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A STUDY OF ORIENTATED CANTILEVER BEAM MOUNTED WITH TIP MASS AND BOUNDED WITH PZT-5A AND PZT-5H FOR VIBRATION ENERGY HARVESTING.

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Abstract

Energy harvesting is extracting energy from ambient sources. Vibration is one the energy source available freely in surrounding environment. Research on vibration energy harvesting have received the attention of many researchers to power wireless sensors and low-power electronic devices from smart materials. In this paper, the effect of PZT-5A and PZT- 5H piezoelectric material on the cantilever beam mounted with tip mass and oriented at 10^0, at the resistance of 1500 ohms is studied. MatLab program is developed considering the Euler Bernoulli beam assumptions, constitutive equations of piezoelectric material and Hamilton's principle. The MatLab program is validated with the previous work on orthonormalisation electromechanical finite element unimorph beam and good agreement is obtained.

Keywords: Orientation, Piezoelectric Material, Vibration, Energy harvesting, FEM.

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Introduction

In the recent years, many researchers are attracted towards the area of energy harvesting from various sources available in the surrounding environment [1].Energy from vibration is one of source which is available freely from machines, human motion having low frequency vibration excitation which can be used to meet the demand without depending on conventional sources. Research in the area of energy harvesting is increasing with the development of smart electronic devices. Limitation in batteries capacity to power the devices and life span for supplying continuous energy for wireless technologies necessitate for energy harvesting. Smart structures based on piezoelectricity are currently generating a demand and piezoelectric material forms the alternate source for the creation of lifelong energy harvesters with compact configuration which will be utilized in engineering and medical applications. Research of piezoelectric elements in energy conversion applications necessitates a thorough understanding of solid mechanics and geometrical shapes [2]. Researchers have focused on analytical and numerical studies of unimorph and bimorph beam which forms the basis for the study of energy harvesting [3-5] due to its compatibility and ease of applications. Authors have studied experimentally, the effect of orientation of beam by attaching harvester to human leg and concluded that optimum results are obtained for 70^0 orientation of harvester with reference to coordinate system [6]. Researchers have focused on orientation of harvester for rotational motion by changing the distance between the rotating center and fixed end of beam [7].A study on configurations of energy harvester for IoT sensor applications in smart cities [8] and optimal location of piezoelectric patch on the length of slat in aircraft applications is carried [9,10]. A less information is available in the literature on the study of orientation of the beam with tip mass on free end and base excitation at fixed end bounded with different piezoelectric materials [11, 12]. The aim of this work is to develop the MatLab program for the study of effect of PZT-5A and PZT 5H material on orientation of the beam with tip mass, to study voltage, current and power frequency response function (FRF) and describe its dynamic behavior .The MatLab program is validated with the previous work on orthonormalisation electromechanical finite element of unimorph beam and good agreement is obtained.

2. Methodology

The Finite element method plays the major role in the complex problem analysis .In vibration based energy harvesting, piezoelectric unimorph beam forms the basic structure. The formulation of beam with piezoelectric (PZT) layer is considered as shown in Fig.1

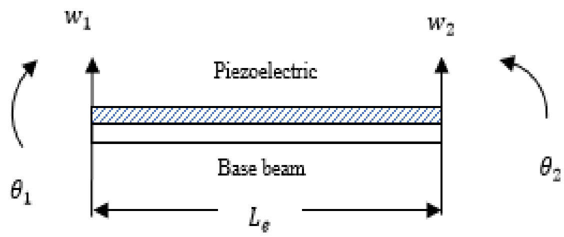

Figure 1 .Element with base beam bounded with PZT

The beam with piezoelectric has two degree of freedom at each node, transverse (w) and rotational (θ)

The displacement function of the beam element is given as

$$w(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \quad (1)$$

Eqn. 1 is the displacement equation with four unknown coefficients. Applying boundary conditions at node 1 and 2 of element, the four coefficients in polynomial equation are solved.

The shape functions are given in equation (2-5)

$$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$$

$where$

$$[N_1] = \left(1 - 3\left(\frac{x}{L_e}\right)^2 + 2\left(\frac{x}{L_e}\right)^3\right) \tag{2}$$

$$[N_2] = \left(L_e\left(x - 2\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3\right)\right) \tag{3}$$

$$[N_3] = \left(3\left(\frac{x}{L_e}\right)^2 - 2\left(\frac{x}{L_e}\right)^3\right) \tag{4}$$

$$[N_4] = \left(L_e\left(-\left(\frac{x}{L_e}\right)^2 + \left(\frac{x}{L_e}\right)^3\right)\right) \tag{5}$$

Strain S(x) is represented by

$$S(x) = -z\frac{d^2\mathrm{w(x)}}{dx^2} = -zB \tag{6}$$

$Where\ B = [B_1\ , B_2\ , B_3, B_4]$

Differentiating Eq. (2-5)

$$\frac{d^2N_1}{dx^2} = \frac{-6}{{L_e}^2} + \frac{12x}{{L_e}^3} = B_1 \tag{7}$$

$$\frac{d^2N_2}{dx^2} = \frac{-4}{L_e} + \frac{6x}{{L_e}^2} = B_2 \tag{8}$$

$$\frac{d^2N_3}{dx^2} = \frac{6}{{L_e}^2} - \frac{12x}{{L_e}^3} = B_3 \tag{9}$$

$$\frac{d^2 4}{dx^2} = \frac{-2}{L_e} + \frac{6x}{{L_e}^2} = B_4 \tag{10}$$

The electromechanical coupling of a piezoelectric can be demonstrated in two ways. In direct piezoelectric effect, an applied mechanical pressure produces a proportional voltage response and in converse effect, an applied voltage produces a deformation of the material.

The direct effect Eq.11 and the converse effect Eq.12 may be modelled as

$$\{D\} = [e]^T\{S\} + [\varepsilon^S]\{E\} \tag{11}$$

$$\{T\} = [c^E]\{S\} - [e]\{E\} \tag{12}$$

The constitutive equation for 1-dimensional form with constant electric field and strain

$$\begin{Bmatrix} T_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{11}^E & -e_{31} \\ e_{31} & \varepsilon_{33}^S \end{bmatrix} \begin{Bmatrix} S_1 \\ E_3 \end{Bmatrix} \tag{13}$$

The base beam and piezoelectric plane stress field equation is given by Eq.14 and 15

$$T_1{}^{(1)} = c_{11}^{(1)} S_1^{(1)} \tag{14}$$

$$T_1{}^{(2)} = c_{11}^{(2)} S_1^{(2)} - e_{31}E_3 \tag{15}$$

$\vartheta(z) = \frac{z - z_n + h_p}{h_p}$ is the shape function over the interval $z_n - h_p \le z \le z_n$

$z_n = \frac{c_{11}^{(1)} {h_s}^2 + c_{11}^{(2)} {h_p}^2 + 2c_{11}^{(1)} h_s h_p}{2(c_{11}^{(1)} h_s + c_{11}^{(2)} h_p}$ = distance from the neutral axis to the top layer of the beam with piezoelectric

$\varphi(z) = \vartheta(z) v_p$ is electrical potential

$\Omega(z) =$ first derivative of shape function $= \frac{d\vartheta(z)}{dz} = 1/h_p$,

The electric field equation

$$E_3 = -\,\Omega(z) v_p \quad (16)$$

$where\ v_p = voltage$

Substituting Equation (6) into Equation (14)

$$T_1{}^{(1)} = -z c_{11}^{(1)} \mathrm{B} \quad (17)$$

Substituting Equation (6) and (16) into Equation (15)

$$T_1{}^{(2)} = -z c_{11}^{(2)} \mathrm{B} + e_{31} \Omega(\mathrm{z}) v_p(t) \quad (18)$$

$$D_3 = -z e_{31} \mathrm{B} - \varepsilon^S{}_{33}\, \Omega(z) v_p \quad (19)$$

The electromechanical piezoelectric cantilever energy harvesting is formulated by the Hamiltonian principle,

$$\int_{t_1}^{t_2} [\delta(\text{ƙ} - \text{ƥ} + \omega) + \delta\varpi]\, dt = 0 \quad (20)$$

Representation of equations in matrix form utilizing extended Hamilton's principle [3]

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{w} \\ \ddot{v_p} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ P_{sr} & P_D \end{bmatrix} \begin{Bmatrix} \dot{w} \\ \dot{v_p} \end{Bmatrix} + \begin{bmatrix} K & P_{rs} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} w \\ v_p \end{Bmatrix} = \begin{Bmatrix} F \\ i_p \end{Bmatrix} \quad (21)$$

Element Stiffness Eq.22, mass matrix Eq.23 is given by

$$[K] = \int z^2 c_{11}^{(1)} [B^T]\, [B] dv^{\,(1)} + \int z^2 c_{11}^{(2)} [B^T]\, [B] dv^{\,(2)} \quad (22)$$

$$[M] = \int \rho^{(1)} [N^T]\, [N] dv^{(1)} + \int \rho^{(2)} [N^T]\, [N] dv^{(2)} \quad (23)$$

Adding tip mass terms to Eq. (23)

$$2 I_0^{tip} x_c [N^T] \frac{d[N]}{dx} + I_0^{tip} [N^T][N] + I_2^{tip} \frac{d[N^T]}{dx} \frac{d[N]}{dx} \quad (24)$$

Offset distance from proof mass centroid [5] is

$$\mathrm{x_c} = \frac{\rho^{tip} b {l_{tip}}^2 h_{tip} + \rho^{(1)} b {l_{tip}}^2 h_s}{2(\rho^{tip} b l_{tip} h_{tip} + \rho^{(1)} b l_{tip} h_s)} \quad (25)$$

Zeroth and second mass moment of inertia [5] is given by

$$\mathrm{I}_0^{tip} = \rho^{tip} b l_{tip} h_{tip} + \rho^{(1)} b l_{tip} h_s \quad (26)$$

$$I_2^{tip} = \left(\rho^{tip} b l_{tip} h_{tip} \left(\frac{{l_{tip}}^2 + {h_{tip}}^2}{12} \right) + \rho^{tip} b l_{tip} h_{tip} \left(z_n - h_p + \frac{h_{tip}}{2} \right)^2 + \left(\frac{l_{tip}}{2} \right)^2 \right) + \left(\rho^{(1)} b l_{tip} h_s \left(\frac{{l_{tip}}^2 + {h_s}^2}{12} \right) + \rho^{(1)} b l_{tip} h_s \left(-z_n - h_p + \frac{h_s}{2} \right)^2 \left(\frac{l_{tip}}{2} \right)^2 \right) \quad (27)$$

Damping matrix C is given as

$$C = \alpha M + \beta K \quad (28)$$

Mechanical Force F is given by

$$F = -Q\ddot{w}_{base} \quad (29)$$

Where

$$Q = \int \rho^{(1)} N^T dV^{(1)} + \int \rho^{(2)} N^T dV^{(2)} \quad (30)$$

Adding tip mass terms to Eq. (29)

$$-I_0^{tip}\, x_c \frac{d[N^T]}{dx} - I_0^{tip}[N^T] \quad (31)$$

Electromechanical coupling is given by

$$P_{sr} = -\int z\Omega\,(z)^T e_{31} \mathrm{B} dV^{(2)} \quad (32)$$

Capacitance matrix is given by

$$P_D = -\int \Omega\,(z)^T \varepsilon_{33} \Omega(\mathrm{z}) dV^{(2)} \quad (33)$$

Transformation matrix is given by

T= [cos (θ) 0 0 0; 0 1 0 0; 0 0 cos (θ) 0; 0 0 0 1]

Formulating the Eq. (21).into global matrix based on electromechanical vector transformation. Voltage, current and power frequency response function obtained by direct non-orthonormalised electromechanical dynamic equation is given by [5]

$$\frac{v_p}{-\omega w_b e^{jwt}} = \left[j\omega C_p - \frac{1}{R_{load}} j\omega \Theta^T * \left[\underline{K} - \omega^2 \underset{.}{M} + j\omega \acute{Ç} \right]^{-1} \Theta \right]^{-1} * j\omega\Theta^T \left[\underline{K} - \omega^2 \underset{.}{M} + j\omega \acute{Ç} \right]^{-1} Q \quad (34)$$

$$\frac{i}{-\omega^2 w_{base} e^{i\omega t}} = \frac{1}{R_{load}} \left\{ \left[C_p i\omega + R_l - \Theta^T i\omega \left[-\underset{.}{M}\omega^2 + \acute{Ç} i\omega + \underline{K} \right]^{-1} \Theta \right]^{-1} * \Theta^T i\omega \left[-\underset{.}{M}\omega^2 + \acute{Ç} i\omega + \underline{K} \right]^{-1} Q \right\} \quad (35)$$

$$\frac{P}{(-\omega^2 w_{base} e^{i\omega t})^2} = \frac{1}{R_{load}} \left\{ \left[C_p i\omega + R_l - \Theta^T i\omega \left[-\underset{.}{M}\omega^2 + \acute{\c{C}} i\omega + \underline{K} \right]^{-1} \Theta \right]^{-1} * \Theta^T i\omega \left[-\underset{.}{M}\omega^2 + \acute{\c{C}} i\omega + \underline{K} \right]^{-1} Q \right\}^2 \quad (36)$$

The dynamic equations are solved assuming system response is linear under harmonic base excitation and beam is excited in transverse direction. The electromechanical piezoelectric cantilever energy harvesting beam equations are formulated by the Hamiltonian principle. The numerical work of the author [5] is extended to outline the key equations and include orientation for the electromechanical dynamic equation.

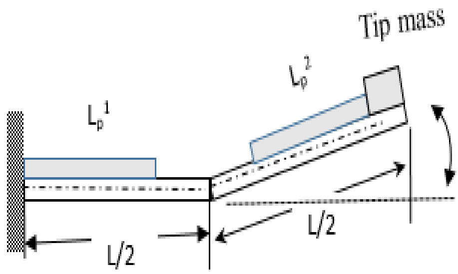

Figure 2 Orientation of Cantilever beam bounded with piezoelectric patches and tip mass at free end**.**

2.1 Model of orientated beam bounded with piezoelectric patches and tip mass

The study on energy harvesting is carried on cantilever beam of length ‘L’. The beam is divided into two parts L/2 each, L_p^1 and L_p^2 is the length of PZT patches, h_b and h_p is the thickness of base beam and PZT patch, θ is the orientation of second half part of beam bounded with PZT patch and mounted with tip mass.

The geometrical and material property of the beam are presented in table 1.Tip mass is mounted on extended length of base beam. The length of the piezoelectric patches Lp^1 and Lp^2 is 20 mm each. The angle of orientations of the beam is 10^0. The analysis of the beam is carried out using MATLAB program in the frequency range of 0 to 1000 Hz. The Cantilever beam is divided into 50 elements.

Table 1 Geometry and material properties[5,13,14]

Length, *l*	50e-3 (m)	Modulus of Elasticity of the PZT -5A and PZT -5H , E_p	66 (Gpa) 62 (Gpa)
Width , *b*	6e-3 (m)	Density, $\rho^{(1)}$	9000 (kg/m3)
Thickness, h_b	0.5e-3(m)	Density of the PZT -5A and PZT -5H layer, $\rho^{(2)}$	7800 (kg/m3)
Thickness of PZT, h_p	0.127e-3 (m)	Piezoelectric constant of PZT -5A and PZT -5H ,*e*31	-12.54 (C/m^2) -19.84 (C/m^2)
Modulus of Elasticity of the base beam, E_b	105 (Gpa)	Permittivity of constant strain of PZT -5A and PZT -5H eps33	$1.593*10^{-8}$ $3.363*10^{-8}$
Length of tip mass	15 (mm)	height of tip mass	10(mm)
Density of tip mass	9000 (kg/m3)	Damping constant	α=4.886; β=1.2433e-05

3. RESULTS

The vibration energy in the environment occur with various frequencies. To exploit the vibration energy, the frequency of the model considered have to operate close to the vibration source. The importance to find the natural frequency is to obtain maximum output. The validation of the model is given in Table 2 and 3

Table 2 Comparison of Natural frequencies for unimorph beam without tip mass

Natural frequency	Literature paper [3]	MatLab results
1^{st}	47.8	47.81
2^{nd}	299.6	299.62
3^{rd}	838.2	838.93

Table 3 Comparison of First natural frequency with tip mass

Literature paper [5]	MatLab results
18.5	18.48

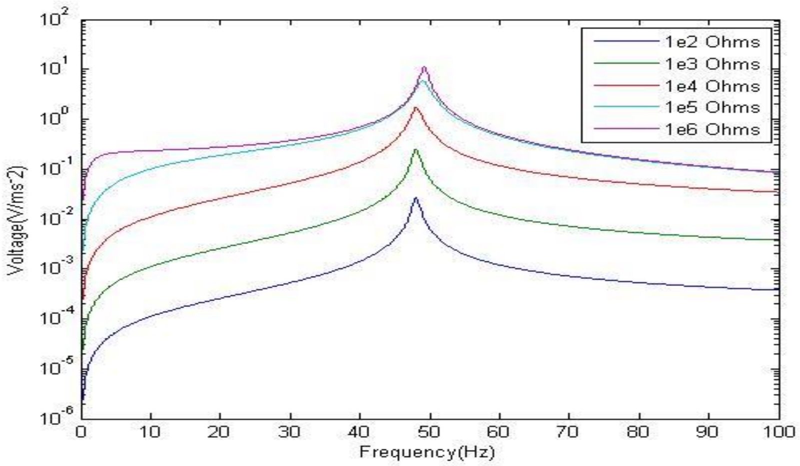

Figure 3.Plot of Voltage FRF

The MatLab program is validated with material and geometrical properties of literature paper [3] and [5] and comparison of natural frequencies of unimorph piezoelectric energy harvester beam without tip mass and with tip mass at 0^0 orientation of beam is carried. The comparison of MatLab Voltage FRF with the literature paper [4] is shown in Fig. 3

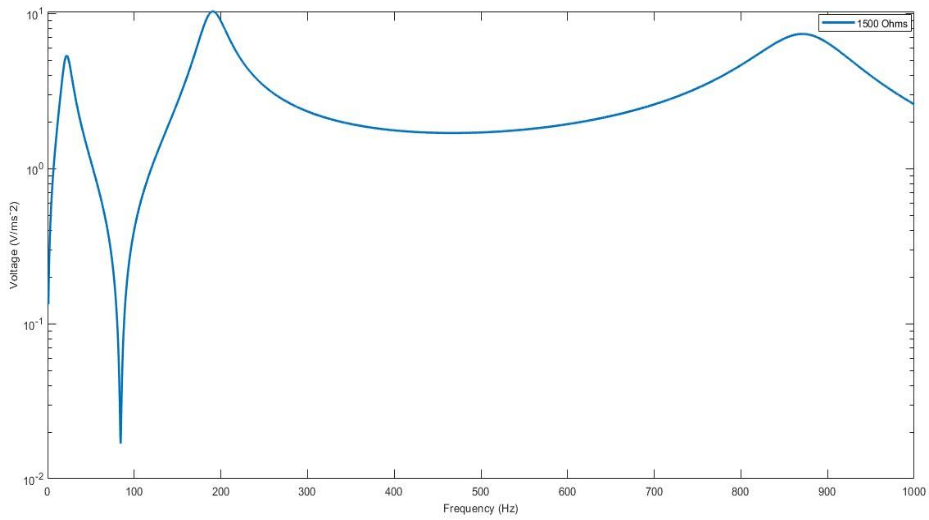

Figure 4. Voltage FRF of PZT-5A

Figure 4.Shows the voltage FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5A.

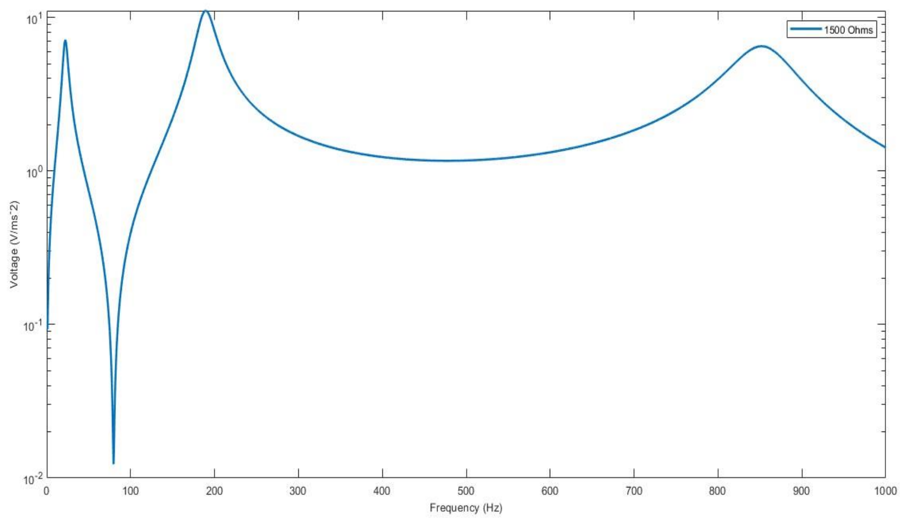

Figure 5. Voltage FRFof PZT-5H

Figure 5.Shows the voltage FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5H.

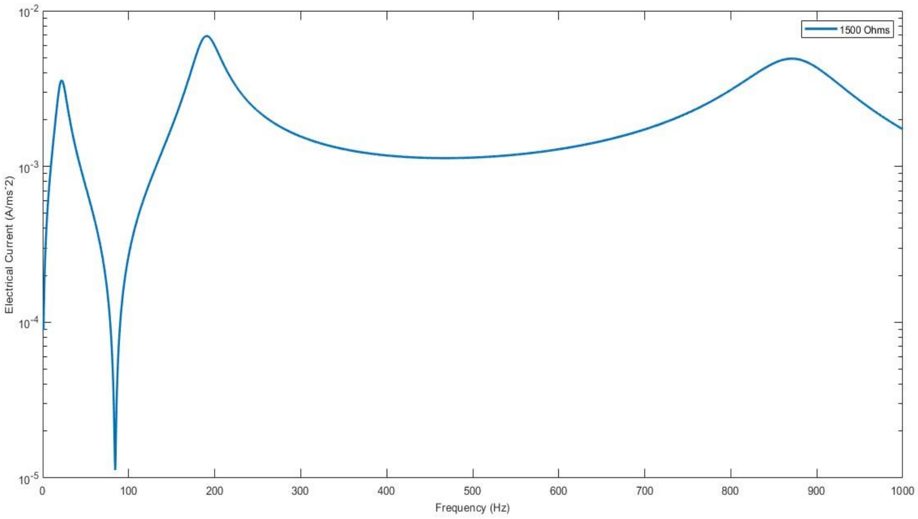

Figure 6.Current FRF of PZT-5A

Figure 6.Shows the current FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5A.

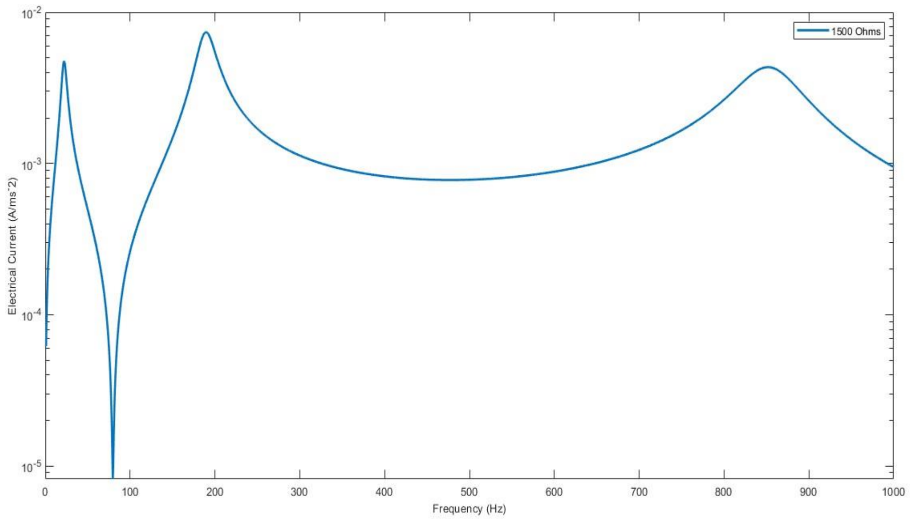

Figure 7.Current FRF of PZT-5H

Figure 7.Shows the current FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5H.

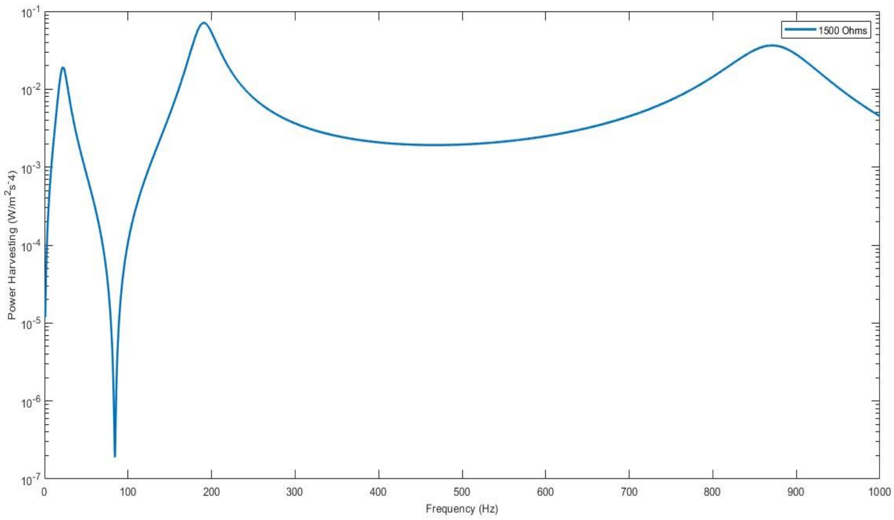

Figure 8.Power FRF of PZT-5A

Figure 8.Shows the power FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5A.

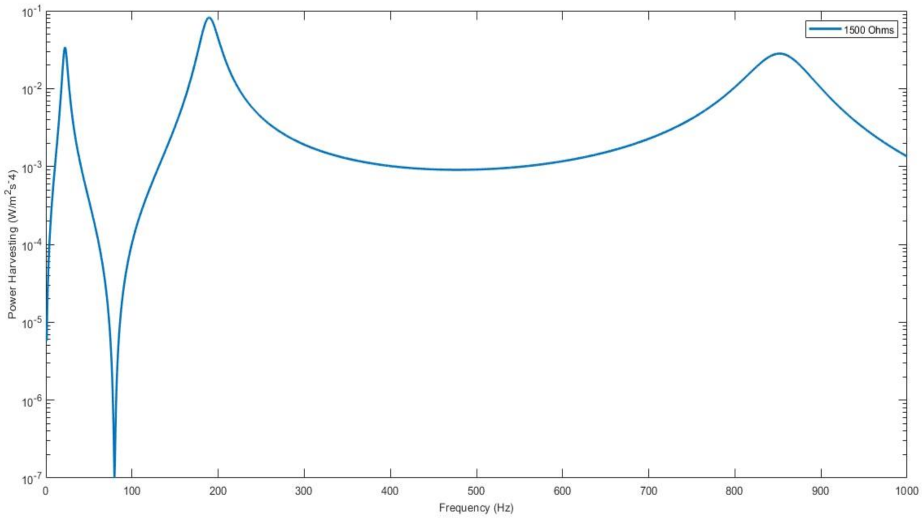

Figure 9.Power FRF of PZT-5H

Figure 9.Shows the power FRF of cantilever beam oriented at 10^0 mounted with tip mass and bounded with PZT -5H.

Figure 4-9 shows the voltage, current and power FRF of cantilever beam orientated at 10^0, bounded with PZT -5A and PZT -5H piezoelectric patches and beam mounted with tip mass at the free end. At resistance of 1500 ohms and frequency range of 0 to 1000Hz.The following observations are made.

The voltage, current is higher for PZT-5A in first and second peak and lower for third peak compared to PZT-5H.

The higher power is achieved for PZT-5H in first and third peak and lower for second peak compared to PZT-5A.

5. CONCLUSIONS

MatLab program is developed using finite element method considering the Euler Bernoulli beam assumptions, constitutive equations of piezoelectric material and Hamilton's principle. The paper focuses on the study of effect of PZT-5A and PZT-5H piezoelectric patches on orientated cantilever beam mounted with tip mass using the direct method with non-orthonormalisation to derive the voltage, current and power frequency response function (FRF).It is observed that for the 10^0 orientation, 0.127mm thickness of piezoelectric material and 1500 ohms resistance. The higher voltage and current is achieved for PZT-5A in first and second peak and lower for third peak compared to PZT-5H. The higher power is achieved for PZT-5H in first and third peak and lower for second peak compared to PZT-5A.

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Nomenclature

$\{D\}$	electric displacement vector
$\{T\}$	stress vector
$[e]$	dielectric permittivity matrix
$[c^E]$	matrix of elastic coefficients at constant electric field strength
$\{S\}$	strain vector
$[\varepsilon^S]$	dielectric matrix at constant mechanical strain
$\{E\}$	electric field vector
I	Moment of inertia
Q	charge
$\ddot{w}_{base}$	Base excitation acceleration
Y	Young's Modulus
ƙ	Kinetic energy
ƥ	Potential energy
ꞷ	Electrical energy
ϖ	External work
C_p	trace of the global capacitance matrix
Θ^T	transformed electromechanical coupling
R_{load}	Resistance
Ḵ, Ṃ, Ḉ	Global stiffness, mass and damping matrix
ω	frequency