DAMPED AND FORCED SIMPLE HARMONIC MOTION AND ITS APPLICATIONS

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Abstract

Damped simple harmonic motion (SHM) refers to the oscillatory behavior of a system in which a restoring force proportional to the displacement acts against a damping force that opposes the motion. This abstract explores the characteristics of damped SHM and its applications across various fields. We developed into the mathematical representation of damped SHM, discussing factors affecting the damping coefficient and its impact on the motion's amplitude and frequency. Additionally, we explore practical applications of damped and forced SHM. The abstract concludes by highlighting the significance of understanding damped SHM's principles in optimizing designs and enhancing system performance across different industries.

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INTRODUCTION

Damped Simple Harmonic Motion (DSHM) is a fundamental concept in physics and engineering that describes the behavior of systems oscillating under the influence of a restoring force and a damping force. Unlike ideal Simple Harmonic Motion (SHM), where a system oscillates indefinitely with constant amplitude and frequency, DSHM takes into account the effects of friction or resistance, which gradually dampen the motion over time. This phenomenon is encountered in various real-world scenarios, from mechanical systems to electrical circuits and even biological processes. In this discussion, we will explore the key characteristics of damped SHM, including the mathematical representation of the motion and the factors influencing its behavior. Moreover, we will delve into the practical applications of DSHM across different domains. From engineering designs that require controlled motion and vibration damping to understanding the behavior of mechanical systems subjected to external forces, the applications of DSHM are both diverse and essential for optimizing system performance. By comprehending the principles of DSHM and its applications, engineers, physicists, and researchers can design more efficient and stable systems, mitigate undesirable vibrations, and enhance the overall functionality of various technological advancements. Through a detailed exploration of DSHM and its real-world significance, we can gain valuable insights into how this concept shapes the world around us.

MATHEMATICAL FORMULATION

A) Damped Simple Harmonic Motion:

The oscillation in which amplitude decrease with the time-damped simple harmonic motion. An example of Damped Simple Harmonic Motion is a simple pendulum

Consider a block of mass M connected to the elastic ring of spring constant c, if we push the block a little and then release it, its angular frequency of oscillation is angular frequency $\omega=\sqrt{\frac{c}{M}}$. On the block an external force will exert a damping force which will reduce the mechanical energy of the block-string system. The energy lost will appear as the heat of the surrounding medium. On the nature of the surrounding medium the damping force depends. When a block is immersed in a liquid, the magnitude of damping increases, and the dissipation energy becomes faster. Thus, the damping force is directly proportional to the velocity and acts opposite to the velocity. If the damping force is f_d, we have,

f_d = - a V

Where the constant b depends on the properties of the medium and the size and shape of the block.

Let's assume O is the position where the block settles after release. Now, if the block is pushed little, the restoring force on the block is f_s = -c x, where x is the displacement of the mass position. Therefore, the force acting on the mass at any time t is, f = -cx-aV

Now, if a (t) is the acceleration of mass M at time t, then by Newton's Law of Motion along the direction of motion, we have ma(t) = -cx(t)–bV(t)

Here, we does not consider the vector notation because we only consider the one-dimensional motion. Therefore, using derivatives we have,

$$m\left(\frac{d^2x}{dt^2}\right) + b\left(\frac{dx}{dy}\right) + \text{cx} = 0$$

This equation describes the motion under the influence of a damping force. Therefore, the above expression is a damped simple harmonic motion expression. The solution is of the form

$$x(t) = Amp^{e^{\frac{-at}{2M}}}\cos(\acute{\omega}\text{t} + \emptyset)$$

Where, Amp is the amplitude and ώ is the angular frequency which is given by,

$$\acute{\omega} = [\sqrt{\frac{c}{M} - \frac{a^2}{4M^2}}]$$

The function x (t) is not strictly periodic because the factor decreases continuously with time. However, if the decrease is small in period T, then the motion is approximately periodic. The amplitude is not constant in a damped oscillator. For small damping, the same expression can be used but by taking amplitude as A

$$E(t) = \frac{1}{2}cAmpe^{-\frac{at}{2M}}$$

This equation shows damping decreases with time.

B) Forced simple harmonic motion:

When a pendulum is displaced from its equilibrium position, it moves in to and fro direction about its mean position. And then this motion dies out due to the opposing force present in the medium. So when this pendulum is forced to oscillate, this can be called forced simple harmonic motion. ω is natural frequency. A child uses his legs to move the swing is an example. So here the external force is applied to maintain oscillations.

An external force is considered here. i.e. f (t) of amplitude f(t), that changes periodically with time.

Thus, the force is applied to a damped oscillator. Therefore this can be represented as,
$f(t) = f_0 \cos\omega_d t$

The forces here are acting on the oscillator are its restoring force, and the external force and a time-dependent driving force become,
ma (t) = -cx(t)-aV(t) + $f_0 \cos\omega_d t$

We know that acceleration=$\frac{d^2x}{dt^2}$. Substitute the value of acceleration in equation II, we get,

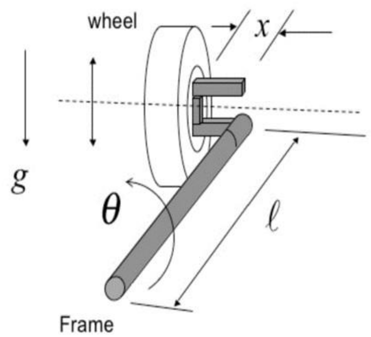
$$M\left(\frac{d^2x}{dt^2}\right) + a\left(\frac{dx}{dy}\right) + cx = f_0 \cos\omega_d t$$

The III equation, its an equation of an oscillator of mass m on which a periodic force of frequency ωd is applied. As we already know that the oscillator will first oscillate with its natural frequency. After applying the external periodic force, the oscillations die out with the natural frequency. So the changed equation after the natural frequency dies out is given as:
x(t) = Amp $\cos(\omega_d + \emptyset)$

where t is the time from when we started applying external force. The Oscillation in which amplitude decrease with time-damped simple harmonic motion

PROBLEM-1
The front suspension of a car contains a torsion rod as in the figure to improve the car's handling.

a) Compute the frequency of vibration of the wheel assembly given that the torsional stiffness is 2000 N m/rad and the mass of wheel assembly is 38 kg and the distance x = 0.26 m.
b) Sometimes owners put different wheels and tires on a car to enhance the appearance or performance. If a thinner tire is put on with a larger wheel raising the mass of 45 kg. What effect does this have on the frequency?



SOLUTION:
a)Ignoring the moment of inertial of the rod, and computing the moment of inertia of the wheel as mx2, the frequency of the shaft mass system is

$$\omega_n = \sqrt{\frac{c}{Mx^2}}$$

$$= \sqrt{\frac{2000N.m}{38\,kg(0.26m)^2}}$$

$$= \sqrt{\frac{2000}{2.5688}}$$

$$= \sqrt{778.573}$$

= 27.9 rad/s

b) The same calculation with 45 kg will reduce the frequency to

$$\omega_n = \sqrt{\frac{k}{mx^2}}$$

$$= \sqrt{\frac{2000N.m}{45kg(0.26m)^2}}$$

$$= \sqrt{\frac{2000}{3.042}}$$

$$= \sqrt{657.46}$$

= 25.6 rad/s

ANSWER:
This corresponds to about an 8% change in unsparing frequency and could influence wheel hop.

PROBLEM-2
You are riding in an automobile of mass 3000kg. Assuming that you are examining the oscillation characteristics of its suspension sags 15cm when the entire automobile is placed on it. Also, the amplitude of the oscillation decreases by 50 % during one complete oscillation. Estimate the value of the spring constant c.

SOLUTION:
Mass of the automobile = 3000 kg

The suspension sags by 15cm
Decrease in amplitude = 50% during one complete oscillation
Let k be the spring constant of each spring then the spring constant of the four springs in parallel is c = 4c since,
F = 4cx
F = mg
mg = 4 cx
c $=\frac{mg}{4x}$
$=\frac{30000}{0.6}$
$=50{,}000$
$=5 \times 10^4$

ANSWER:
The spring constant k is 5 × 104 N

PROBLEM-3
A harmonic oscillator consisting of 50gm mass attached to a massless spring has a quality factor of 200. If it oscillates with an amplitude of 2cm in resonance with a periodic force of frequency 20Hz. Calculate the average energy stored in it.

SOLUTION:
The average energy stored in the oscillator is equal to its maximum potential energy.
$=\frac{1}{2}cx^2$
$=\frac{1}{2}m\omega^2 x_0^2$

Given that,
M = 50gm
ω = 2πn
ω = 2π×20
ω = 40π x0 = 2 cm
The average energy stored or absorbed in the oscillator
$= \frac{1}{2} \times 50 \times (40\pi)^2 (2)^2$
= 1.58×104erg

ANSWER:
The average energy stored or absorbed in the oscillator is 1.58×104erg

PROBLEM-4
A sewing machine needle moves in a path 4cm long and the frequency of its oscillations is 10Hz.what is its displacement and acceleration s after crossing the center of its path?

SOLUTION:
Path length = 4cm
Amplitude $=\frac{Pathlength}{2}$
$=\frac{4}{2}$
= 2 cm
Formula for displacement,
x = Amp sin (ωt +α)
ω = 2πn
= 2π×10
=20πrad/s
x = Amp sin (ωt +α)
$x = 2\sin(20\pi \times \frac{1}{120} + 0)$
$x = 2\sin(\frac{\pi}{6})$
$= 2 \times \frac{1}{2}$
= 1cm
Acceleration
a(t) $=-\omega^2 \times$ t
$= (-20\pi)^2 \times 1$
$=(203.14)^2$
= 3944cm/s^2
ANSWER:
Displacement = 1cm
Acceleration = 3944 cm/s^2

PROBLEM-5
Simple harmonic motion of a particle with an amplitude of 10cm and a period of 6s. At t = 0 and its position x = 5cm going towards x direction, write the equation for the displacement x at time t and also find the acceleration of the particle at t = 4s.

SOLUTION:
Amplitude A = 10cm
Time period T = 6 sec
$\omega = \frac{2\pi}{T}$
$\omega = \frac{2\pi}{6}$
$\omega = \frac{\pi}{3}s$

At t=0,
x = Amp sin (ω t+∅)
5 = 10 sin (ω t+∅)
5 = 10sin∅
$\sin \emptyset = \frac{1}{2}$
$\emptyset = \sin^{-1}\frac{1}{2}$
$\emptyset = \frac{\pi}{6}$

Equation of displacement
$x = 10\sin(\frac{\pi}{3})$
At, t = 4 sec
x = Amp sin (ω + ∅)
$x = 10\sin(\frac{\pi}{3} \times 4 + \frac{\pi}{6})$

$= 10\sin(\frac{9\pi}{6})$

$= 10\sin(\frac{3\pi}{2})$

= -10

Acceleration a = $-\omega^2 x$

$= (\frac{-\pi}{3})^2 \times (-10)$

= 0.1 cm/s

ANSWER:

The acceleration of the particle at t = 4s is 0.1 cm/s

CONCLUSION

In conclusion, damped and forced simple harmonic motion are both important phenomena in physics. Damped harmonic motion involves the reduction of amplitude due to the dissipation of energy, while forced harmonic motion involves the application of an external force to the system. Both types of motion can be modeled mathematically using differential equations and can be observed in various natural and man-made systems, including car suspensions, wind turbines, and diving boards. Understanding these types of motion is crucial for many engineering and scientific applications.

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