Radio Pell Labeling of Graphs

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Abstract

Let G be a connected graph. For any two distinct vertices x and y of G, Define a 1−1 mapping $\emptyset: V(G) \rightarrow N$ such that $d(x,y) + \left(\emptyset(x) + 2\emptyset(y)\right) \geq 1 + dm(G)$, where $dm(G)$ is the diameter of G. The maximum number assigned to any vertex of G is the radio Pell number of $\emptyset$ and it is denoted by $rpn(\emptyset)$. The minimum value of $rpn(\emptyset)$ taken overall radio Pell labelings of G is the radio Pell number of G and it is denoted by $rpn(G)$. In this paper, we investigate the radio Pell number of graphs such as Comb graph $P_n \odot K_1$, Ladder graph L_n, Triangular snake graph T_n, Double Triangular snake graph DT_n, Quadrilateral snake graph Q_n and Double Quadrilateral snake graph DQ_n.

Keywords: Pell labeling, Distance, Diameter, Comb graph, Ladder graph, Triangular graph, Double triangular graph, Quadrilateral graph, Double quadrilateral graph.

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Introduction

In this paper, we consider the graphs are simple, finite and undirected. The vertex set and edge set of G are respectively denoted by V(G) and E(G). Most graph labeling methods trace their origin to one introduced by Rosa[3] in 1967. The Pell numbers are defined by the recurrence relation $P_n = 2P_{n-1} + P_{n-2}, n \geq 2$ where $P_0 = 0$ and $P_1 = 1$. In [4] J.Shiama introduced the concept of radio Pell labeling of graphs. Chartrand et al.[5] introduced the notion of radio labeling of graphs. In this sequel,we introduce the radio Pell labeling of graphs. For standard terminology and notations we follow Harary[1] and Gallian[2].

Definition 1.1. [6] Let G be a graph with p vertices. If there exist a mapping $\emptyset: V(G) \rightarrow \{0,1,2,\dots,P-1\}$ such that the induced function $\emptyset^*: E(G) \rightarrow N$ given by $\emptyset^*(xy) = \emptyset(x) + 2\emptyset(y)$ for every $xy \in E(G)$ are all distinct, where $x, y \geq 0$, then the function $\emptyset$ is called a Pell labeling. A graph which admits Pell labeling is called Pell graph.

Definition 1.2. [7] The distance d(x,y) from a vertex x to a vertex y in a connected graph G is the minimum of the lengths of the $x - y$ paths in G.

Definition 1.3. [7] The diameter $dm(G)$ of G is the greatest eccentricity among the vertices of G.

Definition 1.4. [8] The Comb graph $P_n \odot K_1$ is obtained by joining a single pendant edge to each vertex of a Path P_n.

Definition 1.5. [9] A ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with n vertices and K_2 is a complete graph with two-vertices.

Definition 1.6. [8] A Triangular Snake graph T_n is obtained from a path $x_1, x_2, \dots x_n$ by joining x_i and x_{i+1} to a new vertex y_i for $1 \le i \le n-1$. That is every edge of a path is replaced by a triangle C_3.

Definition 1.7. [10] A Double triangular snake graph DT_n is obtained from a path $x_1, x_2, \dots x_n$ by joining x_i and x_{i+1} to two new vertices y_i and $z_i, 1 \le i \le n-1$. That is, a double triangular snake consists of two triangular snakes that have a common path.

Definition 1.8. [8] A Quadrilateral Snake Q_n is obtained from a path $x_1, x_2, \dots x_n$ by joining x_i and x_{i+1} to two new vertices y_i and z_i respectively and then join y_i and z_i. That is every edge of a path is replaced by a cycle C_4.

Definition 1.9. [10] A Double quadrilateral snake graph DQ_n is obtained from a path $x_1, x_2, \dots x_n$ by joining x_i and x_{i+1} to new vertices $y_i y_i'$ and $z_i z_i'$ respectively and then join $y_i z_i$ and $y_i' z_i'$. That is, a double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path.

1 Main Results

Theorem 2.1. *The radio pell number of a comb graph* $P_n \odot K_1$ *is* $rpn(P_n \odot K_1) = 3n-2, n \ge 2$.

Proof. Let $x_i, 1 \le i \le n$ *be the vertices of a path* P_n *of length n. Join the pendant vertices* y_i *to each* $x_i, 1 \le i \le n$. *The resultant graph is* $P_n \odot K_1$ *whose edge set is* $E(P_n \odot K_1) = \{x_i x_{i+1} / 1 \le i \le n-1\} \cup \{x_i y_i / 1 \le i \le n\}$ *and* $dm(P_n \odot K_1) = n+1$. *Define a function* $\emptyset: V(P_n \odot K_1) \to N$ *by*

$\emptyset(x_i) = n - 2 + i, 1 \le i \le n;$

$\emptyset(y_i) = 2n - 2 + i, 1 \le i \le n.$

Now we verify the radio pell labeling condition $d(x,y) + (\emptyset(x) + 2\emptyset(y)) \ge 1 + dm(P_n \odot K_1)$ *for every pair of vertices of* $P_n \odot K_1$.

case(i) verify the pair $(x_i, y_j), 1 \le i, j \le n$.

$$d((x_i, y_j) + (\emptyset(x_i) + 2\emptyset(y_j)) \ge 1 + (5n + i + 2j - 6) \ge n + 2 = 1 + dm(P_n \odot K_1)$$

case(ii) verify the pair $(x_i, x_j), i \ne j, 1 \le i, j \le n$.

$$d((x_i, x_j) + (\emptyset(x_i) + 2\emptyset(x_j)) \ge 1 + (3n + i + 2j - 6) \ge n + 2$$

case(iii) verify the pair $(y_i, y_j), i \ne j, 1 \le i, j \le n$.

$$d((y_i, y_j) + (\emptyset(y_i) + 2\emptyset(y_j)) \ge 3 + (6n + i + 2j - 6) \ge n + 2$$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ *is a valid radio Pell labeling of* $P_n \odot K_1$. *Therefore,* $rpn(P_n \odot K_1) \le rpn(\emptyset) = 3n - 2$. *Since* $\emptyset$ *is* *injective,* $rpn(\emptyset) \geq 3n-2, n \geq 2$ *for all radio pell labeling* $\emptyset$ *and hence* $rpn(P_n \odot K_1) = 3n-2, n \geq 2$. *Hence the radio pell number of a Comb graph* $P_n \odot K_1$ *is* $rpn(P_n \odot K_1) = 3n-2, n \geq 2$.

Example 2.1. *The radio pell labeling of* $P_5 \odot K_1$ *is in Figure* 2.1

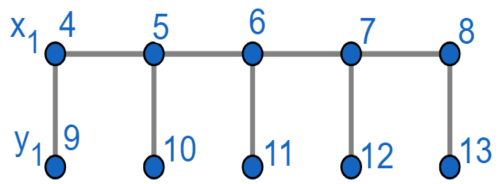

Figure 2.1

Theorem 2.2. *The radio Pell number of a Ladder graph* L_n *is* $rpn(L_n) = 3(n-1), n \geq 3$.

Proof. Let x_i *and* $y_i, 1 \leq i \leq n$ *be the vertices of two paths of length n. Join* x_i *and* $y_i, 1 \leq i \leq n$. *The resultant graph is* L_n *whose edge set is* $E(L_n) = \{x_i x_{i+1}, y_i y_{i+1}, 1 \leq i \leq n-1\} \cup \{x_i y_i / 1 \leq i \leq n\}$ *and* $dm(L_n) = n$. *Define a function* $\emptyset: V(L_n) \rightarrow N$ *by*

$\emptyset(x_i) = n + i - 3, 1 \leq i \leq n;$

$\emptyset(y_i) = 2n - 3 + i, 1 \leq i \leq n.$

Now we verify the radio pell labeling condition $d(x,y) + \big(\emptyset(x) + 2\emptyset(y)\big) \geq 1 + dm(L_n)$ *for every pair of vertices of* L_n.

case(i) verify the pair $(x_i, y_j), 1 \leq i, j \leq n$.

$$d\big((x_i, y_j\big) + \Big(\emptyset(x_i) + 2\emptyset\big(y_j\big)\Big) \geq 1 + (5n + i + 2j - 9) \geq n + 1 = 1 + dm(L_n)$$

case(ii) verify the pair $\big(x_i, x_j\big), i \neq j, 1 \leq i, j \leq n$.

$$d\big((x_i, x_j\big) + \Big(\emptyset(x_i) + 2\emptyset\big(x_j\big)\Big) \geq 1 + (3n + i + 2j - 9) \geq n + 1$$

case(iii) verify the pair $\big(y_i, y_j\big), i \neq j, 1 \leq i, j \leq n$.

$$d\big((y_i, y_j\big) + \Big(\emptyset(y_i) + 2\emptyset\big(y_j\big)\Big) \geq 1 + (6n + i + 2j - 9) \geq n + 1$$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ *is a valid radio Pell labeling of* L_n. *Therefore,* $rpn(L_n) \leq rpn(\emptyset) = 3(n-1)$. *Since* $\emptyset$ *is injective,* $rpn(\emptyset) \geq 3(n-1), n \geq 3$ *for all radio pell labeling* $\emptyset$ *and hence* $rpn(L_n) = 3(n-1), n \geq 3$. *Hence the radio pell number of a Ladder graph* L_n *is* $rpn(L_n) = 3(n-1), n \geq 3$.

Example 2.2. *The radio pell labeling of* L_5 *is in Figure* 2.2

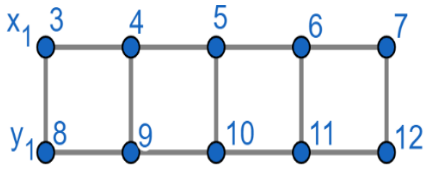

Figure 2.2

Theorem 2.3. *The radio pell number of a triangular snake graph T_n is $rpn(T_n) = 3n - 4, n \geq 3$.*

Proof. Let $x_i, 1 \leq i \leq n$ be the vertices of a path P_n. Join x_i and x_{i+1} to a new vertex $y_i, 1 \leq i \leq n - 1$. The resultant graph is T_n whose edge set is $E(T_n) = \{x_i y_i, y_i x_{i+1}, x_i x_{i+1} / 1 \leq i \leq n-1\}$ and $dm(T_n) = n - 1$. Define a function $\emptyset: V(T_n) \to N$ by

$\emptyset(x_i) = n + i - 3, 1 \leq i \leq n;$

$\emptyset(y_i) = 2n - 3 + i, 1 \leq i \leq n - 1.$

Now we verify the radio pell labeling condition $d(x, y) + \left(\emptyset(x) + 2\emptyset(y)\right) \geq 1 + dm(T_n)$ for every pair of vertices of T_n.

case(i) verify the pair $(x_i, y_j), 1 \leq i, j \leq n$.

$$d\left((x_i, y_j)\right) + \left(\emptyset(x_i) + 2\emptyset(y_j)\right) \geq 1 + (5n + i + 2j - 9) \geq n = 1 + dm(T_n)$$

case(ii) verify the pair $\left(x_i, x_j\right), i \neq j, 1 \leq i, j \leq n$.

$$d\left((x_i, x_j)\right) + \left(\emptyset(x_i) + 2\emptyset(x_j)\right) \geq 1 + (3n + i + 2j - 9) \geq n$$

case(iii) verify the pair $\left(y_i, y_j\right), i \neq j, 1 \leq i, j \leq n$.

$$d\left((y_i, y_j)\right) + \left(\emptyset(y_i) + 2\emptyset(y_j)\right) \geq 2 + (6n + i + 2j - 9) \geq n$$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ is a valid radio Pell labeling of T_n. Therefore, $rpn(T_n) \leq rpn(\emptyset) = 3n - 4$. Since $\emptyset$ is injective, $rpn(\emptyset) \geq 3n - 4, n \geq 3$ for all radio pell labeling $\emptyset$ and hence $rpn(T_n) = 3n - 4, n \geq 3$. Hence the radio pell number of a Triangular snake graph T_n is $rpn(T_n) = 3n - 4, n \geq 3$.

Example 2.3. *The radio pell labeling of T_6 is in Figure* 2.3

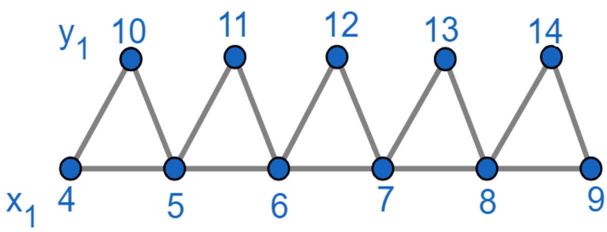

Figure 2.3

Theorem 2.4. *The radio Pell number of a Double Triangular snake graph* DT_n *is* $rpn(DT_n) = 4n - 5, n \geq 3$.

Proof. Let $x_i, 1 \leq i \leq n$ *be the vertices of a path* P_n. *Join* x_i *and* x_{i+1} *to the new vertices* y_i *and* $z_i, 1 \leq i \leq n-1$. *The resultant graph is* DT_n *whose edge set is* $E(DT_n) = \{x_iy_i, x_iz_i, x_ix_{i+1}, y_ix_{i+1}, z_ix_{i+1}/1 \leq i \leq n-1\}$ *and* $dm(DT_n) = n-1$. *Define a function* $\emptyset: V(DT_n) \rightarrow N$ *by*

$\emptyset(x_i) = 3n + i - 5, 1 \leq i \leq n;$

$\emptyset(y_i) = n - 3 + i, 1 \leq i \leq n-1;$

$\emptyset(z_i) = 2n - 4 + i, 1 \leq i \leq n-1.$

Now we verify the radio pell labeling condition $d(x,y) + \big(\emptyset(x) + 2\emptyset(y)\big) \geq 1 + dm(DT_n)$ *for every pair of vertices of* DT_n.

case(i) verify the pair $(x_i, y_j), 1 \leq i, j \leq n$.

$$d\big((x_i, y_j\big) + \Big(\emptyset(x_i) + 2\emptyset\big(y_j\big)\Big) \geq 1 + (5n + i + 2j - 11) \geq n = 1 + dm(DT_n)$$

case(ii) verify the pair $(y_i, z_j), 1 \leq i, j \leq n$.

$$d\big((y_i, z_j\big) + \Big(\emptyset(y_i) + 2\emptyset\big(z_j\big)\Big) \geq 2 + (5n + i + 2j - 11) \geq n$$

case(iii) verify the pair $(x_i, z_j), 1 \leq i, j \leq n$.

$$d\big((x_i, z_j\big) + \Big(\emptyset(x_i) + 2\emptyset\big(z_j\big)\Big) \geq 1 + (7n + i + 2j - 13) \geq n$$

case(iv) verify the pair $\big(x_i, x_j\big), i \neq j, 1 \leq i, j \leq n$.

$$d\big((x_i, x_j\big) + \Big(\emptyset(x_i) + 2\emptyset\big(x_j\big)\Big) \geq 1 + (9n + i + 2j - 15) \geq n$$

case(v) verify the pair $\big(y_i, y_j\big), i \neq j, 1 \leq i, j \leq n$.

$$d\big((y_i, y_j\big) + \Big(\emptyset(y_i) + 2\emptyset\big(y_j\big)\Big) \geq 2 + (3n + i + 2j - 9) \geq n$$

case(vi) verify the pair $\big(z_i, z_j\big), i \neq j, 1 \leq i, j \leq n$.

$$d\big((z_i, z_j\big) + \Big(\emptyset(z_i) + 2\emptyset\big(z_j\big)\Big) \geq 2 + (6n + i + 2j - 12) \geq n$$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ *is a valid radio Pell labeling of* DT_n. *Therefore,* $rpn(DT_n) \leq rpn(\emptyset) = 4n - 5$. *Since* $\emptyset$ *is injective,* $rpn(\emptyset) \geq 4n - 5, n \geq 3$ *for all radio pell labeling* $\emptyset$ *and hence* $rpn(DT_n) = 4n - 5, n \geq 3$. *Hence the radio pell number of a Double Triangular snake graph* $D\,T_n$ *is* $rpn(DT_n) = 4n - 5, n \geq 3$.

Example 2.4. *The radio pell labeling of* DT_5 *is in Figure* 2.4

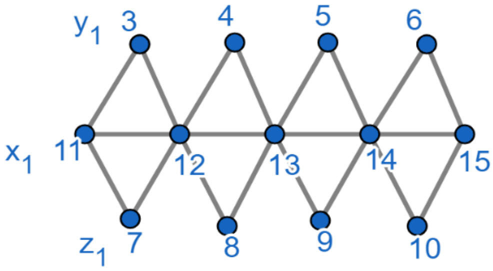

Figure 2.4

Theorem 2.5. *The radio pell number of a Quadrilateral snake graph Q_n is* $rpn(Q_n) = 4n - 3, n \geq 3$.

Proof. Let $x_i, 1 \leq i \leq n$ be the vertices of a path. Let y_i and $z_i, 1 \leq i \leq n-1$ be the two new vertices. Join $y_i z_i, x_i y_i$ and $z_i x_{i+1}, 1 \leq i \leq n-1$. The resultant graph is Q_n whose edge set is $E(Q_n) = \{x_i y_i, y_i z_i, z_i x_{i+1}, x_i x_{i+1} / 1 \leq i \leq n-1\}$ and $dm(Q_n) = n+1$. Define a function $\emptyset: V(Q_n) \to N$ by

$\emptyset(x_i) = 3n + i - 3, 1 \leq i \leq n;$

$\emptyset(y_i) = n - 1 + i, 1 \leq i \leq n-1;$

$\emptyset(z_i) = 2n - 2 + i, 1 \leq i \leq n-1.$

Now we verify the radio pell labeling condition $d(x,y)+(\phi(x)+2\phi(y)) \geq 1+dm(Q_n)$ for every pair of vertices of Q_n.

case(i) verify the pair $(x_i, y_j), 1 \leq i, j \leq n$.

$$d((x_i, y_j) + (\emptyset(x_i) + 2\emptyset(y_j)) \geq 1 + (5n + i + 2j - 5) \geq n + 2 = 1 + dm(Q_n)$$

case(ii) verify the pair $(y_i, z_j), 1 \leq i, j \leq n$.

$$d((y_i, z_j) + (\emptyset(y_i) + 2\emptyset(z_j)) \geq 1 + (5n + i + 2j - 5) \geq n + 2$$

case(iii) verify the pair $(x_i, z_j), 1 \leq i, j \leq n$.

$$d((x_i, z_j) + (\emptyset(x_i) + 2\emptyset(z_j)) \geq 2 + (7n + i + 2j - 7) \geq n + 2$$

case(iv) verify the pair $(x_i, x_j), i \neq j, 1 \leq i, j \leq n$.

$$d((x_i, x_j) + (\emptyset(x_i) + 2\emptyset(x_j)) \geq 1 + (9n + i + 2j - 9) \geq n + 2$$

case(v) verify the pair $(y_i, y_j), i \neq j, 1 \leq i, j \leq n$.

$$d((y_i, y_j) + (\emptyset(y_i) + 2\emptyset(y_j)) \geq 3 + (3n + i + 2j - 3) \geq n + 2$$

case(vi) verify the pair $(z_i, z_j), i \neq j, 1 \leq i, j \leq n$.

$$d((z_i, z_j) + (\emptyset(z_i) + 2\emptyset(z_j)) \geq 3 + (6n + i + 2j - 6) \geq n + 2$$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ is a valid radio Pell labeling of Q_n. Therefore, $rpn(Q_n) \leq rpn(\emptyset) = 4n - 3$. Since $\emptyset$ is injective, $rpn(\emptyset) \geq 4n - 3, n \geq 3$ for all radio pell labeling $\emptyset$ and hence $rpn(Q_n) = 4n - 3, n \geq 3$. Hence the radio pell number of a Quadrilateral snake graph Q_n is $rpn(Q_n) = 4n - 3, n \geq 3$.

Example 2.5. *The radio pell labeling of Q_5 is in Figure 2.5*

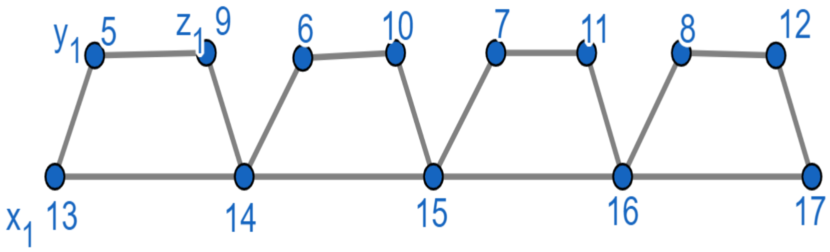

Figure 2.5

Theorem 2.6. *The radio pell number of a double quadrilateral snake graph DQ_n is $rpn(DQ_n) = 6n - 7, n \geq 3$.*

Proof. *Let $x_i, 1 \leq i \leq n$ be the vertices of a path. For $1 \leq i \leq n - 1$, add vertices y_i and y_i' and join them with x_i. Also for $1 \leq i \leq n - 1$, add vertices z_i and z_i' and join them with x_{i+1}. Now join $y_i z_i$ and $y_i' z_i'$. The resultant graph is DQ_n whose edge set is $E(DQ_n) = \{x_i y_i, y_i z_i, z_i x_{i+1}, z_i' x_{i+1}, y_i' z_i', x_i y_i', x_i x_{i+1} / 1 \leq i \leq n - 1\}$ and $dm(DQ_n) = n + 1$. Define a function $\emptyset: V(DQ_n) \to N$ by*

$\emptyset(y_i) = n + i - 3, 1 \leq i \leq n - 1;$

$\emptyset(z_i) = 2n - 4 + i, 1 \leq i \leq n - 1;$

$\emptyset(y_i') = 3n - 5 + i, 1 \leq i \leq n - 1;$

$\emptyset(z_i') = 4n - 6 + i, 1 \leq i \leq n - 1$;

$\emptyset(x_i) = 5n - 7 + i, 1 \leq i \leq n.$

Now we verify the radio pell labeling condition $d(x,y) + (\phi(x) + 2\phi(y)) \geq 1 + dm(DQ_n)$ for every pair of vertices of DQ_n.

case(i) verify the pair $(y_i, z_j), 1 \leq i, j \leq n$.

$$d((y_i, z_j)) + (\emptyset(y_i) + 2\emptyset(z_j)) \geq 1 + (5n + i + 2j - 11) \geq n + 2 = 1 + dm(DQ_n)$$

case(ii) verify the pair $(y_i, y_j'), 1 \leq i, j \leq n$.

$d((y_i, y_j') + (\emptyset(y_i) + 2\emptyset(y_j')) \geq 2 + (7n + i + 2j - 13) \geq n + 2$

case(iii) verify the pair $(z_i, z_j), i \neq j, 1 \leq i, j \leq n.$

$d((z_i, z_j) + (\emptyset(z_i) + 2\emptyset(z_j)) \geq 3 + (6n + i + 2j - 12) \geq n + 2$

case(iv) verify the pair $(y_i, y_j), i \neq j, 1 \leq i, j \leq n.$

$d((y_i, y_j) + (\emptyset(y_i) + 2\emptyset(y_j)) \geq 3 + (3n + i + 2j - 9) \geq n + 2$

Case(v) verify the pair $(y_i, z_j'), 1 \leq i, j \leq n.$

$d((y_i, z_j') + (\emptyset(y_i) + 2\emptyset(z_j')) \geq 3 + (9n + i + 2j - 15) \geq n + 2$

case(vi) verify the pair $(z_i, x_j), 1 \leq i, j \leq n.$

$d((z_i, x_j) + (\emptyset(z_i) + 2\emptyset(x_j)) \geq 2 + (12n + i + 2j - 18) \geq n + 2$

case(vii) verify the pair $(y_i, x_j), 1 \leq i, j \leq n.$

$d((y_i, x_j) + (\emptyset(y_i) + 2\emptyset(x_j)) \geq 1 + (11n + i + 2j - 17) \geq n + 2$

Case(viii) verify the pair $(z_i, y_j'), 1 \leq i, j \leq n.$

$d((z_i, y_j') + (\emptyset(z_i) + 2\emptyset(y_j')) \geq 3 + (8n + i + 2j - 14) \geq n + 2$

case(ix) verify the pair ($z_i', x_j), 1 \leq i, j \leq n.$

$d(z_i', x_j) + (\emptyset(z_i') + 2\emptyset(x_j)) \geq 2 + (14n + i + 2j - 20) \geq n + 2$

Case(x) verify the pair $(z_i, z_j'), 1 \leq i, j \leq n.$

$d((z_i, z_j') + (\emptyset(z_i) + 2\emptyset(z_j')) \geq 2 + (10n + i + 2j - 16) \geq n + 2$

case(xi) verify the pair ($y_i', x_j), 1 \leq i, j \leq n.$

$d(y_i', x_j) + (\emptyset(y_i') + 2\emptyset(x_j)) \geq 1 + (13n + i + 2j - 19) \geq n + 2$

case(xii) verify the pair $(x_i, x_j), i \neq j, 1 \leq i, j \leq n.$

$d((x_i, x_j) + (\emptyset(x_i) + 2\emptyset(x_j)) \geq 1 + (15n + i + 2j - 21) \geq n + 2$

case(xiii) verify the pair ($y_i', y_j'), i \neq j, 1 \leq i, j \leq n.$

$d(y_i', y_j') + (\emptyset(y_i') + 2\emptyset(y_j')) \geq 3 + (9n + i + 2j - 15) \geq n + 2$

case(xix) verify the pair ($z_i', z_j'), i \neq j, 1 \leq i, j \leq n.$

$d(z_i', z_j') + (\emptyset(z_i') + 2\emptyset(z_j')) \geq 3 + (12n + i + 2j - 18) \geq n + 2$

case(xx) verify the pair ($y_i', z_j')$,$1 \leq i, j \leq n.$

$d(y_i', z_j') + (\emptyset(y_i') + 2\emptyset(z_j')) \geq 1 + (11n + i + 2j - 17) \geq n + 2$

Thus the radio pell mean condition is satisfied for all pairs of vertices. Hence, $\emptyset$ *is a valid radio Pell labeling of* DQ_n. *Therefore,* $rpn(DQ_n) \leq rpn(\emptyset) = 6n - 7$. *Since* $\emptyset$ *is injective,* $rpn(\emptyset) \geq 6n - 7, n \geq 3$ *for all radio pell labeling* $\emptyset$ *and hence* $rpn(DQ_n) =$

$6n-7, n \geq 3$. *Hence the radio pell number of a Double Quadrilateral snake graph* DQ_n *is* $rpn(DQ_n) = 6n-7, n \geq 3$.

Example 2.6. *The radio pell labeling of* DQ_5 *is in Figure* 2.6

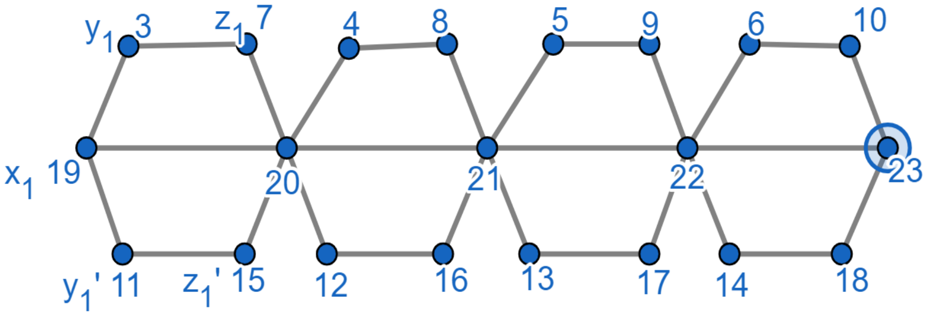

Figure 2.6

2 conclusion

In this paper, we investigate radio pell number of graphs such as Comb graph $P_n \odot K_1$, Ladder graph L_n, Triangular snake graph T_n, Double Triangular snake graph DT_n, Quadrilateral snake graph Q_n and Double Quadrilateral snake graph DQ_n.

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