OPTIMAL SUM RATE FOR DIFFERENT PRE-CODING SCHEMES IN VERY LARGE SCALE MIMO CHANNELS

Avinash S Joshi[1]*, Dr. Gopal A. Bidkar[2]

Article History: **Received:** 10-05-2023 **Revised:** 17-05-2023 **Published:** 25-05-2023

Abstract:
Very large scale MIMO is a novel technique that extends network capacities in multi user environment where base stations are provided with large number of antennas helping many single antenna users on the same frequency. In this paper, we evaluate network capacities for two linear precoding schemes, zero forcing (ZF) and minimum mean squared error (MMSE) with respect to number of base station antennas. At 20 base stations these linear precoding schemes reach 98% of the optimal dirty paper coding (DPC) capacity for the considered channels.

Keywords – capacity, correlation, sum rate, MIMO

[1]*Asst. Professsor, Dept. of ECE, KLS's Vishwanathrao Deshpande Rural Institute of Technology, Haliyal, INDIA. Email: avinash2561971@gmail.com
[2]Professor, Dept. of ECE, Shri Dharmastala Manjunatheshwar College of Engineering and Technology, Dharwad, INDIA. Email:gabidkar@rediffmail.com

***Corresponding Author: -** Avinash S Joshi
*Asst. Professsor, Dept. of ECE, KLS's Vishwanathrao Deshpande Rural Institute of Technology, Haliyal, INDIA. Email: avinash2561971@gmail.com

DOI: 10.53555/ecb/2023.12.5.521

I.INTRODUCTION

MULTIPLE antenna (MIMO) technology for wireless communication is becoming ripe and has been integrated into many advanced standards such as HSPA and LTE [1]. Transceivers equipped with more antennas will benefit in terms of data rate and link reliability with the cost of complexity of hardware and processing of signals at both ends. In single user MIMO systems multiplexing gain diminishes with signal power compared to interference and noise, or in propagation environments with prevailing line of sight or insufficient scatters. SU-MIMO systems need composite multiple antenna terminals.

To get over these disadvantages of SU-MIMO, multi-user MIMO (MU-MIMO) with one antenna terminal and an unlimited number of base stations is investigated in [2]. It is depicted that all the impressions of uncorrelated noise and fast fading disappear, as does in intra cell interference, and the only endure deterrent is the inter cell interference due to pilot contamination. All of these prompt entirely new theoretical research on processing of signals, coding and system design for very large scale MIMO systems.

The assumption of infinite antennas in base station [2] greatly unravel the theoretical analysis which is impractical. From a feasibility point of view the number of antenna array depends on the propagation environment but, the asymptotic consequence of random matrix theory can be noticed even for relatively small dimensions.
In this paper, we study the linear precoding performance in very large MIMO downlink channels. We conceive a single cell environment in which a base station with a very large antenna array serves a number of single antenna users simultaneously.
In sec II we report our system model. In sec IV we report a number of precoding schemes –both the linear zero forcing (ZF) and minimum mean squared error (MMSE) pre-coders as well as optimal dirty paper coding and finally draw conclusion in sec V

II.SYSTEM DESCRIPTION

We conceive the downlink of unit cell MU-MIMO system: the base station contains M antennas that serve K one antenna users. The total power transmitted is compelled to average of P_t. The overall received K x 1 vector y at the users can be described as

$$y = \sqrt{\rho} Hz + n$$

Where H is a compound K x M channel matrix, z is the vector that is transmitted across the M antennas and n is the noise vector with variance of 1. The variable ρ has the transmit energy and channel energy. The total power in H is K and Z satisfies $E\{\|z\|^2\} = 1$. The M x 1 transmit vector z contains pre-coded version the K x 1 data symbol vector x. Through precoding at the transmit side we have
Z= Ux
Where U is a M x K precoding matrix including power allocation to the data symbols. The vector x contains the data symbols from an alphabet χ and energy has unit average. Taken together the energy constraint on x and z yield an energy constraint on U:Tr($U^H U$)=1, where Tr(.) is the trace operator and $(.)^H$ denotes the Hermitian transpose.
To ease the derivation of precoders and their performance, we assumed that the number of users is K=2.

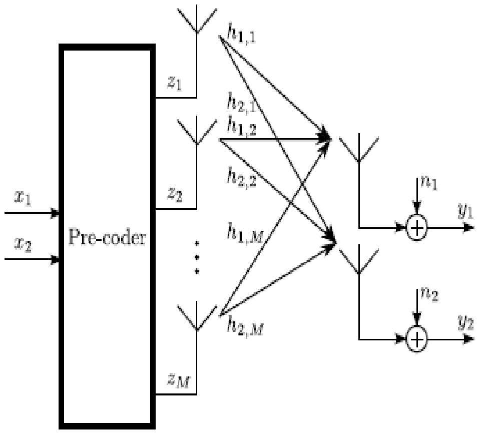

Fig. 1 System model of a MU-MIMO system with M antenna base station and two single antenna users

The input output relation of the channel for this two user case is shown in Fig.1. The Gram matrix associated with H can be expressed as

$$G = HH^H = \begin{pmatrix} 1+g & \delta \\ \delta^* & 1-g \end{pmatrix}$$

Where g gives the power imbalance between the two user channels, and δ is a factor measuring the association between two channels. Since we can permute the rows of H at will, we can assume that 0<g<1. The correlation between the channels to the two users can be expressed as $|\delta|/\sqrt{1-g^2}$.

Further, we require $|\delta| < \sqrt{1-g^2}$ in order having as positive definite matrix G.

I. Pre-Coding Schemes

In this section the derived closed form expressions for DPC capacity and linear precoding sum rates using system model in sec II are presented.

A. Dirty paper coding

The optimum sum rate in the downlink of MU-MIMO system can be attained by the interference pre-subtracting technique called dirty paper coding (DPC). The ideal DPC capacity is

$$C_{DPC} = \begin{cases} \log_2\left[1+\rho+\dfrac{\rho^2(1-g^2-|\delta|^2)^2+4g^2}{4(1-g^2-|\delta|^2)}\right], |\delta|^2 < \delta_{th} \\ \log_2[1+\rho(1+g)], |\delta|^2 > \delta_{th} \end{cases}$$

If $|\delta|^2$ is greater than certain threshold $|\delta|^2$ all power will be allocated to the user with the strongest channel and the DPC capacity becomes same as single user transmission rate

$$C_{DPC} = \log_2(1+\rho(1+g))$$

While ideal sum rate is achieved DPC is too complex to be enforced in practice.

B. Linear Precoding schemes

The precoded matrix U can be written as

$$U = \frac{1}{\sqrt{\gamma}} W \sqrt{P}$$

Where W represents particular linear precoding algorithm, P is the power allocation matrix and γ is used to normalize the total transmit power in z to unity. Therefore, from $Tr(U^H U)=1$, the power normalization factor γ should be

$\gamma = Tr(PW^H W)$

ZF precoding an nihilate the interference by transmitting the signals towards the intended user with nulls in the direction of the other users. The resulting sum rate for ZF precoder is

$$C_{ZF} = \begin{cases} \log_2\left[\dfrac{2+\rho\ (1-g^2-|\delta|^2)^2}{4(1-g^2)}\right], |\delta|^2 < \delta_{th} \\ \log_2\left[1+\dfrac{\rho\ (1-g^2-|\delta|^2)}{1-g}\right], |\delta|^2 > \delta_{th} \end{cases}$$

From above equation we see that the capacity goes to zero when the channel correlation is high.

In MMSE precoding scheme there exists a trade-off between interference suppression against signal power efficiency. The optimal sum rate for MMSE precoder when g=0 is

$$C_{MMSE} = 2\log_2\left[1+\frac{\rho}{2}\left(1-\frac{\rho}{\rho+2}|\delta|^2\right)\right]$$

III.PERFORMANCE COMPARISON

From the sum rate equations above, we see that the DPC capacity and linear precoding sum rates are pompous to correlation and power imbalance between user channels. It is depicted that if the channel correlation approaches to zero i.e. $|\delta|$=0, ZF and MMSE sum rates become equal to DPC capacity. If the channel correlation grows very high, $|\delta|$ approaches $\sqrt{1-g}$ signal power would only be transmitted over the stronger user channel, and the other user would get zero capacity.

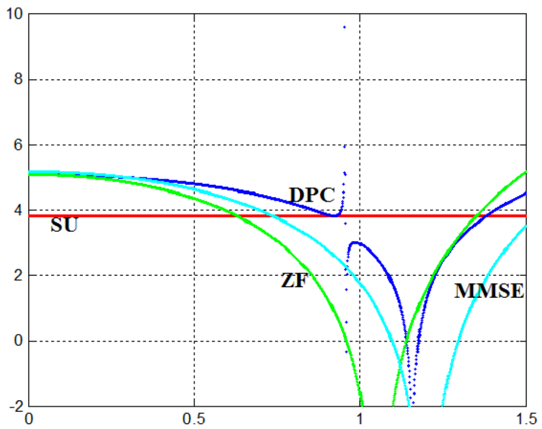

Fig. 2 Sum rates for DPC, ZF, MMSE, single user transmission versus channel correlation factor

Fig 2 shows the DPC capacity , linear precoding sum rate and one user transmission rate as the function of the correlation related factor $|\delta|^2$ for ρ=10dB and g=0.3. We see that the space between DPC capacity and linear precoding sum rates becomes smaller when $|\delta|^2$ decreases. Eventually the linear pre-coding network capacities are the equal to the DPC capacity when $|\delta|^2 = 0$, i.e, when the two user channels are orthogonal. When the channel correlation grows high, ZF capacity diminishes quickly to zero and the DPC capacity diminishes to single-user capacity. We noticed that MMSE sum rate diminishes first and becomes less than the one-user capacity, but then increases after $|\delta|^2$ reaches a certain value. By investigating the power allocation for MMSE, we find the power is only transmitted to the stronger user channel when $|\delta|^2 > 0.7$, hence, as the correlation gets higher, the MMSE pre-coding eventually approaches the single-user transmission. The channel power disparity factor g also has an effect on the capacity. Basically, as g grows, the channel power difference becomes huge and thus the channel correlation $|\delta|/1-g^2$ increases. Consequently, the ZF sum rate diminishes rapidly while the MMSE sum rate and DPC capacity diminish first and then both become equal to single-user capacity. Furthermore, the DPC capacity and linear pre-coding sum rates are less when g is high. Hence, in order to have higher capacity, users with small channel power differences should be served at the same time as claimed by some grouping strategies.

IV.CONCLUSION

In this paper, linear precoding performance is studied for huge MIMO downlink channels. Linear precoding network capacities as high as 98% of DPC capacity was got for two single antenna users. This shows that in realistic propagation environments and less number of antennas we can get benefits using large number of base station antennas.

REFERENCES

[1] E. Dahlman, S. Parkvall, J. Sk¨old, and P. Beming, *3G Evolution HSPA and LTE for Mobile Broadband*. Academic Press, 2008.

[2] T. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3590–3600, Nov 2010.

[3] X. Gao, A. Alayon Glazunov, J. Weng, C. Fang, J. Zhang, and F. Tufvesson, “Channel measurement and characterization of interference between residential femto-cell systems,” in *Proceedings of the 5th European Conference on Antennas and Propagation (EUCAP)*, Rome, Italy, April 2011, pp. 3769–3773.

[4] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. UK: Cambridge University Press, 2003.

[5] C. Peel, B. Hochwald, and A. Swindlehurst, “A vector-perturbation technique for near-capacity multiantenna multiuser communication-part i: channel inversion and regularization,” *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 195–202, Jan 2005.

[6] S. Shi and M. Schubert, “MMSE transmit optimization for multi-user multi-antenna systems,” in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, March 2005, pp. iii/409–iii/412.