A comparative study on carbon nanotubes - nanofluid flow through a tinny pointer under convective type boundary condition

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Abstract

Proposed present learning, its examined the impact of non–linear thermal emission & Biot's numeral upon borderline cover stream end to end a constantly transfering tinny pointer occupied through carbon nano-tubes via bearing in mind liquid such as consistent liquid. The foremost system of PDE's are foremost condensed on the way to conventional non–linear DE's and be there responded statistically via Finite-element-method. The impact of pertinent constraints upon hotness, hydro–dynamic and solutes borderline films be situated scrutinized profundity & the consequences are discovered via graphs & drawings. Likewise, the consequence of said constraints upon the ideals of Sherwood's numeral, Nusselt's numeral & skin–friction quantity is correspondingly scrutinised and the result's are depicted via tabulated numerals. It's perceived that the concentration and temperature sketches escalates with growing ideals of mass of the pointer constraints (a), whereas, swiftness outlines criticises thru enlightening ideals mass of the pointer constraints (a).

Keywords: Carbon Nanotubes; Thin needle; Non–linear thermal radioactivity; Finite-element- method; Biot's numeral.

1. Primer

Trendy recent years the perception of Nano liquids has crooked interested in complementary broad extent for the research community outstanding to its huge series of significances in biomedicine, heat exchangers, cooling of electronic devises, double windowpane, food, transportation, etc. Towards augmenting the thermal comportment(heat tranfer) of overall liquids which are same as ethyline glycoal, water, kerosene, engine oils, we have to add different types of nanoparticles, like, Graphene, silica, silver, gold, copper, alumina, carbon nanotubes, etc. to the base fluids. Decent digit of examination submissions is reported during collected works that are works towords augmentation of heat transferring of the base liquids by accumulating several categories of nanoparticles [Choi 1995, Eastman et al. 2001, Sheremet and Pop 2015, **Chamkha and Ismail 2016, Ghalambaz et al. 2017**]. In which most of the papers focussed on the experimental and theoretical studies of the thermophysical properties of the nanofluids. Of the above mentioned nanoparticles carbon nanotubes has received great importance due to their larger surface area which causes enormous absorption potential. Depending on the quantity of rolled graphene sheets concentric layers, the carbon nanotubes have categorized into two types, namely, carbon nanotubes having single – wall (SWCNTs) and carbon nanotubes with multi – walls (MWCNTs). Due to numerous applications in agriculture and environment sector, filtration procedures, medication, biosensors, tissue engineering, super conductors, waste retrieving, semi – conductors and nanotechnology, several authors [Wang et al. 2013, Karami et al. 2014, Sabiha et al. 2016, Hayat et al. 2017, UlHaq et al. 2017, Imtiaz et al. 2016, Sreedevi et al. 2018, Jyothi et al. 2018] have discussed the natural convention heat transfer improvement of several nanofluids over dissimilar flow coordinate systems, like, rotating stretchable disks, vertical cone, vertical plate, stretching sheet, etc.

The topic of flow through a thin needle has become the most extensive topic in recent times for the researchers because of its involvement and applications in the engineering industries, medicine, hot wire anemometer, problems related to blood flow, transport industry, for determining storm's velocity, lubrication and varnishing of cables, etc. This is significant to identify that the thin needle motion in the fluid stream interrupts the free flow path and this phenomenon is the maximum emphasis to determine the temperature and velocity profiles of the flow area in investigational works. Thin needle was designated as a parabolic revolution about its direction of axis along with the capricious thickness. It is deliberated "thin" if its thicknesses do not higher than the boundary on it or lesser. Keeping above numerous applications of thin needle in mind, first, Lee (1967) has studied the boundary layer flow over a thin needle and presented approximation solution. Narain and Uberoi (1973) deliberated the warmness transmission then stream analysis over a thin vertical needle and noticed similar results in swiftness & hotness as compared with flat plate as the ideals of magnitude of the pointer enhances. Ishak et al. (2007) have provided finite – difference technique to know the characteristics of boundary layer fluid flow over an isothermal constantly stirring thin needle and is moving in the direction of the fluid flow. Ahmad et al. (2008) perceived boundary layer mixed convention assisting and opposing fluid flow through a thin vertical pointer & perceived that the liquid hotness and swiftness are highly swayed by the governing parameters. Afridi et al.(2018) studied the sway of non – linear thermal radiation and entropy generation on the flow over thin needle moving along with free stream, established that the ideals of Bejan parameter depreciates thru upper ideals of radiation parameter.

All the aforesaid studies are focussed on regular fluids. Due to several applications, mentioned above, of nano-fluid stream compared with diverse geometries, Trimbitas et al. (2012) deliberated the influence of dimensions segment of nano-particle and variable wall temperature on heat transfer and fluid flow amplification of $\mathrm{Cu-water}$ made nanofluid over a vertical thin needle. Soid et al. (2017) detected development in the values of rates of heat transfer by the upgrading ideals of nano particle dimension segment parameter among said authors findings of $\mathrm{copper-water}$ nanofluid flow over horizontal thin needle. Ahmad et. al. (2017) perceived the Buongiorno's nanofluid mathematical model flow along a thin needle moving with uniform velocity along the track of the free stream by enchanting Brownians wave & thermo phoresis obsessed by the count. Krishna et al.(2017) presented the warmness transmission individualities of two flows such as Sakiadis flow and Blasius fluid flow's on compared with horizontal continued thin needle with magnetic parameter and heat source/sink and observed increment in Nuseelt number values as the chunkiness of needle size rises. Sulochana et al(2017) analysed the heat and mass transfer appearances of two types of nanofluids, $\mathrm{Al_{50}Cu_{50}-water}$ and $\mathrm{Cu-water}$, over a continuously moving thin horizontal needle by considering uneven heat source or sink and Joule heating. Sandeep et al. (2017) discussed the sway of heat source or sink, thermal radiation, and magnetic field on nanofluid drift over thin film and noticed enhancement in thermal conductivity with growing volume fraction parameter values. Afridi et al(2018) deliberated the influence of entropy generation and volume segment solid nano particle constraint on boundary layer nano fluid flow over thin horizontal needle packed with water based carbon nanotubes and observed reduction in the temperature values of the both $\mathrm{SWCNT-water}$ and $\mathrm{MWCNT-water}$ based nanofluids with diminishing ideals of mass of the pointer. Salleh et al (2018a, 2018b) have described hotness along with mass transference exploration of Buongiorns prototypical nano - fluid stream over thin needle by taking magnetic parameter and Lewis number. Hayat et al(2016) deliberated the consequence of stagntion point stream over thin needle crammed by liquid CO_2 nano tubes centred nanofluid and perceived that the swiftness of the nanofluid escalates as volume fraction of nanoparticle parameter values rises. Sulochana et al(2017) deliberated the heat transfer analysis of two different nanofluids, such as, $\mathrm{Ag-water}$ and $\mathrm{Ag-kerosene}$ based nanofluid flow over horizontal moving needle with joule heating and non – linear thermal radiation. Khan et al. (2018) pondered the entropy generation minimization on heat and mass transfer flow through a thin stirring needle filled by three varieties of nanoparticles such as aluminum oxide, copper and titanium dioxide. Recently, Sheremet et al. (2018) perceived reduction in heat transfer rate with the higher values of Marangoni number in his study on natural convection heat transfer of nanofluid over cubical cavity. Ghalambaz et al. (2018) noticed deterioration in fluid flow and heat transfer rate as the values of hybrid nanoparticles volume fraction rises in his study on square porous cavity. Sheikholeslami et al. (2018) analysed the sway of frictional and thermal entropy generation on turbulent nanofluid flow through circular heat exchanger. Hosseinzadeh et al. (2019), Gireesha et al. (2020), Nayak et al. (2020) and Sudarsana Reddy et al. (2020) professed flow, heat and mass transfer characteristics of different nanofluids by considering various geometries and identified that the addition of nanoparticles augments the rate of heat transfer.

After vigilant observation of the literature, we have identified that no studies have been reported to deliberate the heat and mass transfer analysis of carbon nanotubes – water based nanofluid flow over thin vertical needle moving along the free stream direction occupied by single - wall and multi – wall carbon nanotubes by considering convective type boundary condition into the consideration. The resultant equations are solved with the help of Finite element method and the computer program is implemented in Mathematica 10.0 software to find the numerical solution of the considered problem.

2. Mathematical Analysis

Fig. 1 depicts a steady, convective flow of single - wall and multi – wall carbon nanotubes by taking water as base fluid passing through a constantly moving thin vertical needle along the flow direction under the convective boundary condition and non – linear thermal radiation. The mathematical model of the problem is considered in cylindrical coordinates (x, r) in the axial and radial directions. Here, $r = R(x)$ is taken as the needle's radius and (a) is taken as needle's size. As flow of the thin needle is pondered under the convective boundary condition, it is supposed T_w that, calculated later and is the outcome of convective heating procedure and is categorized by a temperature T_f, h_f, coefficient of heat transfer. Volume fraction of nanoparticles at the surface of the needle is ϕ_w. ϕ_∞ and T_∞ are the ambient fluid volume fraction of nanoparticle and temperature, separately. The needle is deliberated thin if its width does not surpass that boundary layer over it. By considering all these prospects, the main equations for the nanofluid model professed by Tiwari and Das (2007) yield the subsequent form:

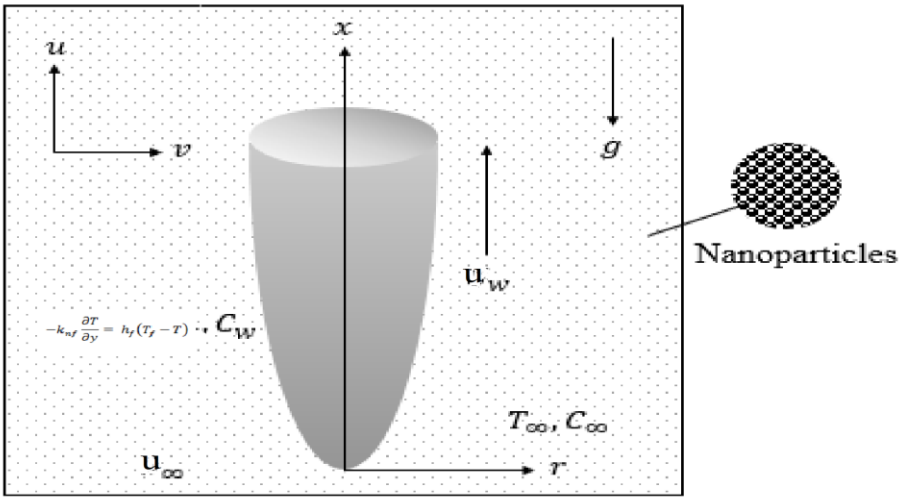

Fig. 1. Schematic diagram of problem

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{nf}}{\rho_{nf}}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{k_{nf}}{(\rho c_p)_{nf}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{16\sigma_{SB}}{3a_R(\rho c_p)_{nf}}\left[T^3\frac{\partial^2 T}{\partial r^2}+3T^2\left(\frac{\partial T}{\partial r}\right)^2\right] \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) \qquad (4)$$

The accompanying boundary conditions are

$$u=u_w,\quad v=V_1(x),\ -k_{nf}\frac{\partial T}{\partial r}=h_f(T_f-T),\qquad C=C_w\ \text{at}\quad r=R(x) \qquad (5)$$

$$u\to u_\infty,\quad T\longrightarrow T_\infty,\qquad C\longrightarrow C_\infty \qquad \text{at}\quad r\to\infty \qquad (6)$$

Here, r and x signifies co – ordinate axis along the surface in the perpendicular direction of the motion and in the direction of the motion, the velocity components along x and r directions are u and v, respectively. The density ρ_{nf}, thermal conductivity k_{nf}, dynamic viscosity μ_{nf}, thermal diffusivity α_{nf}, and heat capacitance $(\rho c_p)_{nf}$ of the nanofluid are specified by:

$$\rho_{nf}=(1-\phi)\rho_f+\rho_{CNT}\,\phi,\qquad k_{nf}=k_f\left(\frac{(1-\phi)+2\phi\left(\frac{k_{CNT}}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2\,k_f}\right)}{(1-\phi)+2\phi\left(\frac{k_f}{k_{CNT}-k_f}\right)\ln\left(\frac{k_{CNT}+k_f}{2\,k_f}\right)}\right),$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\qquad \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\qquad (\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_{CNT},$$

Where, ϕ is the volume fraction of nanoparticles. The thermo physical possession of nanofluid is specified by the subscript nf, properties of the base fluid signifies by f and properties of nanotubes is specified by CNT.

The similarity conversions are taken as:

$$\psi = \vartheta x f(\xi) \qquad \eta = \frac{ur^2}{\vartheta x}\ , \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad S(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Substituting Eqn. (7) into Eqns. (1) – (4), we acquire the subsequent set of ordinary non – linear differential equations,

$$2(\eta f''' + f'') + \left[(1-\phi) + \left(\frac{\rho_s}{\rho_f}\right)\phi\right](1-\phi)^{2.5} f f'' = 0 \tag{8}$$

$$(\eta\theta'' + \theta') + \frac{1}{2}\Pr\frac{A_1}{A_2}\,\theta' f + \frac{1}{A_2} R\,[(\theta_w - 1)^3(3\theta^2(\theta')^2 + \theta^3\theta'' + 3(\theta_w - 1)^2(2\theta(\theta')^2 + \theta^2\theta'') + 3(\theta_w - 1)((\theta')^2 + \theta\,\theta') + \theta''] = 0 \tag{9}$$

$$(\eta\, S'' + S') + \frac{1}{2}\, Sc\ \ S' f = 0 \tag{10}$$

The converted boundary conditions are

$$\eta = 0,\ \ f(a) = \frac{\lambda}{2} a, \quad f'(a) = \frac{\lambda}{2}, \quad \theta'(a) = -\left(\frac{1}{A_2}\right)\xi\,[(1 - \theta(a)],\ \ S(a) = 1.$$

$$\eta \to \infty, \qquad f'(\eta) \to \frac{1-\lambda}{2}\ , \qquad \theta(\eta) \to 0\ , \qquad S(\eta) \to 0. \tag{11}$$

Here, the superscript $'$ represents differentiation with the variable η, and the substantial thermo – physical strictures changing the flow dynamics are defined as

$$\Pr = \frac{\vartheta}{\alpha_f}, \qquad R = \frac{16\sigma_{SB}{T_\infty}^3}{3a_R k_f}, \qquad \lambda = \frac{u_w}{U}\ , \qquad Sc = \frac{\vartheta}{D_m}, \qquad \xi = \frac{-h_f\,\vartheta}{2\,k_f\, r}\ ,$$

$$A_1 = (1-\phi) - \phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right), \quad A_2 = \frac{k_{nf}}{k_f}. \tag{12}$$

The non – dimensional quantities in the present problem are skin – friction coefficient (C_f), local Nusselt number (Nu_x), and local Sherwood number (Sh_x) , which are defined as

$$C_f = \frac{\tau_w}{\rho U_\infty^2}, \qquad Nu_x = \frac{x q_w}{k_f(T_w - T_\infty)}\ \ , \quad Sh_x = \frac{x J_w}{D_m(C_w - C_\infty)} \tag{13}$$

Where, $\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial r}\right)_{r=a} = \mu_{nf}\frac{4cU^2}{\nu_f x} f''(a)$,

$q_w = -k_{nf}\left(\frac{\partial T}{\partial r}\right)_{r=a} = -k_{nf}\frac{2cU(T_w - T_\infty)}{\nu_f x}\theta'(a)$,

$J_w = -D_m\left(\frac{\partial C}{\partial r}\right)_{r=a} = -D_m\frac{2cU(C_w - C_\infty)}{\nu_f x} S'(a)$,

By applying similarity variables (7), we get

${Re_x}^{1/2} C_f = \frac{4a^{1/2}}{(1-\varphi)^{2.5}} f''(a)$, ${Re_x}^{-1/2} Nu_x = -2\, A_2 a^{1/2}\theta'(a)$,

${Re_x}^{-1/2} Sh_x = -2\, a^{\frac{1}{2}}\, S'(a)$.

Where, the local Reynolds number is taken as $Re_x = \frac{Ux}{\nu_f}$.

3. Numerical method

The system of ordinary non - linear differential equations (8) – (10) are highly complex type, and so cannot be solved analytically. The Finite element process (Sudarsana Reddy and Prasada Rao 2012, 2017, 2018, Sreedevi et al. 2017) has been employed to solve the above non – linear equations.

4. Results and Discussion

In this analysis a representative set of both graphical and tabular results for the dimensionless velocity, temperature and concentration, in addition to that, the skin friction coefficient, Nusselt number and Sherwood number for dissimilar values of pertinent parameters of SWCNTs – water and MWCNTs – water based nanofluids through horizontal thin needle under the convective boundary condition are presented in Figs. 2 – 17. We have compared present numerical code results with the results of Ishak et al. (2007) and found excellent agreement which is presented in Table 2.

Figs. 2 – 4 depicted the influence of volume fraction parameter (ϕ) on hydro - dynamic, temperature and concentration outlines of single and multi – walled – water based CNTs nanofluid. It is found that fluid flow velocity diminishes in both SWCNTs − water and MWCNTs – water based nanofluids with rising values of (ϕ), whereas, the temperature and concentration scatterings upsurges as values of (ϕ) are rises and this up surging nature is slightly more in MWCNTs − water nanofluid than SWCNTs − water nanofluid.

The velocity, temperature and concentration sketches are portrayed in Figs. 5 – 7 for diverse values of size of needle (a) in both SWCNTs − water and MWCNTs − water based nanofluids. The velocity sketches depreciate and are more in MWCNTs − water based nanofluid than SWCNTs − water based nanofluid with rising values of (a). However, temperature and concentration sketches intensify as the values of (a) rises and this intensification is higher in MWCNTs − water nanofluid than SWCNTs − water nanofluid.

Fig.8. illustrates the sway of Prandtl parameter (Pr) on thermal boundary layer thickness for both SWCNTs − water and MWCNTs − water based nanofluid and examined that the thickness of thermal boundary layer worsens with improving (Pr) values, further, this worsening in the temperature of the fluid is higher in MWCNTs − water nanofluid than SWCNTs − water nanofluid. The impact of Biot number (ξ) on temperature scatterings is depicted in Fig. 9 and detected depreciation in the temperature of the two nanofluids. This depreciation nature is higher in MWCNTs − water nanofluid than SWCNTs − water nanofluid. It can be professed from the Figs. 10 and 11 that the temperature and concentration scatterings decelerates in the two nanofluids with rising values of Schmidt number (Sc) and this deceleration is higher in MWCNTs − water nanofluid than SWCNTs − water nanofluid.

The effect of non – linear thermal radiation (θ_w) parameter on velocity and temperature outlines for SWCNTs − water and MWCNTs − water based nanofluid are perceived in Figs. 12 – 13. Velocity of the fluid and temperature sketches escalates with greater values of (θ_w). This intensification in the velocity outlines is slightly higher in SWCNTs − water based nanofluid than MWCNTs − water based nanofluid, however, this intensification nature in the temperature sketches is higher in MWCNTs − water based nanofluid than SWCNTs − water based nanofluid.

Figs. 14 and 15 displayed the behaviour of nanofluids on thickness of hydro – dynamic and thermal boundary layer with amplifying values of thermal radiation parameter (R) of SWCNTs − water and MWCNTs − water based nanofluid. It is found that the velocity sketches diminishes with improving values of (R) and this diminishing nature is higher in MWCNTs − water based nanofluid than SWCNTs − water based nanofluid, whereas, the temperature sketches optimizes with upgrading values of (R) and this optimization is more in MWCNTs − water based nanofluid than SWCNTs − water based nanofluid. This is because of the reality that the thermal radiation effect occurrence in fluid flow causes rise in the temperature of the fluid in the entire fluid flow region.

Velocity and temperature sketches for dissimilar values of velocity ratio parameter (λ) are portrayed in Figs. 16 and 17 for both nanofluids. It is examined that velocity sketches diminishes in both nanofluids and this nature is more in $\mathrm{MWCNTs-water}$ based nanofluid than $\mathrm{SWCNTs-water}$ based nanofluid. Nevertheless, the thermal boundary layer thickness elevates in the entire flow regions as (λ) values rises in the two nanofluids and this elevation is more in $\mathrm{MWCNTs-water}$ based nanofluid than $\mathrm{SWCNTs-water}$ based nanofluid.

The dimensionless velocity rates, heat transfer rates and mass transfer rates for two$\mathrm{SWCNTs-water}$ and $\mathrm{MWCNTs-water}$ based nanofluid for dissimilar values of various parameters are computed and revealed in Table 3. It shows the increase in volume fraction parameter (φ), leads to diminish in skin friction coefficient and Sherwood number for both Single and multi – walled – water based nanofluid, whereas, the reduction in heat transfer rates at the boundary is observed in $\mathrm{MWCNTs-water}$ based nanofluid and intensification in $\mathrm{SWCNTs-water}$ based nanofluids. The non – dimensional rate of velocity, heat transfer rates and mass transfer rates for both $\mathrm{SWCNTs-water}$ and $\mathrm{MWCNTs-water}$ based nanofluid worsens with escalating values of R, θ_w, λ, ξ. However, the values of C_f, Nu_x and Sh_x are strengthen for both single and multi – walled – water based nanofluids with intensifying values of (a).

5. Conclusion

This article exhibited the mathematical model for SWCNTs and MWCNTs water based nanofluid along with convective boundary condition. The numerical codes are carried out for both nanofluids and summarized on basis of variation in nanofluids motion and temperature distribution and concentration distributions with in boundary layer and the results are categorized through plots and tables. The important key points of the present study are noticed in the below:

i) The velocity sketches diminishes in both $\mathrm{SWCNTs-water}$ and $\mathrm{MWCNTs-water}$ based nanofluid, whereas the temperature and concentration sketches elaborates with upgrading values (ϕ).
ii) Temperature profiles of both $\mathrm{SWCNTs-water}$ and $\mathrm{MWCNTs-water}$ based nanofluid degenerates as the values of Biot number (ξ) rises.
iii) With improving values of size of the needle parameter (a), the rate of velocity, rate of heat transfer and rate of mass transfer intensifies in both nanofluids.
iv) Both nanofluid velocity and temperature profiles upsurges with boosting values of non – linear radiation parameter (θ_w).
v) The C_f, Nu_x and Sh_x values in the both nanofluids depreciates with the higher values of radiation parameter (R).

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Nomenclature

B_0	Magnetic field strength	Re	Reynolds number
T_∞	Ambient temperature attained	a	Size of needle
T	Fluid temperature (^{0}C)	T_w	Temperature at the cone surface (^{0}C)
C_f	Skin-friction coefficient	ρ_p	Nanoparticle mass density
ρ_f	Density of the base fluid	$(\rho c_p)_{nf}$	Heat capacitance of the nanofluid
$(\rho c_p)_s$	Heat capacitance of nanoparticles	θ_w	Non-linear thermal radiation parameter
f(η)	Dimensionless stream function	$(\rho c_p)_p$	Heat capacitance of the nanofluid
g	Gravitational acceleration	ρ_{nf}	Density of the nanofluid
μ	Fluid viscosity (Kg s/m)	ψ	Stream function (m^2/s)
K^*	Mean absorption coefficient	$(c_p)_f$	Heat capacitance of the base fluid (J/KG K)
σ	Electrical conductivity	a	Constant
k_s	Thermal conductivity of nanoparticle	σ^*	Stephan-Boltzmann constant
M	Magnetic parameter	ε	Pressure parameter
ρ_s	Density of nanoparticle (kg/m^3)	τ_w	Shear stress
Nu_x	Nusselt number	θ(η)	Dimensionless temperature
P	Pressure (Pa)	(x, y)	Cartesian coordinates (m)
Pr	Prandtl number	(u, v)	Velocity components in x- and y-axis (m/s)
q_r	Radiative heat flux	q_w	Wall heat flux
R	Radiation parameter	D_m	Mean fluid concentration
J_w	Wall mass flux	Sc	Schmidth number
λ	Velocity ratio parameter	ξ	Biot number
Greek symbols			
α	Thermal diffusivity of base fluid (m^2/s)	β	Thermal expansion coefficient (1/K)
η	Similarity variable	k_{nf}	Thermal conductivity of nanofluid (W/m K)
μ	Viscosity (Kg s/m)	ν_f	Kinematic viscosity of the base fluid (m^2/s)
φ	Volume fraction of Nanoparticles		
Subscripts			
f	Base fluid	nf	Nanofluid
w	Condition at cone surface	∞	Condition far away from cone surface
CNT	Carbon nanotubes		

Table 1. Thermophysical properties of nanofluids [Afridi et al. (2018)].

Fluid	$\rho\left(\frac{Kg}{m^3}\right)$	$C_p\left(\frac{J}{kgK}\right)$	$k\left(\frac{W}{mK}\right)$	$\sigma(Um)^{-1}$
Pure water	997.1	4179	0.613	0.05

SWCNTs	2600	425	6600	10^6
MWCNTs	1600	796	3000	10^7

Table 2. Comparison of $(f''(a))$ with Ishak et al. (2007) for fixed values of $\phi = 0$, and $\xi = 0$.

Parameter						C_f		Nu_x		Sh_x	
φ	R	θ_w	λ	ξ	a	SWCNT − Water	MWCNT − Water	SWCNT − Water	MWCNT − Water	SWCNT − Water	MWCNT − Water

Parameter	Ishak et al. (2007)	Present Results	Error
a	$f''(a)$	$f''(a)$	
0.01	8.4924	8.4927	0.0003
0.1	1.2888	1.2892	0.0004

0.1	0.1	1.5	-0.1	0.5	0.1	6.9984	7.6995	2.9146	2.9477	1.7040	1.7127
0.4	0.1	1.5	-0.1	0.5	0.1	6.7107	6.7087	2.9432	2.9314	1.7002	1.7002
0.7	0.1	1.5	-0.1	0.5	0.1	6.5083	6.5078	3.0088	2.9252	1.6974	1.6975
0.1	0.01	1.5	-0.1	0.5	0.1	2.8594	2.8594	0.8963	0.8961	1.7824	1.7824
0.1	0.03	1.5	-0.1	0.5	0.1	2.7654	2.7150	0.8629	0.8645	1.7813	1.7739
0.1	0.05	1.5	-0.1	0.5	0.1	2.6524	2.6620	0.8221	0.8154	1.7724	1.7637
0.1	0.1	1.0	-0.1	0.5	0.1	2.8594	2.8594	0.8200	0.8136	1.7824	1.7824
0.1	0.1	1.5	-0.1	0.5	0.1	2.8424	2.7150	0.8157	0.8240	1.7624	1.7739
0.1	0.1	2.0	-0.1	0.5	0.1	2.8104	2.6620	0.7964	0.7881	1.7294	1.7837
0.1	0.1	1.5	-0.7	0.5	0.1	11.329	11.3299	1.6318	1.5844	1.7790	1.7790
0.1	0.1	1.5	-0.5	0.5	0.1	10.937	10.8622	1.6297	1.5822	1.7813	1.7803
0.1	0.1	1.5	-0.3	0.5	0.1	10.544	10.4708	1.6154	1.5622	1.7836	1.7827
0.1	0.1	1.5	-0.1	0.1	0.1	11.918	11.9180	2.8580	2.7086	1.7755	1.7755
0.1	0.1	1.5	-0.1	0.3	0.1	11.908	11.8398	2.0786	2.0438	1.7745	1.7744
0.1	0.1	1.5	-0.1	0.5	0.1	11.818	11.8298	1.6312	1.6344	1.7655	1.7644
0.1	0.1	1.5	-0.1	0.5	0.01	13.429	13.4429	0.8152	0.8539	1.9588	1.9588
0.1	0.1	1.5	-0.1	0.5	0.02	15.200	15.0876	0.9888	0.9829	2.2062	2.2054
0.1	0.1	1.5	-0.1	0.5	0.03	16.453	16.3330	1.1506	1.1046	2.3807	2.3798

Table 3. The values of skin friction coefficient, Nusselt and Sherwood number for different values of governing parameters.

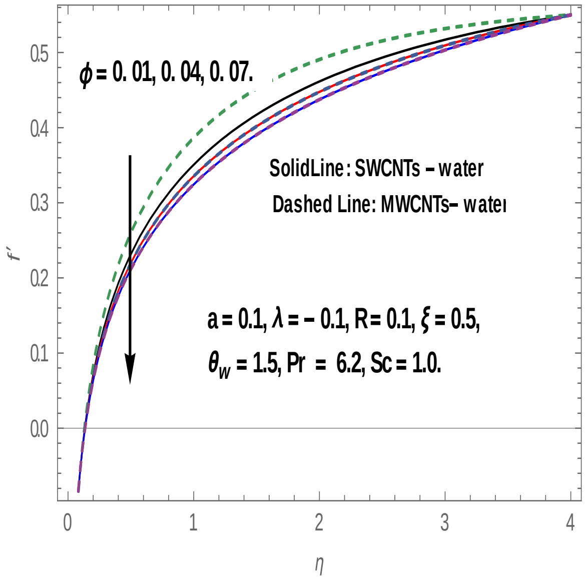

Fig.2. Effect of (φ) on Velocity profiles.

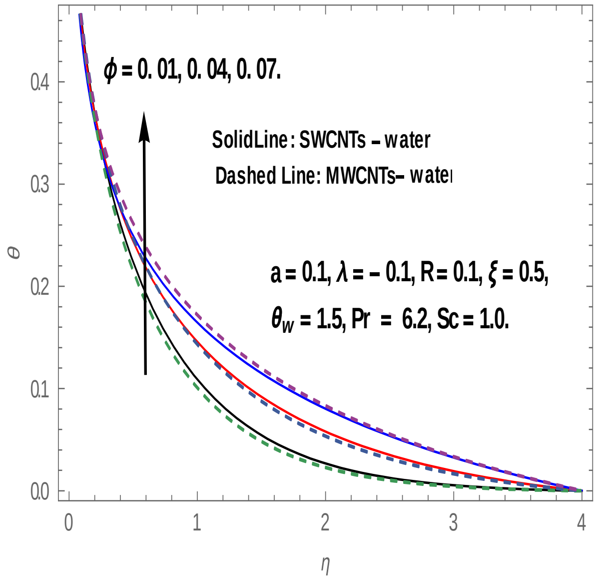

Fig.3. Effect of (φ) on Temperature profiles.

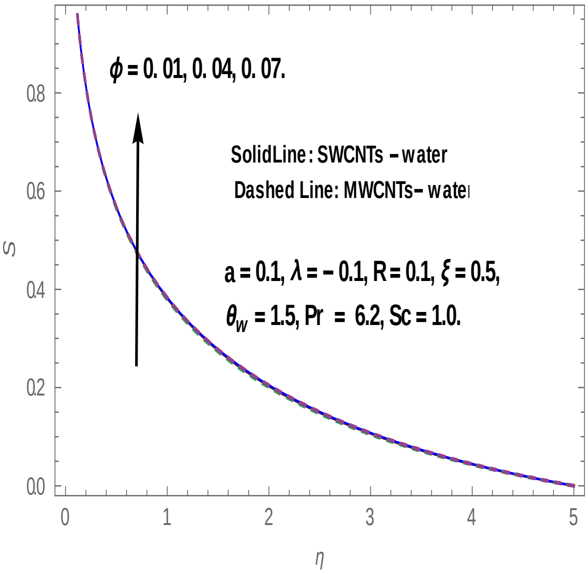

Fig.4 Effect of (φ) on Concentration profiles.

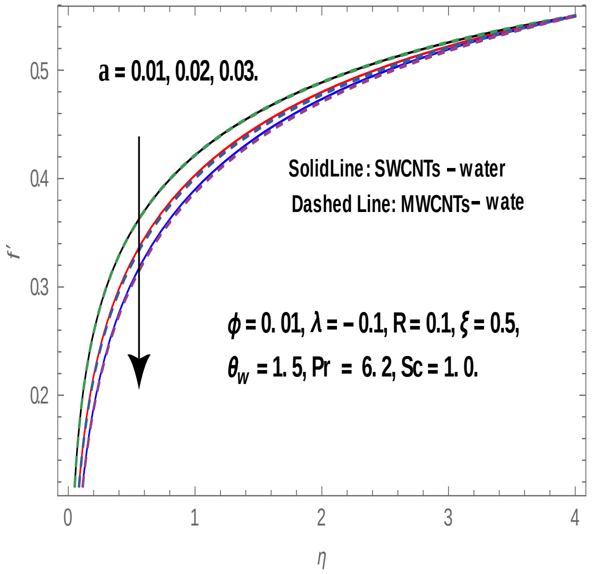

Fig.5. Effect of (a) on Velocity profiles.

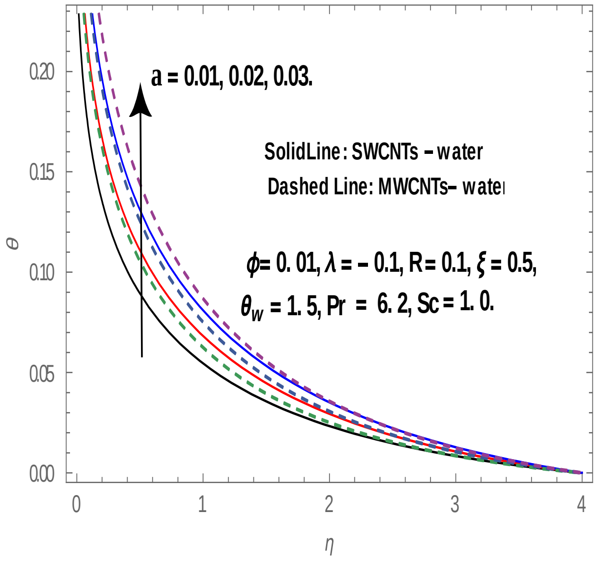

Fig.6. Effect of (a) on Temperature profiles.

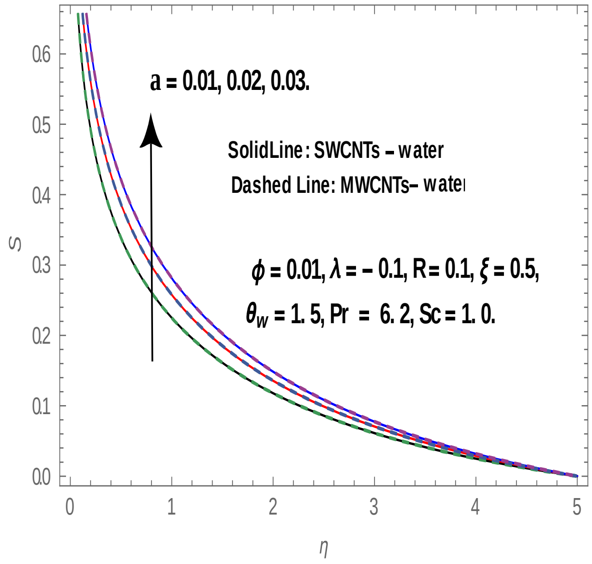

Fig.7. Effect of (a) on Concentration profiles.

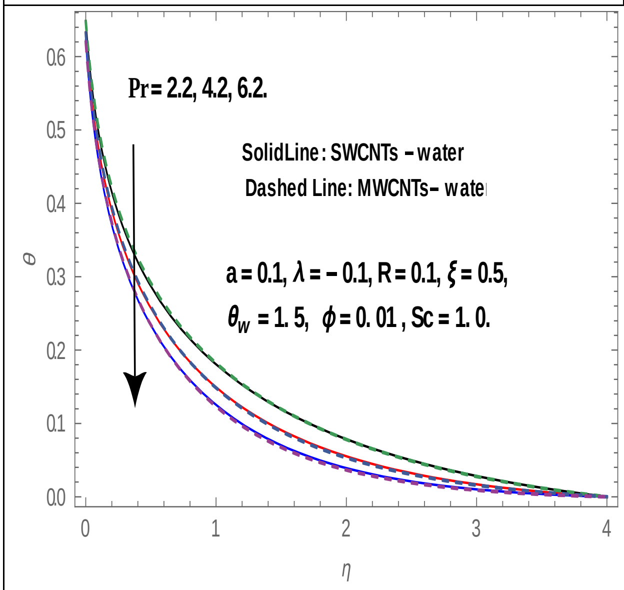

Fig.8. Effect of (Pr) on Temperature profiles.

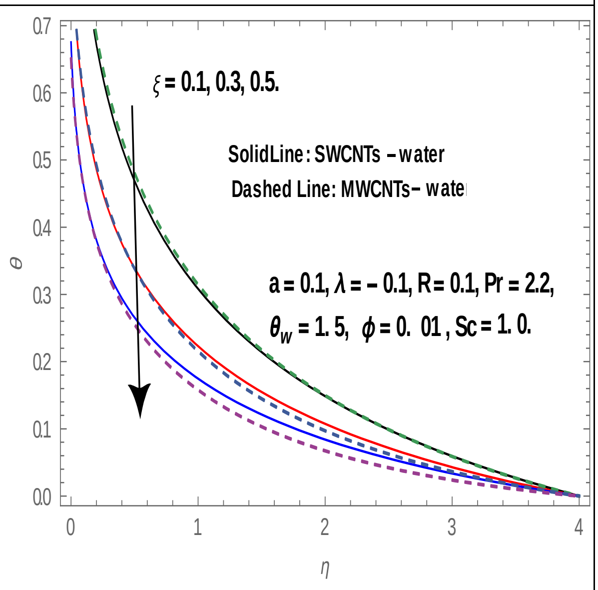

Fig.9. Effect of (ξ) on Temperature profiles.

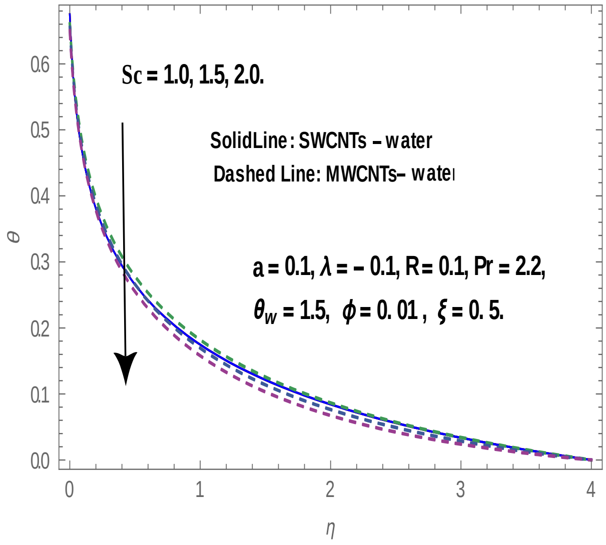

Fig.10. Effect of (Sc) on Temperature profiles.

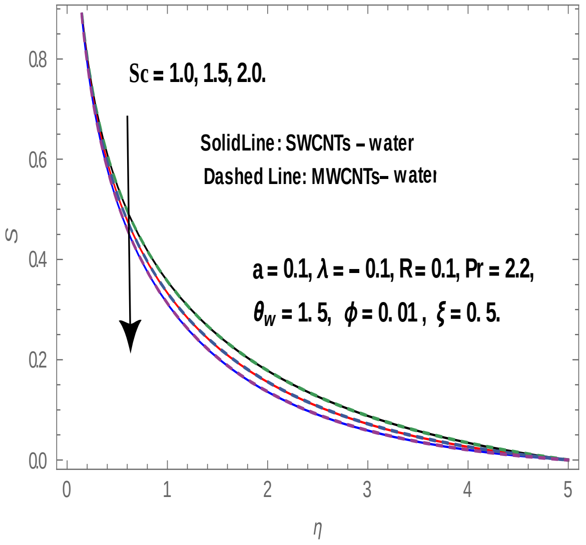

Fig.11. Effect of (Sc) on Concentration profiles.

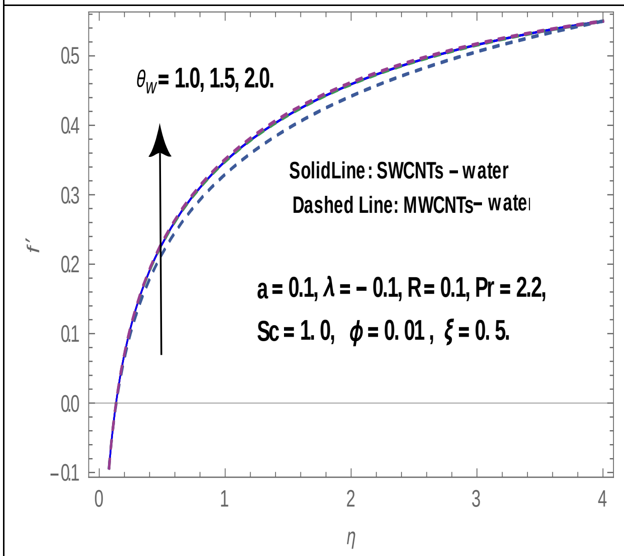

Fig.12. Effect of (θ_w) on velocity profiles.

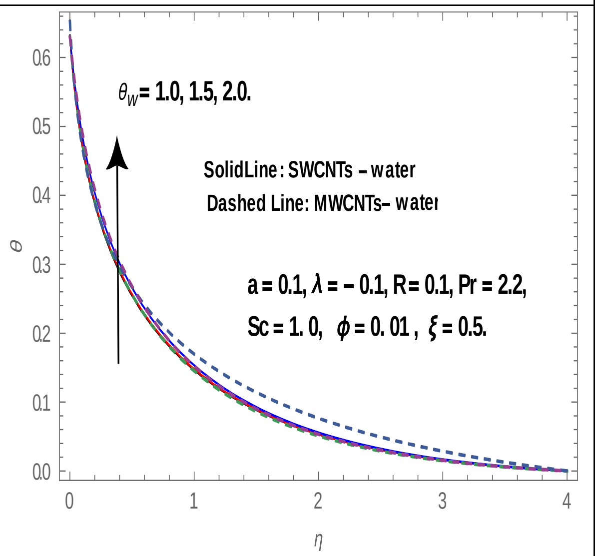

Fig.13. Effect of (θ_w) on Concentration profiles.

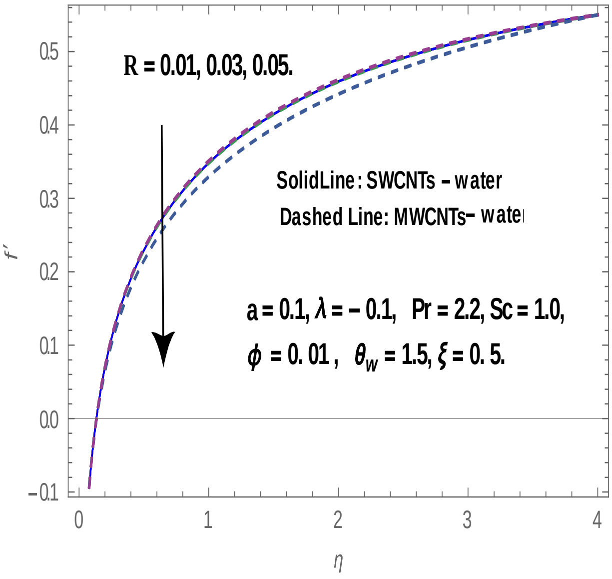

Fig.14. Effect of (R) on Velocity profiles.

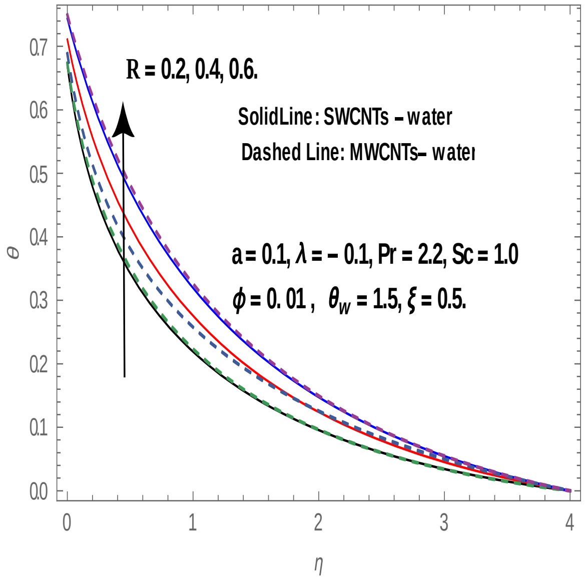

Fig.15. Effect of (R) on Temperature profiles.

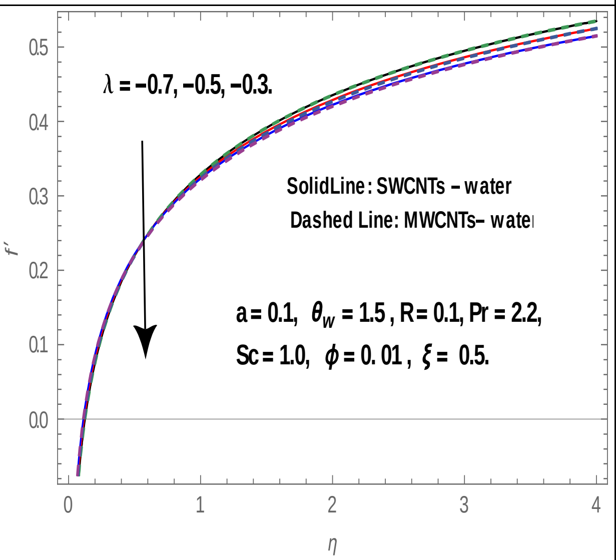

Fig.16. Effect of (λ) on Velocity profiles.

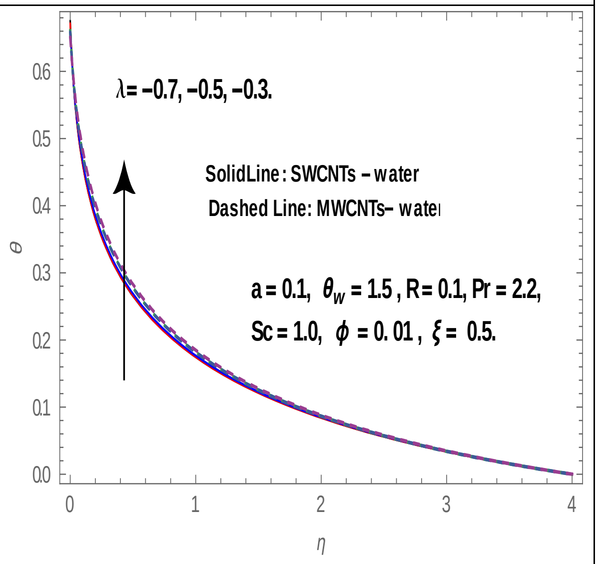

Fig.17. Effect of (λ) on Temperature profiles.