Investigation on rg-Closed Type Sets in TOS

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Abstract

This article introduces a new type of sets known as rg-closed sets in TOSs. rg-open sets in TOS generalised closed sets to form this new class. This new class strictly falls between the classes of closed sets and rg-closed collections in TOS.

Keywords: TOS, rg-closed set (irg, drg, brg-closed sets), r*g*-closed set (ir*g*, dr*g*, br*g*-closed sets).

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1. Introduction

Leopoldo Nachbin conducted the first research on TOSs [1]. In 1970, the superclass of sets known as r* g*-closed sets was invented by Levine [19]. A brand-new category of sets [14] introduced by MKRS Veera Kumar. Which should come before the r*g*-closed sets and closed set classes, respectively. In [15], presented the research on i-closed, d-closed, and b-closed sets for the first time in 2001.

A Topological ordered space (TOS) is known as a triple $(X, \tau, \leq)$, in which X is a non-empty set, τ is a topology on X, and $\leq$ is a partial order on X[5].

Definition 1.1: For any $\alpha \epsilon X, \{\beta \epsilon X / \alpha \leq \beta\}$ will be represented by $[\alpha, \rightarrow]$. If P = i(P), where i(P) = $\bigcup_{\alpha \epsilon P}[\alpha, \rightarrow]$, then “a subset P of a” TOS $(X, \tau, \leq)$ is referred to as an increasing set [5, 6, 7].

Definition 1.2: For any $\alpha \epsilon X, \{\beta \epsilon X / \beta \leq \alpha\}$ will be represented by $[\leftarrow, \alpha]$. If P = d(P), where d(P) = $\bigcup_{\alpha \epsilon P}[\leftarrow, \alpha]$, then “a subset P of a” TOS $(X, \tau, \leq)$ is known to be decreasing[5, 6, 7].

An increasing (resp. a decreasing) set is the complement of a decreasing (resp. an increasing) set. C(P) denotes the complement of ‘P’ in X.

icl (P) = $\cap\{F/F$ is an increasing closed subset of X containing P with F = i(F)$\}$.

dcl (P) = $\cap\{F/F$ is a decreasing closed subset of X containing P with F = d(F)$\}$.

bcl (P) = $\cap\{F/F$ is a closed subset of X containing P with F=i(F)=d(F)$\}$.

IO(X) (resp. DO(X), BO(X)) represents the set of all increasing (or decreasing, both increasing and decreasing) open subsets of a TOS$(X, \tau, \leq)$.

For a subset P of a space$(X, \tau, \leq)$, cl(P) (resp. dcl(P), bcl(P)) denote the increasing (resp. decreasing, both increasing and decreasing) closure of P[5, 6, 7].

2. TOS with rg-closed sets

Definition 2.1: A topological space (X, τ) has a subset P is referred to as rg-closed [29] set if cl(P) ⊆ R whenever P ⊆ R and R is regular open in (X, τ).

Definition 2.2: A topological space (X, τ)has a subset P is known as "r*g*-closed set" [29], if rcl(P) ⊆ R whenever P ⊆ R and R is g-open in (X, τ).

Theorem 2.3: Every "r*g*-closed set" is a "rg-closed set".

Proof: suppose P ⊆ R and R is "regular open". Now, R is regular open implies R is open.
W. k. t every closed set is a generalized closed set(g-closed set).
So, every open set is a "g-open set".
Since, R is open we have R is "g-open".
Therefore, rcl(P) ⊆ R, whenever P ⊆ R and R is g-open.
Since P ⊆ R, there exists an open set G we have P ⊆ G ⊆ cl(G) ⊆ R.
Since P ⊆ cl(G) ⇒ cl(P) ⊆ cl(cl(G)) = cl(G) ⊆ R. ⇒cl(P) ⊆ R. Therefore cl (P) ⊆ R, whenever P ⊆ R and R is regular open.

The following illustration demonstrates that an rg-closed set does not always have to be an r*g*-closed set.

Example 2.4: Let X = $\{\alpha, \beta, \gamma\}$, τ = $\{\emptyset, X, \{\alpha\}, \{\beta\}, \{\alpha, \beta\}\}$ and ≤ = $\{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\beta, \gamma), (\alpha, \gamma)\}$. $(X, \tau, \leq)$ is a TOS. r*g*-closed sets are $\emptyset$, X, $\{\gamma\}$, $\{\beta, \gamma\}$, $\{\alpha, \gamma\}$. rg-closed sets are $\emptyset$, X, $\{\gamma\}$, $\{\alpha, \beta\}$, $\{\beta, \gamma\}$, $\{\alpha, \gamma\}$. Let P = $\{\alpha, \beta\}$. P is an rg-closed set but not a r*g*-closed set.

3. Results between i(r*g*), d(r*g*) and b(r*g*) closed type sets

The following definitions are provided.

Definition 3.1: If ircl(P) ⊆ R whenever P ⊆ R and R is g-open in (X, τ), then a subset P of $(X, \tau, \leq)$ is known as "i(r*g*)-closed set".

Definition 3.2: if drcl (P) ⊆ R whenever P ⊆ R and R is g-open in (X, τ), then a subset P of $(X, \tau, \leq)$ is called d(r*g*)-closed set.

Definition 3.3: if brcl (P) ⊆ R whenever P ⊆ R and R is g-open in (X, τ), then a subset P of $(X, \tau, \leq)$ is called b(r*g*)-closed set.

Theorem 3.4: Every i(r*g*)-closed set is an i(rg)-closed set.

Proof: Every r*g*-closed set is also a rg-closed set, as far as we can tell. Every closed set in i(r*g*) is likewise closed in i(rg), therefore this holds. The following illustration demonstrates that a set need not be i(r*g*)-closed in order to be i(rg)-closed.

Example 3.5: Let X = $\{\alpha, \beta, \gamma\}$, τ = $\{\emptyset, X, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \gamma\}\}$ and ≤ = $\{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\beta, \gamma)\}$. Clearly $(X, \tau, \leq)$ is a TOS. i(r*g*)-closed sets are $\emptyset$, X, $\{\beta, \gamma\}$. i(rg)-closed sets are $\emptyset$, X, $\{\gamma\}$, $\{\beta, \gamma\}$. Let P = $\{\gamma\}$. P is i(rg)-closed set but not i(r*g*)-closed set.

Theorem 3.6: Any set that is d(r*g*)-closed is d(rg)-closed.

Proof: We know, every r*g*-closed set is an rg-closed set. Thus, every d(r*g*)-closed set is a d(rg)-closed set. The following illustration demonstrates that a d(rg)-closed set need not always be a d(r*g*)-closed set.

Example 3.7: Let X = $\{\alpha, \beta, \gamma\}$, τ = $\{\emptyset, X, \{\alpha, \beta\}\}$ and ≤ = $\{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\alpha, \gamma)\}$. $(X, \tau, \leq)$ is a TOS. $\emptyset$, X, $\{\alpha, \gamma\}$ are d(r*g*)-closed sets. $\emptyset$, X, $\{\alpha\}$, $\{\alpha, \beta\}$, $\{\alpha, \gamma\}$ are d(rg)-closed sets. Let P = $\{\alpha\}$. P is a d(rg)-closed set but not a d(r*g*)-closed set.

Theorem 3.8: Every b(r*g*)-closed set is a b(rg)-closed set.

Proof: To the best of our knowledge, every set that is r*g*-closed is also rg-closed. A b(rg)-closed set is equivalent to a b(r*g*)-closed set. A b(rg)-closed set need not necessarily be a b(r*g*)-closed set, as the following example shows.

Example 3.9: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\gamma, \beta)\}$. Clearly, $(X, \tau, \leq)$ is a TOS. $\emptyset$, X are b(r*g*)-closed sets. $\emptyset$, X, $\{\alpha, \beta\}$ are b(rg)-closed sets. Let P = $\{\alpha, \beta\}$. P is a b(rg)-closed set but not a b(r*g*)-closed set.

Theorem 3.10: Every b(r*g*)-closed set is i(r*g*)-closed set.

Proof: As far as we know, a balanced set is always an increasing set. Then, all b(r*g*)-closed sets are i(r*g*)-closed sets. The following illustration demonstrates that i(r*g*)-closed set need not always be a b(r*g*)-closed set.

Example 3.11: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\beta, \alpha), (\gamma, \beta), (\gamma, \alpha)\}$. $(X, \tau, \leq)$ is a TOS. b(r*g*)-closed sets are $\emptyset$, X. i(r*g*)-closed sets are $\emptyset$, X, $\{\alpha, \beta\}$. Let P = $\{\alpha, \beta\}$. P is i(r*g*)-closed set but not be a b(r*g*)-closed set.

Theorem 3.12: Every "b(r*g*)-closed set" is a "d(r*g*)-closed set".

Proof: As we know, "Every balanced set is a decreasing set". Any set that is "b(r*g*)-closed" is also a "d(r*g*)-closed". The following illustration demonstrates that a "d(r*g*)-closed" set does not always have to be a "b(r*g*)-closed set".

Example 3.13: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\beta, \alpha), (\gamma, \beta), (\gamma, \alpha)\}$. Clearly, $(X, \tau, \leq)$ is a TOS. b(r*g*)-closed sets are $\emptyset$, X. d(r*g*)-closed sets are $\emptyset$, X, $\{\beta, \gamma\}$. Let P = $\{\beta, \gamma\}$. P is a d(r*g*)-closed set but not a b(r*g*)-closed set.

Theorem 3.14: i(r*g*)-closed and d(r*g*)-closed are independent notions. The following example will demonstrate this.

Example 3.15: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\alpha, \gamma)\}$. Clearly, $(X, \tau, \leq)$ is a TOS. i(r*g*)-closed sets are $\emptyset$, X. d(r*g*)-closed sets are $\emptyset$, X, $\{\beta, \gamma\}$. Let P = $\{\beta, \gamma\}$. P is a d(r*g*)-closed set but not an i(r*g*)-closed set.

Example 3.16: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\beta, \alpha), (\gamma, \beta), (\gamma, \alpha)\}$. $(X, \tau, \leq)$ is a TOS. i(r*g*)-closed sets are $\emptyset$, X, $\{\beta, \gamma\}$. d(r*g*)-closed sets are $\emptyset$, X. Le t P = $\{\beta, \gamma\}$. P is i(r*g*)-closed set but not be a d(r*g*)-closed set.

Theorem 3.17: Every b(rg)-closed set is an i(rg)-closed set.

Proof: As we know, a balanced set is always an increasing set. Hence, all b(rg)-closed sets are i(rg)-closed sets. Generally, the next example demonstrates that i(rg)-closed sets do not have to be b(rg)-closed sets.

Example 3.18: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\gamma, \beta)\}$. Clearly, $(X, \tau, \leq)$ is a TOS. b(rg)-closed sets are $\emptyset$, X, $\{\alpha, \beta\}$. i(rg)-closed sets are $\emptyset$, X, $\{\beta\}$, $\{\alpha, \beta\}$, $\{\beta, \gamma\}$. Let P = $\{\alpha\}$. P is i(rg)-closed set but not a b(rg)-closed set.

Theorem 3.19: Every b(rg)-closed set is a d(rg)-closed set.

Proof: Every balanced set is a decreasing set, as we are aware. Every b(rg)-closed set is a d(rg)-closed set, hence this is true. The following illustration demonstrates that a d(rg)-closed set does not always have to be a b(rg)-closed set.

Example 3.20: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\alpha, \gamma\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\beta, \gamma), (\alpha, \gamma)\}$. Clearly $(X, \tau, \leq)$ is a TOS. "b(rg)-closed sets" are $\emptyset$, X. "d(rg)-closed sets" are $\emptyset$, X, $\{\beta\}$, $\{\alpha, \beta\}$. Let P = $\{\beta\}$. P is a "d(rg)-closed set" but not a "b(rg)-closed set".

Theorem 3.21: i(rg)-closed and d(rg)-closed are distinct concepts. In this case, demonstrated by the example that follows.

Example 3.22: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\beta\}, \{\alpha, \beta\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\alpha, \gamma)\}$. $(X, \tau, \leq)$ is a TOS. "i(rg)-closed sets" are $\emptyset$, X, $\{\gamma\}$, $\{\beta, \gamma\}$. "d(rg)-closed sets" are $\emptyset$, X, $\{\alpha, \beta\}$, $\{\alpha, \gamma\}$. Let P = $\{\alpha, \gamma\}$. P is a "d(rg)-closed set" but not "i(rg)-closed set". Let Q = $\{\gamma\}$. Q is "i(rg)-closed set" but not a "d(rg)-closed set".

4. TOS with irg-closed type sets

Theorem 4.1: A set P ∪ Q is "irg-closed" if P and Q are "irg-closed sets".

Proof: If P ∪ Q ⊆ R and R is "regular-open", then P ⊆ R and Q ⊆ R.

But P and Q are "irg-closed" and therefore icl(P) ⊆ R and icl(Q) ⊆ R.

Therefore, (icl (P) ∪ icl (Q)) ⊆ R ⟹ icl (P ∪ Q) ⊆ R.

Hence, P ∪ Q is irg-closed.

Example 4.2: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\beta\}, \{\alpha, \beta\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta)\}$. $(X, \tau, \leq)$ is a TOS. Take, P = $\{\gamma\}$, Q = $\{\alpha, \beta\}$. If (P ∪ Q) = $\{\gamma\} \cup \{\alpha, \beta\} = \{\alpha, \beta, \gamma\}$ ⊆ R = X and R is regular-open, then $\{\gamma\}$⊆ R and $\{\alpha, \beta\}$ ⊆ R. But P and Q are irg-closed and therefore icl (P) ⊆ R and icl (Q) ⊆ R. Therefore, (icl (P) ∪ icl (Q)) ⊆ R, and hence icl(P ∪ Q) ⊆ R. Hence P ∪ Q is irg-closed.

Theorem 4.3: Suppose that Q ⊆ P ⊆ X, Q is an "ig-closed open" subset of X and Q is an "irg-closed set" to P. Then, Q is "irg-closed" to X.

Proof: Let Q ⊆ R and let R be regular-open. We have Q ⊆ (P ∩ R).

But Q is an irg-closed set relative to P.

Hence icl_P(Q) ⊆ (P ∩ R). →(1)

Note that P ∩ R is regular-open in P.

But icl_P(Q) = icl(Q) ∩ P →(2)

From (1) and (2),

(P ∩ icl(Q)) ⊆ (P ∩ R)

Consequently P ∩ icl (Q) ⊆ R. Hence, P ∩ (icl(Q) ∪ C(icl(Q))) ⊆ R ∪ C(icl(Q)).

That is P ∩ X ⊆ (R ∪ C(icl(Q))).

So P ⊆ (R ∪ C(icl(Q))) = G, say →(3)

But then G is an open set. Since P is "ig-closed" in X, from (3) we have

icl(P) ⊆ (R ∪ C(icl(Q))) = G →(4)

But icl(Q) ⊆ icl(P) →(5)

From (4) and (5) we have icl(Q) ⊆ (R ∪ C(icl(Q))).

Hence icl(Q) ⊆ R because icl(Q) ∩ C(icl(Q)) = ∅ ⟹ Q is irg-closed relative to X.

Corollary 4.4: Let P be an "ig-closed", open set. Suppose that Q is an "i-closed set". Then P ∩ Q is an "irg-closed set" relative to X.

Proof: We have that P ∩ Q is closed in P. Hence icl(P ∩ Q) = P ∩ Q in P. Let P ∩ Q ⊆ R, Where R is regular-open in P. That is icl(P ∩ Q) ⊆ R. Hence P ∩ Q is an irg-closed set in the ig-closed P. By the theorem [4.3], P ∩ Q is an irg-closed set relative to X.

Theorem 4.5: If a set P is "irg-closed" then icl(P)\P contains no nonempty "regular-closed set".

Proof: Suppose that P is "irg-closed". Let S be a "regular-closed" subset of icl(P)\P.
Then S ⊆ (icl(P) ∩ C(P)) and so P ⊆ C(S). But P is irg-closed.
Therefore icl(P) ⊆ C(S). → (1)
Consequently S ⊆ C(icl(P)) → (2)
We have already S ⊆ icl(P) → (3)
From (2) and (3), S ⊆ (icl(P) ∩ C(icl(P))) = ∅
Thus S = ∅. Therefore icl(P)\P contains no nonempty "regular-closed set".

Corollary 4.6: Let P be an "irg-closed set". If P is "regular-closed" then cl(int(P))\P is "regular-closed".

Proof: Let P be an "irg-closed". If P is "regular-closed" i.e., cl(int(P)) = P. Then cl(int(P))\P = P\P = ∅. But, ∅ is always regular-closed. As a result, cl(int(P))\P is "regular-closed".
On the other hand, imagine that cl(int(P))\P is "regular-closed". However, P is irg-closed. Additionally, the regular-closed set cl(int(P))\P is contained in icl(P)\P. By the above theorem [4.5], cl(int(P))\P = ∅. Hence cl(int(P)) = P. Therefore P is "regular-closed".

Theorem 4.7: If P is "ig-closed" then P is "irg-closed".

Proof: Suppose that P ⊆ R, Where R is regular-open. Now R regular-open ⟹ R is open. Thus P ⊆ R and R is open. But P is ig-cosed. Hence icl(P) ⊆ R. Therefore, P is irg-closed.

The following illustration demonstrates that an "irg-closed" set need not always be an "ig-closed set".

Example 4.8: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\beta\}, \{\alpha, \beta\}\}$ and $\leq = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma), (\alpha, \beta), (\gamma, \beta)\}$. Clearly $(X, \tau, \leq)$ is a TOS. "irg-closed" sets are $\emptyset$, X, $\{\alpha, \beta\}$, $\{\beta, \gamma\}$. "ig-closed" sets are $\emptyset$, X, $\{\beta, \gamma\}$. Let P = $\{\alpha, \beta\}$. Clearly, P is an irg-closed set but not an ig-closed set.

Theorem 4.9: If P is "irg-closed" and P ⊆ Q ⊆ icl(P) then icl(Q)\Q contains no nonempty "regular-closed sets".

Proof: Suppose P is "irg-closed" and P ⊆ Q ⊆ icl(P).
Since P ⊆ Q ⟹ C(Q) ⊆ C(P) → (1)
Since Q ⊆ icl(P) ⟹ icl(Q) ⊆ icl(icl(P)) ⊆ icl(P) → (2)
That is icl(Q) ⊆ icl(P).
From (1) and (2), (icl(Q) ∩ C(Q)) ⊆ (icl(P) ∩ C(P)).
Which implies (icl(Q)\Q) ⊆ (icl(P)\P). Now P is irg-closed. Hence, icl(P)\P has no nonempty regular-closed subsets neither does icl(Q)\Q.

Theorem 4.10: Assume that P is "irg-closed" in X and P ⊆ Y ⊆ X. If Y is open in X, Then P is "irg-closed" relative to Y.

Proof: Assume that R is regular-open in X and that P ⊆ Y ∩ R. Therefore, P ⊆ R and hence icl(P) ⊆ R. This implies that (Y ∩ icl(P)) ⊆ Y ∩ R. Therefore P is irg-closed to Y.

Theorem 4.11: Let X be a regular space. Prove that every compact subset of X is an irg-closed set.

Proof: Suppose that P ⊆ R where R is "regular-open". Now that R is "regular-open" implies R is "open". But P is compact in the regular space X. Hence, ∃ an open set O ∋ P ⊆ O ⊆

cl(O) ⊆ R. Since P ⊆ cl(O) ⟹ icl(P) ⊆ icl(cl(O)) = cl(cl(O)) = cl(O) ⊆ R. That is icl(P) ⊆ R. Hence P is irg-closed in X.

5. ir*g*-closed type sets in TOS:

Theorem 5.1: A set P ∪ Q is "ir*g*-closed" if P and Q are "ir*g*-closed sets".

Proof: If P ∪ Q ⊆ R and R is "g-open", then P ⊆ R and Q ⊆ R. But P and Q are "ir*g*-closed" and therefore ircl (P) ⊆ R and ircl (Q) ⊆ R. Therefore, (ircl(P) ∪ ircl(Q)) ⊆ R, and hence ircl(P ∪ Q) ⊆ R. Hence P ∪ Q is "ir*g*-closed".

Example 5.2: Let $X = \{\alpha, \beta, \gamma\}$, $\tau = \{\emptyset, X, \{\alpha\}, \{\beta\}, \{\alpha, \beta\}\}$ and ≤ = {(α, α), (β, β), (γ, γ), (α, β), (β, γ)}. Clearly $(X, \tau, \leq)$ is a TOS. Take P = $\{\gamma\}$, Q = $\{\beta, \gamma\}$. If (P ∪ Q) = $\{\gamma\} \cup \{\beta, \gamma\} = \{\beta, \gamma\}$ ⊆ R = X and R is "g-open", then $\{\gamma\}$ ⊆ R and $\{\alpha, \beta\}$ ⊆ R. But P and Q are "ir*g*-closed" and therefore ircl (P) ⊆ R and ircl (Q) ⊆ R. Therefore, (ircl(P) ∪ ircl(Q)) ⊆ R, and hence ircl(P ∪ Q) ⊆ R. Hence P ∪ Q is "ir*g*-closed".

Theorem 5.3: If a set P is "ir*g*-closed" then ircl(P)\P contains no nonempty "regular-closed set".

Proof: Suppose that P is "ir*g*-closed". Let S be a "regular-closed" subset of ircl(P)\P.

Then S ⊆ (ircl(P) ∩ C(P)) and so P ⊆ C(S). But P is ir*g*-closed.

Therefore ircl(P) ⊆ C(S). →(1)

Consequently S ⊆ C(ircl(P)) →(2)

We have already S ⊆ ircl(P) →(3)

From (2) and (3) S ⊆ (ircl(P) ∩ C(ircl(P))) = ∅.

Thus S = ∅. Therefore, ircl (P)\P contains no nonempty regular-closed set.

Corollary 5.4: If P is an "ir*g*-closed set", then P is "regular-closed" ⟺ cl(int (P))\P is "regular closed".

Proof: Make P an irg-closed. If cl(int(P)) = P, then P is "regular-closed". If so, cl(int(P))\P = P\P = ∅. However, ∅ is always "regular closed". cl(int(P))\P is hence "regular-closed".

Assume, on the other hand, that cl(int(P))\P is regular-closed. P is, however, irg-closed. The regular-closed set cl(int(P))\P is also contained in ircl(P)\P. The statement "cl(int(P))\P = ∅." is based on the aforementioned theorem. As a result, cl(int(P)) = P. As a result, P is "regular closed".

Theorem 5.5: If P is ir*g*-closed and P ⊆ Q ⊆ ircl(P), then ircl(Q)\Q does not include any nonempty "regular-closed sets".

Proof: If P is "ir*g*-closed" and P ⊆ Q ⊆ ircl(P).

Since C(P)⊆C(Q) follows from P⊆Q → (1)

Q ⊆ ircl(P) implies that ircl(Q) ⊆ ircl(ircl(P)) = ircl(P)

In this case ircl(Q) ⊆ ircl(P) → (2)

From (1) & (2) (ircl(Q) ∩ C(Q)) ⊆ (ircl(P) ∩ C(P))

Implies (ircl(Q)\Q) ⊆ (ircl(P)\P).

P is now ir*g*-closed. As a result, neither ircl(P)\P nor ircl(Q)\Q have any nonempty regular-closed subsets.

Theorem 5.6: Let P ⊆ Y ⊆ X and suppose that P is "ir*g*-closed" in X. Then P is "ir*g*-closed" relative to Y, provided Y is "open" in X.

Proof: Let P ⊆ Y ∩ R and suppose that R is "g-open" in X. Then P ⊆ R and hence ircl(P) ⊆ R. This implies that (Y ∩ ircl(P)) ⊆ Y ∩ R. Thus P is ir*g*-closed relative to Y.

Conclusion

We presented "(r*g*)*closed sets" and "(r*g*)*open sets" in this study and looked at some of their features. This class of sets can be extended to other topological spaces, such as fuzzy and bi-topological spaces, and used to examine the concepts of "continuity, compactness, and connectedness".

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