TWO-LEVEL TRADE CREDIT POLICY APPROACH FOR A PRODUCTION INVENTORY MODEL UNDER GREENING DEGREE DEPENDENT DEMAND AND RELIABILITY

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Abstract:

In this era, people are concerned about protecting the environment from human pollution, and businesses are turning to green technology to source environmentally friendly goods. As a result, it is challenging for the business manager to seize the market by offering the most incredible green quality at a reasonable price in a specific economy. In this model, demand depends on the selling price and the greening level of the product. Trade credit is one of the most powerful promotional strategies for pushing a product, as it indirectly reduces the product's selling price. Therefore in this research paper, an EPQ model is formulated under two trade credit policies with the effect of reliability. In this two-level trade credit policy manufacturer offers a permissible delay period to the retailer, and the retailer provides a partial trade credit to the customers. In the production system, reliability is also an essential factor. Production is always greater than demand; therefore, shortages are not allowed. Deterioration is considered at a constant rate. The main objective of this paper is to minimize the total cost. An algorithm has been given to solve the numerical example. To validate the proposed model, seven numerical examples have been solved for seven cases of trade credit. Mathematica 12 software is used to find the global minimum solution to the optimal cycle length (T), the time (t_1) the production will end, and the convexity of total cost function concerning production time and cycle time has been shown for all the cases. The optimal solution is obtained in the third case where cycle length (T) exceeds the manufacturer's permissible delay time (N) and is less than the customer's trade credit period. Finally, a sensitivity analysis has been made for the numerical example 3. The paper concludes with the conclusion, and an outlook of possible future directions is depicted.

Keywords: Partial trade credit policy, EPQ Model, Green sensitive and price dependent demand, reliability, Deterioration.

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1. Introduction:

1.1 Partial trade credit

The traditional model assumes that the retailer must pay as soon as the products are procured. In practice, to increase sales and market share, the manufacturer may offer the retailer a payment delay (known as trade credit). If the payment is made within the trade credit term, there is usually no interest levied on the outstanding amount. As a result, the retailer has the option to sell the goods, deposit the money so it can earn interest. If the retailer does not pay within the trade credit period on previously negotiated terms and circumstances, the manufacturer might charge a high interest rate. Retailers benefit financially because they can receive interest on the income generated during the allowable wait period.

1.2 Price and green sensitive demand

Most of the inventory models are based on the hypothesis that demand is constant over the entire inventory cycle whilenow a days, businesses are implementing green technology to purchase green products to save the environment from pollution as people become more aware of the need to protect the environment from pollution produced by humans. As a result, the business manager faces a difficult problem in capturing the market by supplying the greatest green quality at a reasonable price in a specific time frame economy.

2. Literature review

In the inventory model, Goyal (1985) first introduced the concept of trade credit in which the manufacturer offers some delay period in payment to the retailer.Huang (2003) developed an EOQ model based on Goyal's (1985) model, in which the supplier grants the retailer a permissible delay period (retailer credit period), and the retailer grants his or her customers a trade credit period (customer credit period) that is shorter than the retailer credit period. Huang and Hsu (2008) expanded on Huang's (2003) model, assuming that the retailer receives full trade credit from the supplier but only gives half trade credit to his or her consumers. Tenget. al (2012) developed an economic order quantity model in which demand is taken non-decreasing. Sarkar et al. (2015) gives an inventory model for fixed life time products with variable deterioration under trade credit policy. In this paper manufacturer offers full trade credit to the retailer but retailer offers partial trade credit to his customers. A joint pricing inventory model for degrading goods with expiration dates and partial backlogs is provided by Tiwari et al. (2018) under two levels of partial trade credits in the supply chain. A time-dependent demand inventory model for decaying items using preservation technology, a trade credit facility, and partial backlogging was described by Shaikh (2019).A fuzzy green inventory model with non-instantaneous decaying products is presented by Tiwari et al. (2019) that assists firms in maximising total annual profit under various trade-credit policy scenarios while also reducing carbon emissions for a cleaner environment. The report explores all possible scenarios that could arise in carbon emission cost green inventory models under various permitted payment delays. For non-instantaneous decaying goods, Rapolu and Kandpal (2020) investigated the issue of simultaneously optimising pricing, advertisement frequency, preservation technology investment, and ordering policies. According to this policy, three potential instances are each given its own consideration by the supplier who grants a certain credit time for account settlement. Mahata et al. (2020) proposed a vendor-retailer-customer chain system in which the supplier and retailer both provide partial trade credit to the subsequent downstream credit-risk member under credit perid dependent demand rate. Singhet al. (2021) studied a two-echelon supply chain, which included a producer and a supplier with optimal trade-credit policies. They used the idea of volume agility to make the manufacturing process flexible.

Jabbarzadeh et al. (2021) developed a model in which credit period offered by supplier is less than or equal to the length of time in which no shortage happens under fuzzy environment.A neutrosophic economic order quantity (EOQ) model was developed by Mohanta et al. in 2022 under the concept that consumer demand is sensitive to retail price and marketing efforts. A partial trade credit policy is adopted by the retailer and the supplier.According to Moradi et al. (2022), it is important to use both defective production and payment delays because both of these issues have a direct impact on the profit made.

Grundey and Zaharia (2008) concentrated on environmental marketing and ecological labelling; in which customers have the opportunity to obtain green products and services in exchange for compromising their preferences in favour of green business practises. Yang (2013) et al. gives a supply chain for deteriorating items with multi-retailer and price-sensitive demand. Ghosh and Shah (2015) developed supply chain coordination difficulties that arise from green supply chain activities, as well as the influence of cost sharing contracts on supply chain stakeholders' crucial choices. In 2018, Giri et al. proposed an inventory model in which green sensitive consumer demand has been taken with revenue sharing contract. A supply chain for environmentally friendly products that takes into account recycling, reverse logistics, and remanufacturing with demand that is reliant on carbon emissions was established by Rani et al. in 2019.By presuming that, in situations of price- and eco-sensitive demand, the buyer has greater decision-making power than the provider Giri et al. (2020) develop a coordinated model with a cost-sharing contract, a non-coordinated decentralised model, and an integrated centralised model.In a market where consumers are concerned about the environment, Ghosh et al. (2020) investigate the effects of product green procurement costs and government laws on a single firm and duopoly. To determine the best values for product greening level, pricing, and profitability, they research corporate strategy. Yadav and Khanna (2021) considered expiration date of the perishable product with price dependent demand. Moreover, Yuvan et al. (2021) thought about a quantity-flexibility deal with ecofriendly demand in the automotive business. They made the erroneous assumption that the retailer sets the retail price and the automaker decides the green level. Manna and Bhunia (2022) developed a model with green sensitive demand and green production. The related differential equations of inventory levels in the proposed model are shown in interval form since interval-valued demand and faulty rates were taken into account.Giri and Dash (2022) studied a single retailer single manufacturer model for imperfect production with green sensitive and advertisement dependent demand.After receiving each shipment from the manufacturer, the store conducts a faultless screening procedure to identify the defective goods. According to this eco-friendly demand, Bhavani et al. (2022) examined a sustainable inventory model of non-instantaneous degrading goods under partially backlogged shortages. In this study, carbon emissions under cap and tax systems as well as emission releases owing to inventory creation, deterioration, and holding were estimated.

Research gap

At the present time, people are becoming more aware of the need to protect the environment from pollution produced by humans. Therefore businesses are implementing green technology to purchase green products to save the environment. In this paper EPQ model for deteriorating items with partial trade credit policy under green sensitivity demand has been studied.

Table 1 provides an understanding of the variance between the previous work and the work of this publication. Several researchers have successfully created inventory models for a variety of situations, but very few have been able to create a model with green sensitive demand.

Contribution of this study

The contribution of this study is defined as follows:

- In this era, demand of the environment friendly product has become high. So to make the model more realistic demand is taken price and green sensitive.
- Two-level trade credit policies are implemented in order to enhance the company profile and provide quick and easy access to funding. According to this policy, supplier offer a trade credit period to the manufacturer and manufacturer offer a partial trade credit period to the customer.
- Reliability factor is also taken into consideration in order to lower quality control failure rates, expedite product changes, etc.

Research Article	Type of credit period	Reliability	Demand Type	Deterioration
Dye (2012)	Two level	×	Time dependent	✓
Teng et al. (2013)	Full trade credit	×	Time dependent	×
Singh and Rathore (2014)	Full trade credit	✓	Time dependent	✓
Sarkar et al.(2014)	NA	✓	Price and Time dependent	×
Shah and Vaghela(2016)	NA	×	Time and advertisement dependent	✓
Shaikh (2017)	Alternative	×	Time and price dependent	✓
Sharma et al. (2018)	NA	×	Selling price dependent	✓
Mahapatra et al.(2019)	NA	✓	Price and stock dependent demand	✓
Maji et al. (2020)	NA	✓	Reliability and selling price dependent	×
Kumar et al. (2020)	Full trade credit	✓	Advertisement, time and selling price dependent	✓
Duary et al. (2021)	NA	×	Displayed stock level and selling price	✓
Tayal et al. (2021)	Full trade credit	×	Price and stock dependent	✓

Singh and Ummeferva (2022)	NA	×	Price and Green sensitive	✓
Handa et. al (2022)	Two level trade credit	×	Constant	✓
This paper	Partial trade credit	✓	Green quality and selling price dependent	✓

NA- Not Available

Table1 Recent research works in the inventory literature

3. Notations and Assumptions

3.1Notations

Notations	Descriptions
A	Ordering cost($/unit)
c_h	Holding cost($/unit)
c_d	Deterioration cost($/unit)
p	Selling price($/unit)
s	Production cost($/unit)
a	Market capacity(Constant)
b	Scale parameter of demand sensitivity with selling price(Constant)
β	Green level (green quality of the product) ($/unit)
θ	Deterioration rate(Constant)
r	Reliability rate to produce good items(%)
P	Production rate(Units)
M	Trade credit period of the manufacturer provided by the supplier(Months)
N	Trade credit period of the customers provided by the manufacturer(Months)
$q(t)$	Inventory level at any time t(Units)
γ	γ of the total purchase price to be paid by the customer to the manufacturer
I_e	Manufacturer earned total interest ($/unit/Months)
I_p	Manufacturer paid total intrest ($/unit/Months)
Decision variables	
t_1	Time at which production will end (Months)
T	Cycle Length(Months)
TC(T)	Total Cost($)

3.2 Assumptions

Demand of the product is D = $\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]$ where a>0, b>0. It is a linear function with respect to green quality and selling price dependent, which is more applicable than fixed demand in real world situations.

1) The manufacturer produces the items with reliability.
2) Deterioration is considered in this model and the rate of deterioration is assumed as constant.
3) The infinite planning horizon is considered.
4) The policy of partial trade credit is used in this model to strengthen the business relationship between the retailer and the customer. In partial trade credit policy, the manufacturer provides this facility to the retailer and the retailer to the customers.

4. Mathematical Model

This business model which starts at the producer's production plant, and initially its production starts at t=0. The time interval from t=0 to t=t_1 affects the model in three ways, production as well as aggregate demand from customers and deterioration. Production is stopped by the producer at time t = t_1. Since the overall customer demand and the rate of deterioration affect inventory during the time period [t_1, T], it is necessary to understand manufacturer inventory model in a mathematical way by the following first order linear differential equations:

$$\frac{dq}{dt}+\theta q(t)=-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP \quad 0<t<t_1 \tag{1}$$

$$\frac{dq}{dt}+\theta q(t)=-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right] \quad t_1<t<T \tag{2}$$

Solving the equations (1) and (2), with the help of boundary condition q (0) =0 and q (T) =0, we get

$$q(t)=\frac{-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP}{\theta}\left[1-e^{-\theta t}\right] \tag{3}$$

$$q(t)=\frac{\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta}\left[e^{\theta(T-t)}-1\right] \tag{4}$$

Using the continuity condition at t=t_1, one can get

$$t_1=\frac{1}{\theta}\log\left[1+\frac{\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[e^{\theta T}-1\right]}{rP}\right] \tag{5}$$

Now the total cost of the inventory is calculated using the following components:

4.1Ordering Cost: The cost associated with the required quantity, material's type is called the ordering cost. If A be the ordering cost per cycle. ThenOrderingCost=A(6)

4.2Holding cost: The expense of keeping goods in the warehouse from the time they are received until all of the things are sold is known as holding cost. If the holding cost per unit time is c_h, then from time 0 to time T, the total holding cost for storing the items in the warehouse will be

$$HC=c_h\left[\int_0^{t_1} q(t)\,dt+\int_{t_1}^{T} q(t)\,dt\right]$$

$$=c_h\left[\left[\frac{-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP}{\theta}\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\frac{\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta}\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right] \quad (7)$$

4.3Deterioration cost: Deterioration is a naturalphenomenonwhich occurs in many edible items. If the deterioration cost per unit time is c_d, then the total deterioration cost will be

$$DC=c_d\theta\left[\int_0^{t_1} q(t)\,dt+\int_{t_1}^{T} q(t)\,dt\right]$$

$$=c_d\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right] \quad (8)$$

There are two unique possibilities may emerge, according to the considered credit period durations,

Scenario 1: N < M

In this scenario, the customer's credit duration (N) given by the manufacturer is less than the manufacturer's credit period (M) given by the supplier. There are four possible outcomes in this situation.

Case-1: M≤ t_1

When the time $t_1$is greater than the Manufacturer's credit period (M). Therefore manufacturer earned interest in the time period [0,M] and he pay the interest from the duration [M,T].In this case, the manufacturer's interest payable and earned can be computed as follows:

$$\text{Interest paid}=\frac{sI_p}{T}\left[\int_M^{t_1} q(t)\,dt+\int_{t_1}^{T} q(t)\,dt\right]$$

$$=\frac{sI_p}{T\theta}\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]+\left(rP-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\right)\left[t_1-M+\frac{e^{-\theta t_1}-e^{-\theta M}}{\theta}\right]\right] \quad (9)$$

$$\text{Interest earned}=\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_0^M t\,dt+pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_N^M t\,dt}{T}$$

$$=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2T}\left[M^2+(\gamma-1)N^2\right] \quad (10)$$

Therefore manufacturer’s average cost is

TC_1 = Ordering Cost + Holding cost + Deteriorating cost + Interest paid –Interest earned

$$TC_1=\frac{1}{T}\left[\begin{array}{l} A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right] \\ +\frac{sI_p}{\theta}\left[\begin{array}{l}-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right] \\ +\left(rP-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\right)\left[t_1-M+\frac{e^{-\theta t_1}-e^{-\theta M}}{\theta}\right]\end{array}\right]-\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2}\left[M^2+(\gamma-1)N^2\right]\end{array}\right] \quad (11)$$

Case-2: $t_1 < M \leq T$

In this case manufacturer's trade credit period is greater than the time t_1. Therefore manufacturer earned the interest during the time period [0, M] and he has to pay the interest in the time duration [M,T]. For this case, the amount of interest payable and interest earned may be computed as follows:

$$\text{Interest paid}=\frac{sI_p}{T}\left[\int_M^T q(t)\,dt\right]$$

$$=\frac{sI_p\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta^2 T}\left[e^{\theta(T-M)}-\theta(T-M)-1\right] \quad (12)$$

$$\text{Interest earned}=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\gamma\int_0^N t\,dt+pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_N^M t\,dt}{T}$$

$$=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2T}\left[M^2+(\gamma-1)N^2\right] \quad (13)$$

Therefore manufacturer's average cost is

TC_2 = Ordering Cost +Holding cost + Deteriorating cost + Interest paid –Interest earned

$$TC_2=\frac{1}{T}\left[\begin{array}{l} A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right] \\ +\frac{sI_p\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right]-\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2}\left[M^2+(\gamma-1)N^2\right]\end{array}\right] \quad (14)$$

Case-3: $N < T \leq M$

In this situation, the cycle duration (T) exceeds the maximum delay time allowed by the manufacturer (N). As a result, the manufacturer is not required to pay interest.i.e. IP = 0 (15)

Furthermore, under a partial trade credit policy, the manufacturer might earn interest on the income received from consumers. This interest can be calculated as follows:

$$\text{Interest earned}=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\gamma\int_0^N t\,dt+pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_N^T t\,dt+\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_T^M dt}{T}$$

$$=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2T}\left[2MT+(\gamma-1)N^2-T^2\right] \quad (16)$$

Therefore manufacturer's average cost is

TC_3 = Ordering Cost + Holding Cost + Deteriorating cost + Interest paid –Interest earned

$$TC_3=\frac{1}{T}\left[\begin{array}{l}A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right]\\-\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{2}\left[2MT+(\gamma-1)N^2-T^2\right]\end{array}\right] \quad (17)$$

Case-4: T ≤ N < M

If the cycle time T is grater then both the customer's credit and manufacturer's allowed delay in payment period. Whenever such a situation arises, the supplier does not have to pay any interest by the manufacturer.

Interest paid = 0 (18)

$$\text{Interest earned}=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{T}\left[\gamma\int_0^T t\,dt+\gamma T\int_T^N dt+T\int_N^M dt\right]$$

$$=pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[M+(\gamma-1)N-\frac{\gamma T}{2}\right] \quad (19)$$

Therefore manufacturer's average cost is

TC_4 = Ordering Cost + Holding cost + Deteriorating cost + Interest paid –Interest earned

$$TC_4=\frac{1}{T}\left[\begin{array}{l}A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right]\\-TpI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[M+(\gamma-1)N-\frac{\gamma T}{2}\right]\end{array}\right] \quad (20)$$

Scenario 2: M < N

In this scenario, the manufacturer's credit duration (M) from the supplier is shorter than the customer's credit period (N) from the manufacturer. Three cases arise in this circumstance, which are given follows:

Case-5: M< N ≤ t_1

In this case manufacture's trade credit and the customers trade credit both are less than the time t_1. Therefore the manufacturer earned the interest from the time period [0, M] and he has to pay the interest in the time period [M, T]. Interest payable amount and Interest earn amount for this case is calculated as follows:

$$\text{Interest paid}=\frac{sI_p}{T}\left[\int_M^{t_1}q(t)\,dt+\int_{t_1}^T q(t)\,dt\right]$$

$$=\frac{sI_p}{T\theta}\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]+\left(rP-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\right)\left[t_1-M+\frac{e^{-\theta t_1}-e^{-\theta M}}{\theta}\right]\right] \quad (21)$$

$$\text{Interest earned}=\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_0^M t\,dt}{T}$$

$$=\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]M^2}{2T} \quad (22)$$

Therefore manufacturer's average cost is

TC_5= Ordering Cost + Carrying cost + Deteriorating cost + Interest paid –Interest earn

$$TC_5=\frac{1}{T}\left[\begin{array}{l}A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right]\\+\frac{sI_p}{\theta}\left[\begin{array}{l}-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\\+\left(rP-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\right)\left[t_1-M+\frac{e^{-\theta t_1}-e^{-\theta M}}{\theta}\right]\end{array}\right]-\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]M^2}{2}\end{array}\right] \quad (23)$$

Case-6: $t_1 < M < N \leq T$

Interest payable amount and interest earned amount for this case is calculated as follows:

$$\text{Interest paid}=\frac{sI_p}{T}\left[\int_M^T q(t)\,dt\right]$$

$$=\frac{sI_p\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta^2 T}\left[e^{\theta(T-M)}-\theta(T-M)-1\right] \quad (24)$$

$$\text{Interest earned}=\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\int_0^M t\,dt}{T}$$

$$=\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]M^2}{2T} \quad (25)$$

Therefore manufacturer's average cost is

TC_6= Ordering Cost + Holding cost + Deteriorating cost + Interest paid-Interest earn

$$TC_6=\frac{1}{T}\left[\begin{array}{l}A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right]\\+\frac{sI_p\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right]-\frac{pI_e\gamma\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]M^2}{2}\end{array}\right] \quad (26)$$

Case-7: $T \leq M < N$

If the cycle length (T) is less than from both customer's credit period (N) and manufacturer's allowed delay payment, whenever such a situation arises, the supplier does not have to pay any interest by the manufacturer.

Interest paid = 0 (27)

$$\text{Interest earned}=\frac{pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]}{T}\left[\gamma\int_0^T t\,dt+\gamma T\int_T^M dt+T\int_M^N dt\right]$$

$$= pI_e\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[N+(\gamma-1)M-\frac{\gamma T}{2}\right] \quad (28)$$

Therefore manufacturer's average cost is

TC$_7$ = Ordering Cost + Holding cost + Deteriorating cost + Interest paid —-Interest earned

$$TC_7=\frac{1}{T}\left[\begin{array}{l} A+\left(\frac{c_h}{\theta}+c_d\right)\left[\left[-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]+rP\right]\left[t_1+\frac{e^{-\theta t_1}-1}{\theta}\right]-\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[T-t_1+\frac{1-e^{\theta(T-t_1)}}{\theta}\right]\right] \\ -pI_eT\left[\left(\frac{\beta}{1+\beta}\right)a-bp\right]\left[N+(\gamma-1)M-\frac{\gamma T}{2}\right] \end{array}\right] \quad (29)$$

5. Solution Methodology

5.1 Algorithm

In this model seven case has been arises. To find the optimal solution of the problem Mathematica 12 software has been used. The following steps are performed in the Mathematica 12 software

Step1: Define the total cost function for case first.

Step 2: Put the numerical value of all the parameter.

Step 3: Use the command to minimize the total cost function.

Step 4: Compile and execute.

Step 5: Check the output.

Step 6: There will be two possibilities i.e. solution will be feasible or not. If it is feasible go the step 7 or if it's not feasible go the step 8.

Step 7: Repeat the process (from step 1 to 6) for all remaining six cases. In all seven cases check in which case total cost is minimum. That will be the optimal case of this problem.

Step 8: If the solution is not feasible then change the input values and repeat the step from 3 to 8. This process should be follow until the feasible solution is not obtained.

6. Numerical Example

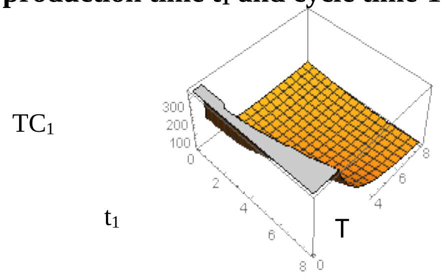
Seven examples for seven cases have been selected to describe each case of specified inventory model, and all of the solutions are shown in the table.

Condition 1: N < M

Example1. (M $\leq$ t$_1$) Ordering cost A =100, Holding cost c_h=10, Deterioration cost, c_d= 2, Selling price p = 10, Production rate P = 150, Reliability rate r = 0.01, Production cost s = 10, Deterioration rate θ = 0.02, green quality of the product = 0.01, Demand parameter a = 100, Scale parameter of demand sensitivity with selling price b = 0.02,Manufacturer's trade credit period M = 3.7, Customer's trade credit period N = 3.5, Interest earned by the manufacturer I_e = 0.1, Interest paid by the manufacturer I_p =0.2,γ = 0.03.

The solution is t_1=4.34848 months, T= 6.270266 months and TC$_1$=$27.5740.

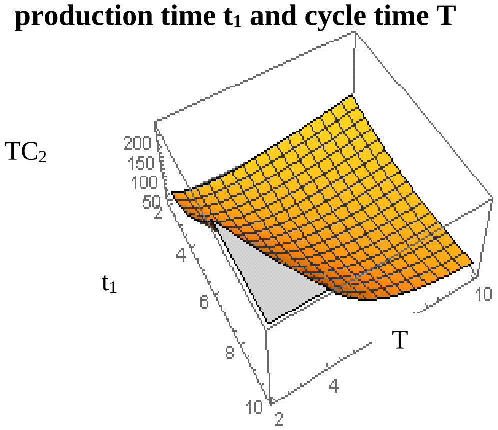
Convexity of the total cost function w.r.t. production time t$_1$ and cycle time T



Example2. (t$_1$ < M $\leq$ T) takes the same parameter as used in example 1 except for the parameter customer's trade credit period N = 3, manufacturer's trade credit period M = 3.2

The solution is t_1=3.06237months, T=4.70412months and TC$_2$=$38.0520.

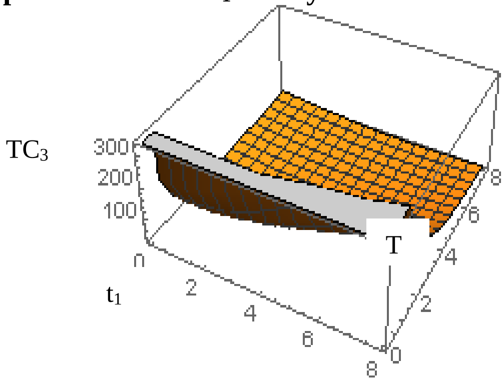
Convexity of the total cost function w.r.t. production time t$_1$ and cycle time T



Example3. (N< T $\leq$ M) takes the same parameter as used in example 1 except for the parameter customer's trade credit period N = 3.5, manufacturer's trade credit period M = 7

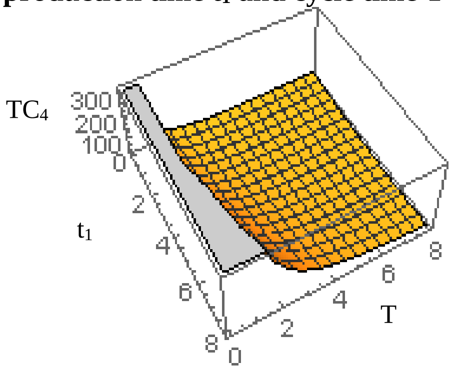
The solution is t_1=4.52355months, T= 6.96612months and TC$_3$=$23.8459.

Convexity of the total cost function w.r.t. production time t_1 and cycle time T



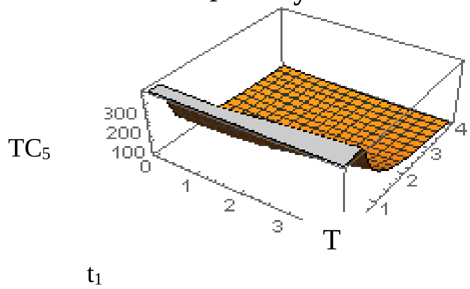
Example4. (T $\leq$ N < M) takes the same parameter as used in example 1 except for the parameter customer's trade credit period N = 8, manufacturer's trade credit period M = 8.2
The solution is t_1=4.97549 months, T= 7.64408months and TC_4=$25.9470

Convexity of the total cost function w.r.t. production time t_1 and cycle time T



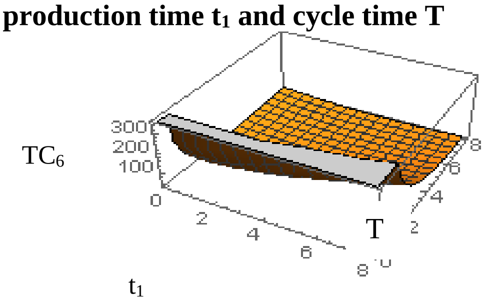
Condition 2: M < N
Example5. (M < N $\leq$ t_1) takes the same parameter as used in example 1 except for the parameter manufacturer's trade credit period M = 3.5, customer's trade credit period N = 3.7
The solution is t_1=4.38395months, T= 6.75607months and TC_5=$27.7892

Convexity of the total cost function w.r.t. production time t_1 and cycle time T



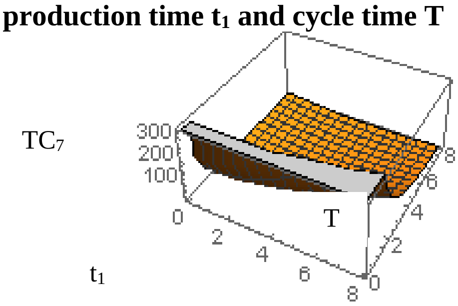
Example6.(t_1 < M < N $\leq$ T) takes the same parameter as used in example 1 except for the parameter manufacturer's trade credit period M = 5.5, customer's trade credit period N = 5.7
The solution is t_1=4.51176months, T= 6.94839months and TC_6=$26.6111.

Convexity of the total cost function w.r.t. production time t_1 and cycle time T



Example7. (T $\leq$ M < N) takes the same parameter as used in example 1 except for the parameter manufacturer's trade credit period M = 8, customer's trade credit period N = 8.7
The solution is t_1=4.97549months, T= 7.64408months and TC_7=$26.1370.

Convexity of the total cost function w.r.t. production time t_1 and cycle time T



Examples	Circumstances	t_1	T	Total cost
Case 1	M $\leq$ t_1	4.34848	6.70266	27.5740
Case 2	t_1 < M $\leq$ T	3.06237	4.70412	38.0520
Case 3	N < T $\leq$ M	4.52355	6.96612	23.8459
Case 4	T $\leq$ N < M	4.97549	7.64408	25.9470
Case 5	M < N $\leq$ t_1	4.38395	6.75607	27.7892
Case 6	t_1 < M < N $\leq$ T	4.51176	6.94839	26.6111
Case 7	T $\leq$ M < N	4.97549	7.64408	26.1370

Table 2: Solution for the different cases

7. Sensitivity Analysis and Observations

7.1 Sensitivity Analysis

The optimal result is derived using this data set to understand this model in real life, and also a sensitivity analysis on the parameters that may affect the model. Processes of sensitivity were defined as percentage fluctuations in the aforementioned ideal values. The analysis was carried out by adjusting the main parameters from -20 to +20 percent one by one while keeping all other parameters constant, as given in table.

Parameter	% changes	t_1	T	Total cost
A	+20	4.94934	7.60493	26.5912
	+10	4.74071	7.29226	25.2486
	-10	4.29663	6.62454	22.3743
	-20	4.05835	6.26504	20.8227
θ	+20	4.5549	6.98023	23.822
	+10	4.53917	6.97311	23.8341
	-10	4.50803	6.95927	23.8576
	-20	4.49261	6.95255	23.8691
c_d	+20	4.52209	6.96393	23.8555
	+10	4.52282	6.96502	23.8507
	-10	4.52428	6.96722	23.8411
	-20	4.52501	6.96831	23.8364
P	+20	3.39214	6.24306	27.2437
	+10	3.87285	6.54542	25.7345
	-10	5.47034	7.60511	21.3629
	-20	6.99616	8.68456	17.9681
c_h	+20	4.19706	6.47443	26.1543
	+10	4.35128	6.70689	25.0209
	-10	4.71765	7.25766	22.6243
	-20	4.93856	7.5888	21.3502
γ	+20	4.52278	6.96495	23.8409
	+10	4.52316	6.96554	23.8434
	-10	4.52394	6.96670	23.8484
	-20	4.52433	6.96728	23.8510
a	+20	5.91212	7.59251	20.3795
	+10	5.14801	7.20746	22.3628
	-10	3.99351	6.83749	24.8677
	-20	3.52941	6.80839	25.4449
r	+20	3.39099	6.241	27.2548
	+10	3.87156	6.5433	25.745
	-10	5.46866	7.60283	21.3713
	-20	6.99424	8.68222	17.9748
b	+20	4.47768	6.95181	23.9453
	+10	4.50054	6.95888	23.896
	-10	4.54671	6.97354	23.7951
	-20	4.57003	6.98115	23.7436
N	+20	4.47768	6.95181	23.2004
	+10	4.50054	6.95888	24.0156
	-10	4.54671	6.97354	23.6916
	-20	4.57003	6.98115	23.5528
I_e	+20	4.44904	6.85405	23.3357
	+10	4.48563	6.90909	23.5918
	-10	4.56288	7.02524	24.098
	-20	4.60371	7.08659	24.3479
β	+20	5.89188	7.58918	20.4328
	+10	5.14057	7.20413	22.3816
	-10	3.99791	6.83819	24.8606
	-20	3.53637	6.80809	25.4392
p	+20	4.47768	6.95181	23.9453
	+10	4.50054	6.95888	23.896
	-10	4.54671	6.97354	23.7951
	-20	4.57003	6.98115	23.7436

Table 3: Sensitivity for different parameters

7.2 Observations& Managerial Insights

From the table, the following observations have been found.

1. An increase in the inventory holding cost parameter (c_h)implies better storage condition time at which the production will end (t_1), cycle length (T) decreases while the total cost

increases. Also, similar to the common practice, increase in the ordering cost Deterioration cost (c_d), and results in an increase in the total cost.

2. If the selling price (p) increases, time at which the production will end (t_1), cycle length (T) decreases and the total cost also increases. In order to capture demand in such a case, it is suggested to reduce the selling price as it will encourage the customers to purchase the product.
3. If the reliability of the product increases, the total cost of the product increases.
4. If the customer's trade credit (N) increases, time at which the production will end (t_1), cycle length (T) decreases and the total cost also decreases. It suggests that if the credit time is longer, then the total cost is also lower.
5. If the demand parameter a decreases, time at which the production will end (t_1), cycle length (T) decreases while the total cost increases.
6. If the parameter (γ) decreases, time at which the production will end (t_1), cycle length (T) increases and the total cost also increases.
7. If the interest earned by the manufacturer (I_e), increases, time at which the production will end (t_1), cycle length (T) decreases and the total cost also decreases.

Conclusion

In this research paper an economic production quantity model has been developed under partial trade credit policy with the effect of reliability. Demand is depend on the price and greening level of the products. In this paper one numerical example has been developed for the seven cases. The minimum cost is obtained for case -3 in which cycle length is greater than the customer's trade credit period and less than the credit period of the manufacturer. Finally a sensitivity analysis has been made for the numerical example 3 and some useful results have been obtained from the table of sensitivity.

For further future extension demand can be changed such price and stock dependent, credit linked demand etc. Also to reduce the deterioration, preservation technology can be used. This model is not applicable when the demand is not green sensitivity.

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