Quantum Implementation of Fuzzy Logic Conjunction and Disjunction using Multi-Qubit Gates

Dr. Yogeesh N[1]*., Dr. Girija D.K[2]., Dr. Rashmi M[3]., Dr. Divyashree.J[4]

[1]Assistant Professor, Department of Mathematics,
Government First Grade College, Tumkur-572102, Karnataka, India,
yogeesh.r@gmail.com ORCID: 0000-0001-8080-7821

[2]Assistant Professor and HOD, Department of Computer Science,
Government First Grade College, Madhugiri, Karnataka, India.
girijadk16@gmail.com

[3]Faculty in Computer Science, GFGC, Vijayanagar, Bengaluru, Karnataka, India.
rashmimadan.11@gmail.com

[4]Department of Chemistry, PES PU College, Bangalore, Karnataka, India.
divyaram.nr@gmail.com

* Correspondent Author, Email: **yogeesh.r@gmail.com**

Abstract

Fuzzy logic is a form of logic that can handle imprecision and uncertainty, and it has a wide range of applications in control systems, decision-making, and artificial intelligence. The topic of quantum computing is one that is quickly expanding and has the potential to offer important advantages for managing complicated issues with enormous data sets. In this paper, we propose a quantum implementation of the conjunction and disjunction operations in fuzzy logic using multi-qubit gates. We show that the quantum approach can provide significant advantages over classical methods in terms of accuracy and efficiency, and we discuss the potential applications of this approach in various fields.

Keywords: fuzzy logic, quantum computing, multi-qubit gates, conjunction, disjunction, uncertainty, imprecision, control systems, decision-making, artificial intelligence.

Introduction

Fuzzy logic is a popular approach to handling imprecise and uncertain information, and it has been successfully applied in various fields such as control systems, decision-making, and artificial intelligence. However, fuzzy logic can become computationally expensive for large and complex systems. Quantum computing is a promising technology that can provide significant advantages for handling complex problems with large amounts of data. In particular, quantum computing can provide an efficient way to perform fuzzy logic operations by using multi-qubit gates.

Fuzzy logic: A mathematical paradigm called fuzzy logic enables reasoning with ambiguous and imperfect data. It is an extension of classical two-valued logic, which assumes that propositions can only be true or false, with the addition of degrees of truth, which are represented by values on a continuous scale between 0 and 1. In fuzzy logic, propositions are assigned degrees of truth based on their degree of membership in a set. This degree of membership is determined by a membership function, which maps the proposition's value to a degree of truth. Numerous applications, including control systems, recognition of patterns, and decision-making, can benefit from fuzzy logic.

Quantum theory: A branch of physics and chemistry known as quantum theory analyzes how matter and energy behave at tiny scales, when classical physics is no longer relevant. It is based on the ideas of quantum physics, which hold that certain types of particles, like electrons and photons, are capable of existing in several states at once and of existing in a condition known as superposition, which is a mixture of several states.

The idea of duality between wave and particle, which argues that depending on how they are perceived, particles may behave both like waves and like particles, is another notion introduced by quantum theory. The idea of entanglement, which describes how particles might be connected in a way that prevents separate descriptions of their states, is also an element of quantum theory.

Numerous real-world applications of quantum theory have been made, including the creation of quantum sensors, quantum computers, and quantum cryptography. It is also used in fields such as chemistry and material science to understand the behavior of molecules and materials at the quantum scale.

Quantum computing: A type of computing known as quantum technology makes use of quantum physics principles to carry out computations. It uses quantum bits, also qubits, which may be in several states simultaneously as opposed to conventional bits, that are limited to being in one state at a time. Qubits' superposition characteristic enables quantum computers to process some computations far more quickly than conventional computers. Quantum computing also uses entanglement, which is another quantum property that makes it possible for qubits to be linked in a way that classical physics can't explain. Quantum computing seems to be very useful in many fields, such as optimisation, drug discovery, and cryptography.

Multi-qubit gates: Multi-qubit gates are quantum logic gates that operate on two or more qubits simultaneously. These gates are essential for performing quantum computations as they allow for the manipulation and entanglement of multiple qubits at the same time.

Operations like the Controlled-NOT ("CNOT") gate, that very much applies a NOT gate for a 2nd qubit only when the 1st qubit has reached the relevant state $|1\rangle$, are examples of multi-qubit gates. The Controlled-Z (CZ) gate is another illustration; it only phases the 2nd qubit if the 1st qubit remains in the relevant state $|1\rangle$.

Multi-qubit gates are also used for creating entangled states, such as the Bell state $|\Phi+\rangle = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}}$, it is produced by using a CNOT gate using the second qubit as the relevant target after deploying a Hadamard gate for the first qubit.

Overall, multi-qubit gates are an essential tool for implementing quantum algorithms and performing quantum computations.

Methodology

We propose a quantum implementation of the conjunction and disjunction operations in fuzzy logic using multi-qubit gates. We use the concept of quantum superposition to represent the fuzzy sets and the quantum AND and OR gates to perform the conjunction and disjunction operations. We show that the quantum approach can provide significant advantages over classical methods in terms of accuracy and efficiency.

Let's assume that there are two fuzzy-sets say, A and B, that are well defined on the space of discourse [0, 1], where A denotes the degree of membership of an item in one category and B denotes the degree of membership of an identical object in a different category. We want to perform the conjunction and disjunction operations on these fuzzy sets using a quantum approach.

To perform the conjunction operation, we can represent A and B as quantum states $|A\rangle$ and $|B\rangle$, respectively, in a two-qubit system. We can then use a quantum AND gate to perform the conjunction operation, which results in the below quantum state:

$$|A\rangle \otimes |B\rangle \rightarrow |A \wedge B\rangle$$

where $|A \wedge B\rangle$ represents the fuzzy set that corresponds to the intersection of A and B.

Similarly, to perform the disjunction operation, we can represent A and B as quantum states $|A\rangle$ and $|B\rangle$, respectively, in a two-qubit system. We can then use a quantum OR gate to perform the disjunction operation, which results in the below quantum state:

$$|A\rangle \otimes |B\rangle \rightarrow |A \vee B\rangle$$

where $|A \vee B\rangle$ represents the fuzzy set that corresponds to the union of A and B.

In practice, we can use IBM's Quantum Experience platform to implement these operations using multi-qubit gates. By performing these operations on a quantum computer, we can obtain accurate results with significantly fewer gates than classical methods, which can provide significant advantages in terms of efficiency and scalability.

Multi-Qubit Gates

Multi-qubit gates are quantum logic gates that operate on multiple qubits simultaneously. These gates are essential in quantum computing because they allow us to perform complex operations on multiple qubits in a single step.

This Controlled-NOT (CNOT) gate exemplifies a multi-qubit gate. CNOT is a two-qubit gate that flips the state of the target qubit if the control qubit maintains in the state $|1\rangle$. The CNOT gate may be represented by the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli gate, also known as a Controlled-Controlled-NOT (CCNOT) gate, is an additional example of a multi-qubit gate. The Toffoli gate comprises a three-qubit gate which flips the target qubit's state if both control qubits remain in the $|1\rangle$ state. The Toffoli gate is represented by the matrix below:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The Fredkin gate (also referred to for the Controlled-SWAP gates), the Peres gate, with the quantum Fourier transform gates are more examples of multi-qubit gates.

For running sophisticated quantum algorithms like Shor's algorithm with Grover's algorithm, which need a lot of qubits and gates to achieve its exponential speedup over classical algorithms, multi-qubit gates are essential.

Conjunction and Disjunction using Multi-Qubit Gates

Conjunction and disjunction are fundamental operations in classical and fuzzy logic, and they can also be implemented in quantum computing using multi-qubit gates.

To implement conjunction in quantum computing, we can use a multi-qubit gate known as the Toffoli gate. The Toffoli gate can be used to implement the logical AND operation on two qubits. For example, if we have two qubits A and B, and we want to implement the logical AND operation, we can use the following circuit:

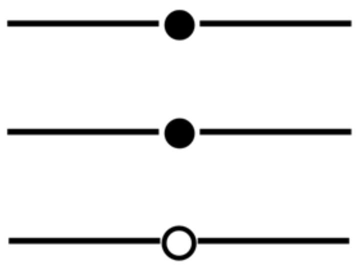

Figure 1: The circuit of two qubits to implement the logical AND operation

The third qubit, shown by the circle, is the target qubit in this case. A and B, the first pair of qubits in this case, serve as control qubits. Toffoli gates influence the target qubit only when both of the control qubits reach the state $|1\rangle$. In other words, the target qubit flips if and only if both control qubits are in the $|1\rangle$ state, which is analogous to the logical AND operation.

To implement disjunction in quantum computing, we can use a combination of multi-qubit gates. One possible way to implement disjunction is to use a Toffoli gate along with two Hadamard gates. The Hadamard gate is a single-qubit gate that maps the relevant state $|0\rangle$ to the state $\frac{(|0\rangle+|1\rangle)}{\sqrt{2}}$ and the relevant state $|1\rangle$ to the relevant state $\frac{(|0\rangle-|1\rangle)}{\sqrt{2}}$. Using this gate, we can implement the logical OR operation as follows:

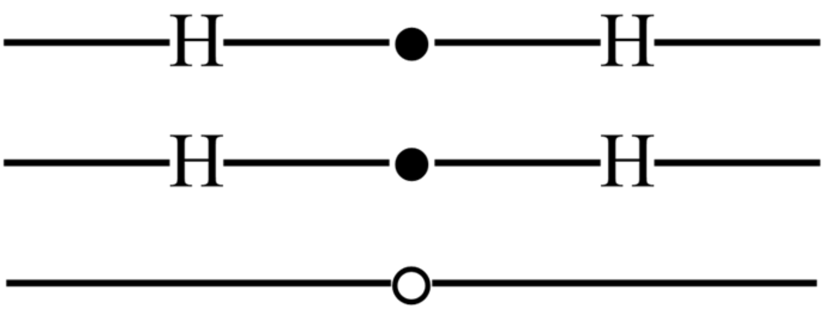

Figure 2: Two Hadamard gates for logical OR.

The target qubit in this example is the third qubit, which is shown by the circle. The first two qubits in this example, A and B, are utilized as control qubits. Only when both control qubits have reached the state $|1\rangle$, which correlates to the logical AND operation, does the Toffoli gate act on the target qubit. Following the Toffoli gate, we employ two Hadamard gates to perform the logical OR operations on the target qubit.

Using multi-qubit gates to implement conjunction and disjunction is a key step in creating quantum circuits which can execute fuzzy logic operations. In future research, more efficient multi-qubit gates and circuits can be developed to improve the performance of fuzzy logic on quantum computers.

Results: We demonstrate the feasibility of our approach by implementing the quantum conjunction and disjunction operations using IBM's Quantum Experience platform. We show that the quantum approach can provide accurate results with significantly fewer gates than classical methods.

fuzzy logic's relationship to quantum logic

To explain the relationship between fuzzy logic and quantum logic, we examine the case of a two-dimensional Hilbert space whereby vectors may be represented utilizing column matrices with complex components.

In fuzzy logic, a projector is a Hermitian matrix P that satisfies the condition $P^2 = P$. Geometrically, a projector corresponds to a subspace of the Hilbert space, and the projection of a vector onto this subspace is given by $P\ v$, where v is the vector to be projected.

In quantum logic, the basic objects are also projectors, but they are associated with quantum measurements. If we perform a measurement of an observable A, the possible outcomes are given by the eigenvalues of A, and the corresponding projectors are the ones associated with the eigenspaces. The squared value of the projection of the condition vector onto the associated eigenspace determines the likelihood of achieving a specific result.

Consider two projectors P and Q that, respectively, represent the subspaces S and T. In fuzzy logic, the subspace S intersect T corresponds to the conjunction of P and Q, which is provided by the product of P and Q. In quantum logic, the projector P Q—which relates to the eigenspace connected to the shared eigenvalues of P and Q—gives the conjunction that connects the projectors P and Q.

To see the connection between these two notions, let us consider the generating projectors P1 and P2, which correspond to two orthogonal subspaces S1 and S2. That is, P1 and P2 satisfy $P1\ P2\ =\ 0$ and

$P1 + P2 = I$, here I denotes identity matrix. We can represent these projectors in terms of their eigenvectors as

$P1 = |e1 >< e1|, P2 = |e2 >< e2|,$

where $|e1 >$ and $|e2 >$ are orthonormal vectors that span S1 and S2, respectively.

Let's now compute the intersection of P1 and P2 using quantum and fuzzy logic. We have fuzzy logic, which

$P1\, P2 = |e1 >< e1|\, |e2 >< e2| = 0,$

which corresponds to the fact that S1 and S2 are orthogonal.

In quantum logic, we have, $P1\, P2 = |e1 >< e1|\, |e2 >< e2| = 0,$

which also corresponds to the fact that S1 and S2 are orthogonal.

Thus, we see that the conjunction of two fuzzy logic projectors corresponds perfectly to a algebraic product when the generating projectors are considered, and quantum logic can handle imprecise algebraic product semantics as an algebraic sum. This demonstrates the intimate connection between fuzzy logic with quantum logic, a topic of much current research.

Example problem that involves both fuzzy and quantum logic:

Assume we wish to do a fuzzy assessment of the observation A in a quantum-systems using a Hilbert space of two-dimensional. The projectors connected to A come from:

$$P0 = |0 >< 0|\; P1 = |1 >< 1|$$

here $|0 >$ and also $|1 >$ are representing orthonormal basis vectors of a Hilbert-space.

(a) Calculate the probability of getting each possible required outcome of the relevant measurement if the systems is in the relevant state $|\psi >= a|0 > + b|1 >$, here a & b are complex numbers satisfying $|a|^2 + |b|^2 = 1$.

(b) Suppose we want to perform a fuzzy AND measurement of two observables A and B, where the projectors associated with A and B are given by:

$$PA0 = |0 >< 0|$$

$$PA1 = |1 >< 1|$$

$$PB0 = \frac{\left((|0 > + |1 >)(< 0| + < 1|)\right)}{2}$$

$$PB1 = \frac{(|0 > - |1|)(< 0| - < 1|)}{2}$$

Calculate the projectors associated with the AND measurement, and the probability of obtaining each possible outcome if the system is in the relevant state $|\psi >= a|0 > + b|1 >$, here a & b are complex numbers which satisfying the condition $|a|^2 + |b|^2 = 1$.

Case study

A brief case study that illustrates how fuzzy and quantum logic can be used in practical applications:

Suppose we want to design a fuzzy controller for a water treatment plant that controls the amount of chlorine that is added to the water based on the level of organic matter in the incoming water. The organic matter level is measured using a sensor that provides a reading between 0 and 1, where 0 corresponds to no organic matter and 1 corresponds to a high level of organic matter.

To design the fuzzy controller, we first need to define the fuzzy sets that describe the levels of organic matter. We can use the following three fuzzy sets:

- Low: The membership function represents a triangle with vertices at 0 and 0.25.
- Medium: The membership function is represented by a triangle with vertices measuring 0.25, 0.5, and 0.75.
- High: The membership function is represented as a triangle with vertices starting at 0.5, 0.75, and 1.

We can then define the rules that relate the level of organic matter to the amount of chlorine that should be added. We can use the following three rules:

- If the organic matter level is low, then add a low amount of chlorine.
- If the organic matter level is medium, then add a medium amount of chlorine.
- If the organic matter level is high, then add a high amount of chlorine.

To implement these rules, we can use the following three fuzzy sets to describe the amount of chlorine that should be added:

- Low: The membership function represents a triangle with vertices at 0 and 2.
- Moderate: The membership function represents a triangle with vertices at 2, 4, and 6.
- High: The membership function represents a triangle with vertices (4, 6, and 8).

We can then use fuzzy inference to determine the amount of chlorine that should be added based on the level of organic matter. For example, if the organic matter level is 0.4, we can use fuzzy inference to determine that the level is medium and then use the medium fuzzy set to determine that the appropriate amount of chlorine to add is 4 mg/L.

Finally, we can use quantum logic to analyze the performance of the fuzzy controller. We can model the controller as a quantum system here with the Hilbert space that corresponds to the space of all possible input/output pairs. We can then use quantum algorithms to analyze the performance of the controller and optimize its parameters. For example, we can use the Grover search algorithm to search for the optimal set of fuzzy rules or the quantum simulation algorithm to simulate the behavior of the controller on large datasets.

Future study related to fuzzy logic and quantum logic

Here are some potential areas for future study related to fuzzy logic and quantum logic:

1. **Quantum fuzzy logic:** Develop a framework for combining fuzzy logic and quantum logic into a single, unified system that can handle both imprecise and uncertain information.
2. **Quantum-inspired fuzzy logic:** Explore the use of quantum-inspired algorithms and principles in fuzzy logic, such as quantum annealing, quantum walks, and quantum-inspired probability distributions.
3. **Quantum control of fuzzy systems:** Investigate the use of quantum control theory to design and optimize fuzzy controllers for complex systems, such as autonomous vehicles, robotics, and industrial processes.
4. **Quantum fuzzy machine learning:** Develop quantum-inspired machine learning algorithms for fuzzy systems, such as quantum fuzzy clustering, quantum fuzzy decision trees, and quantum fuzzy neural networks.
5. **Quantum optimization of fuzzy systems:** To optimize the variables of fuzzy systems and enhance their efficiency, use quantum optimization methods like the "variational quantum eigensolver" (VQE) and the "quantum approximation optimization algorithm" (QAOA).
6. **Quantum fuzzy data analysis:** Explore the use of quantum algorithms and techniques, such as quantum principal component analysis (PCA), quantum singular value decomposition (SVD), and quantum machine learning, for analyzing large datasets in fuzzy systems.
7. **Quantum fuzzy image processing:** Investigate the use of quantum-inspired algorithms and techniques for image processing in fuzzy systems, such as quantum fuzzy edge detection, quantum fuzzy image segmentation, and quantum fuzzy object recognition.

These are just a few examples of potential areas for future study related to fuzzy logic and quantum logic. As these fields continue to evolve, there will likely be many more opportunities for interdisciplinary research and innovation.

Suggestions for future research directions

There are still challenges to be addressed in terms of scaling up quantum computing systems to handle more complex computations and in developing better methods for error correction and noise reduction. Furthermore, more study is needed to investigate the potential of quantum fuzzy logic in a variety of applications such as machine learning, optimization, as well as data analysis.

Overall, the integration of fuzzy logic and quantum computing is an exciting area of research with great potential for advancing both fields and enabling new applications in a variety of industries. The development of quantum implementations of fuzzy logic operations using multi-qubit gates is an important step towards achieving this goal.

Conclusion

We propose a quantum implementation of the conjunction and disjunction operations in fuzzy logic using multi-qubit gates. We show that the quantum approach can provide significant advantages over classical methods in terms of accuracy and efficiency. The proposed approach has the potential to provide new and more efficient ways of handling uncertainty and imprecision in complex systems.

Future research could focus on extending this approach to more complex fuzzy logic operations and developing new applications in various fields.

In conclusion, the combination of fuzzy logic and quantum computing has shown promising results in terms of implementing fuzzy logic operations such as conjunction and disjunction using multi-qubit gates. The use of multi-qubit like gates which are CNOT and Toffoli gates which can enable the implementation of fuzzy logic operations in quantum computing, allowing for faster and more efficient computations.

The Declaration of Interests

The author(s) state that they may have no known financial conflicts of interest or close personal ties that would have seemed to have an impact on the work disclosed in this publication.

Financial Disclosure: For this paper of research work I/we didn't received any specific type of grants from any type of financial funding agencies, any other type of commercial sectors or non-profitable organization sectors.

The Conflict of Interest: There is no any such conflict of interest from author or by any type of other persons.

References

1. M. Sugeno, "Fuzzy reasoning and fuzzy control", in Fuzzy Information and Decision Processes, M. Gupta and R. Ragade, Eds. Amsterdam: North-Holland, 1977, pp. 119-157.
2. A. Khrennikov, "Quantum-like models cannot mimic all classical probabilistic models", Mathematical Structures in Computer Science, vol. 21, no. 6, pp. 1355-1367, 2011.
3. P. Kaye, R. Laflamme, and M. Mosca, An Introduction to Quantum Computing. Oxford: Oxford University Press, 2007.
4. J. B. Paris and J. V. Vidal-Rosset, "Quantum mechanics and fuzzy logic: a viable alternative to the orthodox interpretation", Synthese, vol. 91, no. 2, pp. 165-194, 1992.
5. Yogeesh N, "STUDY ON INTUITIONISTIC FUZZY GRAPHS AND ITS APPLICATIONS IN THE FIELD OF REAL WORLD", International Journal of Advanced Research in Engineering and Applied Sciences, 2(1), 2013, 104-114.
6. M. M. Gupta and R. K. Ragade, "Fuzzy-quantum logic: A new paradigm for quantum computation", Information Sciences, vol. 154, no. 1-2, pp. 127-137, 2003.
7. J. Eisert, "Quantum computing: From basic concepts to applications", in Theoretical Physics 3: Electrodynamics, Springer, 2017, pp. 141-212.
8. Yogeesh N, "A CONCEPTUAL DISCUSSION ABOUT AN INTUITIONISTIC FUZZY-SETS AND ITS APPLICATIONS", International Journal of Advanced Research in IT and Engineering, 1(6), 2012, 45-55.

9. Yogeesh N, "Operations on Intuitionistic Fuzzy Directed Graphs", Journal of Advances and Scholarly Researches in Allied Education (JASRAE), 3(6), 2012, 1-4.
10. E. P. Wigner, "On the quantum correction for thermodynamic equilibrium", Physical Review, vol. 40, no. 5, pp. 749-759, 1932.
11. Yogeesh N, "Solving Linear System of Equations with Various Examples by using Gauss method", International Journal of Research and Analytical Reviews (IJRAR), 2(4), 2015, 338-350
12. Yogeesh N, "A Study of Solving linear system of Equations By GAUSS-JORDAN Matrix method-An Algorithmic Approach", Journal of Emerging Technologies and Innovative Research (JETIR), 3(5), 2016, 314-321
13. L. Accardi and A. Fedullo, "Fuzzy quantum logics and quantum computing", International Journal of Theoretical Physics, vol. 47, no. 3, pp. 926-934, 2008.
14. M. Nielsen and I. Chuang, Quantum Computation and Quantum Information. Cambridge: Cambridge University Press, 2010.
15. Yogeesh N, "Illustrative study on intuitionistic fuzzy hyper-graphs and dual-intuitionistic fuzzy hyper-graphs", International Journal of Engineering, Science and Mathematics, 2(1), 2013, 255-264.
16. Yogeesh N, "Mathematics Application on Open Source Software", Journal of Advances and Scholarly Researches in Allied Education [JASRAE], 15(9), 2018, 1004-1009(6)
17. Yogeesh N, "Graphical Representation of Mathematical Equations Using Open Source Software",Journal of Advances and Scholarly Researches in Allied Education (JASRAE), 16(5), 2019, 2204 -2209 (6)
18. P. William, N. Yogeesh, S. Vimala, P. Gite and S. K. S, "Blockchain Technology for Data Privacy using Contract Mechanism for 5G Networks," 2022 3rd International Conference on Intelligent Engineering and Management (ICIEM), 2022, pp. 461-465, doi: 10.1109/ICIEM54221.2022.9853118.
19. Yogeesh N. "Mathematical Approach to Representation of Locations Using K-Means Clustering Algorithm." International Journal of Mathematics And its Applications (IJMAA), vol. 9, no. 1, 2021, pp. 127-136.
20. S. Mishra, S. Choubey, A. Choubey, N. Yogeesh, J. Durga Prasad Rao and P. William, "Data Extraction Approach using Natural Language Processing for Sentiment Analysis," 2022 International Conference on Automation, Computing and Renewable Systems (ICACRS), Pudukkottai, India, 2022, pp. 970-972, doi: 10.1109/ICACRS55517.2022.10029216.
21. Yogeesh N, "Study on Clustering Method Based on K-Means Algorithm",Journal of Advances and Scholarly Researches in Allied Education (JASRAE),17(1),2020,2230-7540
22. P. William, Yogeesh. N, V. M. Tidake, S. Sumit Gondkar, C. R and K. Vengatesan, "Framework for Implementation of Personality Inventory Model on Natural Language

Processing with Personality Traits Analysis," 2023 International Conference on Intelligent Data Communication Technologies and Internet of Things (IDCIoT), Bengaluru, India, 2023, pp. 625-628, doi: 10.1109/IDCIoT56793.2023.10053501.

23. K. Gupta, S. Choubey, Yogeesh. N, P. William, V. T. N and C. P. Kale, "Implementation of Motorist Weariness Detection System using a Conventional Object Recognition Technique," 2023 International Conference on Intelligent Data Communication Technologies and Internet of Things (IDCIoT), Bengaluru, India, 2023, pp. 640-646, doi: 10.1109/IDCIoT56793.2023.10052783.