}$ for the gaining part, and the middle element is chosen for the sharing part, where $t[0, 1]$ denotes the proportion of the best and worst classes that is obtained in an divisible format.

Thus by effectively determining the worst, best and middle elements in the group, the decision problem occurs due to knapsack problem is eliminated and the other variants of knapsack problem that cause problem in optimization is solved by reducing the population size in Discrete gain acquisitive knowledge optimization. The population size change throughout the optimization process because it is one of the most crucial parameters of the optimization algorithm. First, a large population size is required for the exploration of optimization problem solutions, but a decrease in population size is needed to improve the quality of solutions and the efficiency of the algorithm without any computational complexity.

There is a notion of generic-polynomial-time reduction with respect to which the distributional bounded halting problem is complete within class of distributional NP problems. Hence a formula is derived to reduce the population size which is given in the equation

$$j_{gn+1} = round[(j_{min} - j_{max}) * \left(\frac{gener}{gener^{max}}\right) + gener^{max}]$$ ----------- (12)

where $gn + 1$ stands for the modified population size in the next generation. Given that j_{min} and j_{max} are minimum and maximum population size. Applying equation (12) to Discrete gain acquisitive knowledge optimization has the main benefit of eliminating the worst or most impractical solutions from the initial stage of the optimization process without affecting exploration potential. In a later stage, it highlights the tendency toward exploitation by removing the poorest solutions from the search space thereby solves the knapsack problem.

6. Layer splitting color set approach

In Layer splitting color set, the subset sum and subnomial membership problems which are considered as NP-complete is solved by splitting the elements in the group based on color setting process. Consider a target integer t and a set S of m integers in which the set of all subset sums is given in the equation $\sum(S) = \left\{\sum_{\beta \in Z} |Z \subseteq S\right\}$ ----------- (13)

The set $Z \oplus Y = z + y \mid \beta \in Z, \gamma \in Y$ is their join, and U_j is the upper bound of the elements. Z and Y, for two sets $Z, Y \subseteq [0, U_j]$. This join is time computable $O(U_j \log U_j)$

hence the time complexity is reduced by Layer splitting color set approach without any decision problem. In layer splitting color set approach, elements are divided into $logm$ layers. The last layer $Slog\ m$ has elements with weights in $[0, t/2^{logm-1}]$, while layer Sj is the set of elements with weights in $[t/2j, t/2^{j-1}]$ for $0 < j < log\ m$. This gives the assurance that only $2j$ elements from layer can be selected as a solution. It is sufficient to compute their $MaxConv\ O(log\ m)$ times if it quickly compute (Sj) for all j. To determine the exponential time complexity, colour setting approach is used that compute (Sj) in e $O(T(t) + m)$ time. Hence, for all j computes $II(Sj)$ in $O(T(tlogt\ log^3(2^{j-1}/\rho)))$ time for $Sj \subseteq [t/2j, t/2^{j-1}]$ and for all $\rho \in [0, 1/4]$, where each entry of (Sj) is correct with a probability of at least $1 - \rho$. The set S with m disjoint subset is given by $S = Z1 \cup \ldots\ldots \cup Zm$ where $m = {}^{s}/_{\log(s/\rho)}$. After that, using $O(log(s/\rho))$ elements calculate (Zj) for each partition. $O(T(log(s)t/s)\ log3(s/\rho))$ time is needed for each Zj. Since MinConv requires at least linear time $T(m) = \delta(m)$, therefore need $O(T(t)\ log3(s/\rho))$ time for all $Zj(m)$. This method saves us a lot of time compared to simply computing $(Z1)\ max\ldots max\ (Zm)$, as there are at most $logm$ rounds and need to compute MaxConv with numbers of order $O(2^g t\ log(s/\rho)/s)$. The following equation shows how difficult it would be to join them

$$\sum_{g=1}^{logm} \frac{m}{2^g} T\left(\frac{2^g \log\left(\frac{s}{\rho}\right)t}{slogt}\right) = O(T(tlogt)logm) \qquad \text{------------ (14)}$$

The generic case complexity is proved to be low by the effective selection of δ in any set S hence there exist an integer constant $j \geq 0$ such that, given the choice of the enumeration of Z^* and the definition of generic computability, the binary sequence of S agrees with the initial segment of Z of length $s(k)$ in at least $3s(k)/4$ positions for any $k \geq j$. Pick m so that $km > j$. Thus, it is demonstrated that, for any $\mu > 0$, a set of measures with a maximum of μ can cover the set of all elements that are generally computable by. As a result, it follows that the measure of the set of languages that can be computed by δ is zero. Overall, it is found that the algorithm's generic case complexity is $O(T(t\ log\ t)\ log3(l/\rho))$, with some logarithmic factors omitted if assume that $\mu > 0$ exists and that $T(m) = \delta(m^{1+\mu})$ is true. Hence layer splitting color set approach solves subset sum and subnomoid membership problems and their optimization variants are eliminated by low generic case complexity. Overall, the DO based decision problems have been defined with theoretical proof along with considering generic case complexity. Then, a novel DGAKO-LSCS approach has been proposed to solve subset sum, knapsack and subnomoid membership problems that solve both decision problems and optimization variants by reducing the time complexity. The effectiveness of the proposed DGAKO-LSCS approach has been proved by generic case complexity calculation. The result obtained from Acquisitive Knowledge Optimization with Layered Color Set for DO based Decision problems in Group theory has been explained in detail in the next section.

7. Result and discussion

The results obtained from the proposed DGAKO-LSCS approach has been provided in this section. The results showed that proposed DGAKO-LSCS approach efficiently solved the DO based decision making problems and the effectiveness of the proposed approach is also proved by comparing it with other existing approaches such as Generic Algorithm (GA), Binary Particle Swarm Optimization (BPSO), Binary version of Artificial B- Colony (BABC) and GTOA.

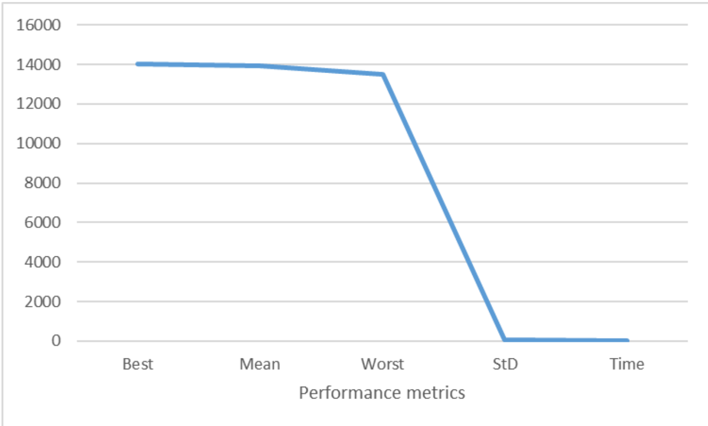

Figure B : Performance metrics of proposed DGAKO-LSCS approach

The performance metrics evaluated from the proposed DGAKO-LSCS approach such as best class, worst class, mean, standard deviation and time has been shown in the figure B. The best class prediction of elements, worst class prediction of elements, and mean of the proposed DGAKO-LSCS approach has optimum values of 14051, 13500 and 13950. This optimum values are provided by Discrete gain acquisitive knowledge optimization that effectively perform decision making to detect the best, worst and mean values of elements in the group and also reduces the computational complexity by minimizing the element dimensions. The standard deviation and time of the proposed DGAKO-LSCS approach is reduced to 50.6 and 0.103 seconds by Layer splitting color set approach in which the time complexity is minimized via splitting the elements in the sets and this approach's effectiveness is verified with generic case complexity that prove the proposed approach has a low time complexity and standard deviation of 0.103 seconds and 50.6. The result obtained from the proposed DGAKO-LSCS approach has been compared with existing approaches such as GA, BPSO, BABC and GTOA in terms of best class, worst class, mean, standard deviation and time.

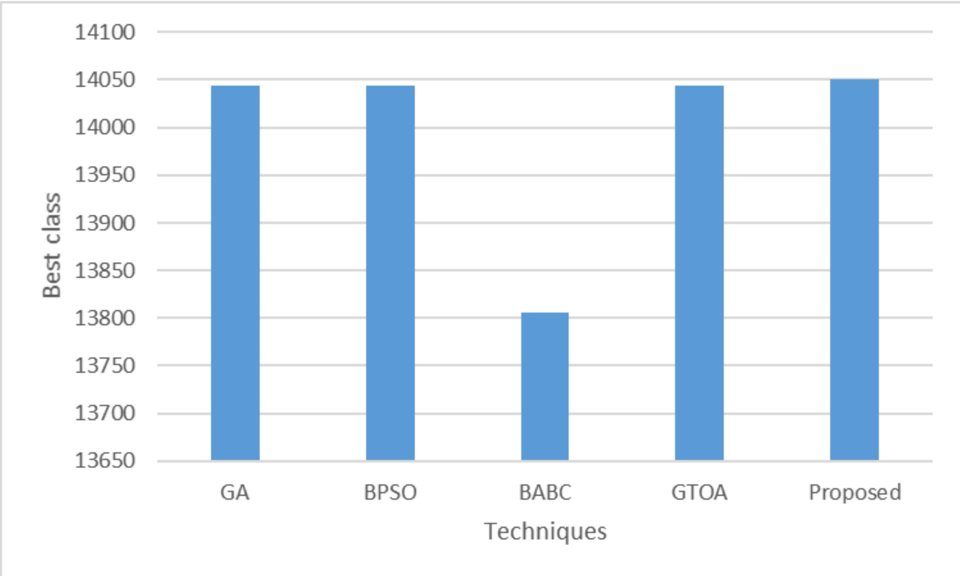

Figure C: Best class Comparison of worst

The comparison of best class of proposed DGAKO-LSCS approach with other existing approaches. The best class of proposed approach is compared with existing techniques such as GA, BPSO, BABC and GTOA. The best class of proposed DGAKO-LSCS approach has the optimum value of 14050 whereas the best class of GA, BPSO, BABC and GTOA are 14044, 14044, 13860 and 14044 respectively. The best class of proposed DGAKO-LSCS approach is high whereas the best class of BABC is low.

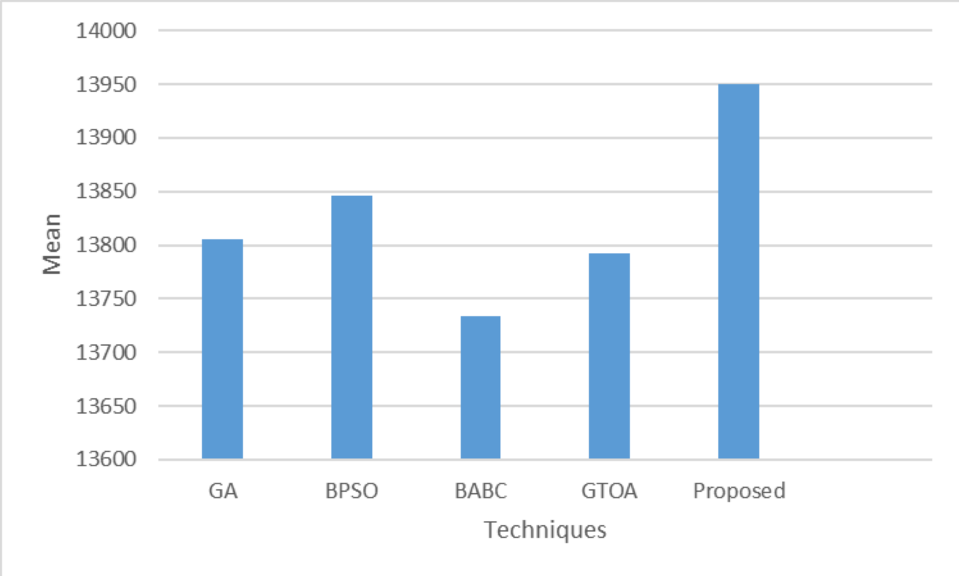

Figure D : Mean comparison of proposed DGAKO-LSCS approach

The comparison of mean of proposed DGAKO-LSCS approach with other existing approaches. The mean of proposed approach is compared with existing techniques such as GA, BPSO, BABC and GTOA. The mean of proposed DGAKO-LSCS approach has the optimum value of 13950 whereas the mean of GA, BPSO, BABC and GTOA are 13806, 13846, 13734, and 13792 respectively. The mean of proposed DGAKO-LSCS approach is high whereas the mean of BABC is low.

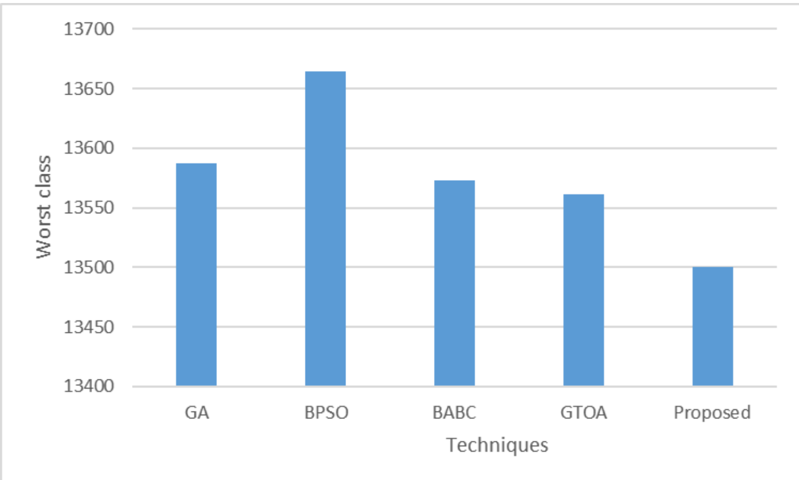

Figure E : Worst class comparison of proposed DGAKO-LSCS approach

The comparison of worst class of proposed DGAKO-LSCS approach with other existing approaches and compared with existing techniques such as GA, BPSO, BABC and GTOA. The worst class of proposed DGAKO-LSCS approach has the optimum value of 13500 whereas the worst class of GA, BPSO, BABC and GTOA are 13587,13664,13573,13561 and 13690 respectively. The worst class of proposed DGAKO-LSCS approach is low due to its effective decision making tendency whereas the worst class of BPSO is high.

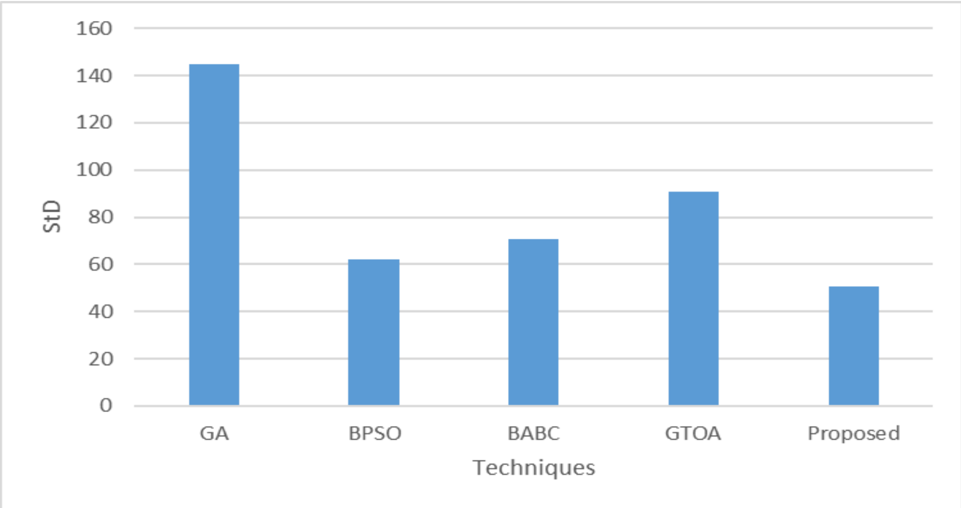

Figure F: Standard deviation comparison of proposed DGAKO-LSCS approach

The comparison of Standard deviation of proposed DGAKO-LSCS approach with other existing approaches. Standard deviation of proposed approach is compared with existing techniques such as GA, BPSO, BABC and GTOA. Standard deviation of proposed DGAKO-LSCS approach has the minimum value of 50.6 whereas the Standard deviation of GA, BPSO, BABC and GTOA are 144.91, 62.21, 70.76 and 90.57 respectively. Standard deviation of proposed DGAKO-LSCS approach is low whereas the Standard deviation of GA is high.

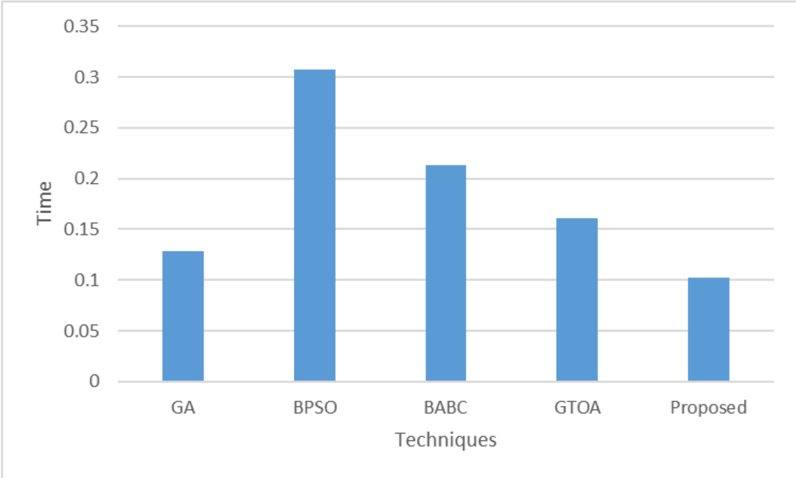

Figure G : Time complexity comparison of proposed DGAKO-LSCS approach

The comparison of time complexity of proposed DGAKO-LSCS approach with other existing approaches. The time complexity of proposed approach is compared with existing techniques such as GA, BPSO, BABC and GTOA. The time complexity of proposed DGAKO-LSCS approach has the minimum value of 0.102 seconds whereas the time complexity of GA, BPSO, BABC and GTOA are 0.129, 0.307, 0.213 and 0.161 seconds respectively. The time complexity of proposed DGAKO-LSCS approach is low whereas the time complexity of BPSO is high.

Table 1 : Comparison of proposed DGAKO-LSCS approach with existing techniques

Techniques	Best	Mean	Worst	StD	Time
GA	14044	13806	13587	144.91	0.129
BPSO	14044	13846	13664	62.21	0.307
BABC	13806	13734	13573	70.76	0.213
GTOA	14044	13792	13561	90.57	0.161
Proposed	14051	13950	13500	50.6	0.102

Table 1 depicts the comparison of proposed DGAKO-LSCS approach with existing techniques such as GA, BPSO, BABC and GTOA in terms of best class, worst class, mean, standard deviation and time. From table 1, it is noticed that proposed DGAKO-LSCS approach has optimum best class, worst class and mean values as well as have low standard deviation and time complexity. Overall Acquisitive Knowledge Optimization with Layered

Color Set for DO based Decision problems in Group theory outperforms existing techniques such as GA, BPSO, BABC and GTOA with optimum best class, worst class and mean values of 14051, 13500 and 13950 by Discrete gain acquisitive knowledge optimization and have low standard deviation and time complexity of 50.6 and 0.103 seconds by Layer splitting color set approach.

8. Conclusion

Acquisitive Knowledge Optimization with Layered Color Set for DO based Decision problems in Group theory has been presented in this research to solve the DO based decision making problems such as subset sum problem, knapsack problems and Submonoid memebership problem by performing two phases of operation. In the first phase, the knapsack problem and its optimization variants are solved by Discrete gain acquisitive knowledge optimization that arrange the elements in the group in ascending order based on objective function and reduces the number of elements in the group in order to attain high quality solution with optimal best case elements of 14051 and worst case elements of 13500. Then, in the second phase, subset sum and submonoid membership problems are solved by Layer splitting color set approach in which the elements are divided into $logm$ layers that reduce the time complexity to 0.102 seconds with low standard deviation of 50.6 and the effectiveness of this approach is proved by determining the generic case complexity. Thus the result obtained showed that the proposed approach solves the DO based optimization problem with optimal mean value of 13950.

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