ON CARTESIAN PRODUCT OF COMPLEX DOUBT Q-FUZZY SUBRING

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Abstract

In this paper, we introduce of complex doubt $q-$ fuzzy subring and discuss its various algebraic aspects. This paper analyze the properties of the product two of complex doubt $q-$ fuzzy subrings, and some properties related to it are established.

Keywords: Fuzzy Set; Complex fuzzy set, Complex Doubt $q-$fuzzy set, Complex Doubt $q-$Fuzzy subring.

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1. Introduction

Zadeh [18] initiated the study of fuzzy sets in 1965. Biwas [6] gave the idea of anti fuzzy subgroups in 1990. Malik and Mordeson [11] commenced the idea of fuzzy homomorphism of rings in 1992. Gang and Yun [8] explained the notion of fuzzy factor rings in 1998. Ramot et al. [16] launched the notion of complex fuzzy in 2002. Liu[10], initiated by the new concept of fuzzy invariant subgroups and fuzzy ideals in 1982. This concept becomes more effective for resaercher and quite different from fuzzy complex numbers innovated by Buckley [7]. Prasanna, Premkumar et.al [14], described by the new notation On $\kappa - Q$-Anti Fuzzy Normed Rings in 2021. , Zhang et al. [19] introduced the various operation of complex fuzzy sets in 2009. Azam et al [5] defined anti fuzzy ideal and developed the quotient ring with respect to anti fuzzy ideal in 2013. Al-Husban and Salleh [3] presented the concept of complex fuzzy subgroups in 2016. Alsarahead and Ahmed [1] proposed the idea of π-fuzzy subgroup and complex fuzzy subgroup. They also described image and inverse image of complex fuzzy subgroup under group homomorphism in 2017. Moreover, fundamental algebraic attributes of complex fuzzy subrings were established by the same authors [2] in 2017. These concept are different from fuzzy subgroups and fuzzy ideal introduced and discuused by Rosenfeld [17] and Liu [9] respectively. Kellil [9] defined the product of fuzzy subrings in 2018 Al-Tahan and Davvaz [4] analyzed the concept of complex fuzzy H_v-subgroup and complex anti fuzzy H_v-subgroup in 2018. Prasanna , Premkumar et.al.[15], introduced the concept of A Study on Complex Anti Fuzzy Subring in 2021. In 2018, On Fundamental Attributes on Homomorphism of μ-anti- Fuzzy Subgroups, developed by Nagaraj and Premkumar [13] . Muhammad Gulzar et.al.[12], introduced the concept of On some characterization of Q-complex fuzzy sub-rings in 2021.

This paper is organized as: Section 2 contains the introductory definition of complex fuzzy subrings and related result which play a key role for our further discussion. In section 3, we prove that the product of two complex Doubt q −fuzzy subrings is complex Doubt $q -$ fuzzy subring and develop some results of the product of two complex Doubt $q -$ fuzzy subrings.

Preliminaries

Definition (2.1) [18]: A fuzzy set δ of a nonempty set P is a mapping

$$\delta : P \to [0, 1].$$

Definition (2.2): Let A be complex fuzzy sets of set P, with membership function $\theta_A(x) = \eta_A(x)e^{i\varphi_A(x)}$. The complex fuzzy complement of A is specified by a function

$$\theta_{A^c}(x) = \eta_{A^c}(x)e^{i\varphi_{A^c}(x)} = \{1 - \eta_A(x)\}e^{i\{2\pi - \varphi_A(x)\}}$$

Definition (2.3): A homogeneous complex fuzzy set A of ring S is called complex anti fuzzy subring of S if

1. $\theta_A(x - y) \leq \max\{\theta_A(x), \theta_A(x)\}$, for all $x, y \in S$
2. $\theta_A(xy) \leq \max\{\theta_A(x), \theta_A(x), \}$, for all $x, y \in S$

Product of Complex Doubt Q −Fuzzy Subring

Definition 3.1 :

Let A and B be two complex Doubt $q -$ fuzzy subring of S_1 and S_2 respectively. Then the product of A and B is defined as

$\theta_{A\times B}\big((x, y), q\big) = \eta_{A\times B}((x, y), q)e^{i\varphi_{A\times B}((x,y),q)} = \max\{\eta_A(x, q), \eta_B(y, q)\}e^{imax\{\varphi_A(x,q),\varphi_B(y,q)\}}$, Where $\in S_1$, $y \in S_2$ and $q \in Q$.

Theorem 3.2:

Let A and B be two complex Doubt $q -$ fuzzy subring of S_1 and S_2 respectively. Then $A \times B$ is complex Doubt $q -$ fuzzy subring of S_1 and S_2.

Proof:

Let $x_1, x_2 \in S_1$, $y_1, y_2 \in S_2$ and $q \in Q$ be an elements.

Then $(x_1, y_1), (x_2, y_2) \in S_1 \times S_2$. Now $\theta_{A\times B}\left(\big((x_1, y_1) - (x_2, y_2)\big), q\right) = \theta_{A\times B}\big((x_1 - x_2, y_1 - y_2), q\big)$

$= \max\{\eta_A\big((x_1 - x_2), q\big), \eta_B\big((y_1 - y_2), q\big)\}\, e^{imax\{\varphi_A((x_1-x_2),q),\varphi_B((y_1-y_2),q)\}}$

$= \max\{\eta_A\big((x_1 - x_2), q\big)e^{i\varphi_A((x_1-x_2),q)}, \eta_B\big((y_1 - y_2), q\big)e^{i\varphi_A((y_1-y_2),q)}\}$

$= \max\{\theta_A\big((x_1 - x_2), q\big), \theta_B\big((y_1 - y_2), q\big)\} \leq \max\{max\{\theta_A(x_1, q), \theta_A(x_2, q)\}, max\{\theta_B(y_1, q), \theta_B(y_2, q)\}\}$

$= \max\{max\{\theta_A(x_1, q), \theta_B(y_1, q)\}, max\{\theta_A(x_2, q), \theta_B(y_2, q)\}\}$

$\theta_{A\times B}\left(\big((x_1, y_1) - (x_2, y_2)\big), q\right) = \max\{\theta_{A\times B}\big((x_1, y_1), q\big), \theta_{A\times B}\big((x_2, y_2), q\big)\}$

Further, $\theta_{A\times B}\left(\left((x_1,y_1)(x_2,y_2)\right),q\right)=\theta_{A\times B}\left((x_1x_2,y_1y_2),q\right)$
$=\max\{\eta_A((x_1x_2),q),\eta_B((y_1y_2),q)\}\,e^{imax\{\varphi_A((x_1x_2),q),\varphi_B((y_1y_2),q)\}}$
$=\max\{\eta_A((x_1x_2),q)e^{i\varphi_A((x_1x_2),q)},\eta_B((y_1y_2),q)e^{i\varphi_A((y_1y_2),q)}\}$
$=\max\{\theta_A((x_1x_2),q),\theta_B((y_1y_2),q)\}\le\max\{max\{\theta_A(x_1,q),\theta_A(x_2,q)\},max\{\theta_B(y_1,q),\theta_B(y_2,q)\}\}$
$=\max\{max\{\theta_A(x_1,q),\theta_B(y_1,q)\},max\{\theta_A(x_2,q),\theta_B(y_2,q)\}\}$
$\theta_{A\times B}\left(\left((x_1,y_1)(x_2,y_2)\right),q\right)\le\max\{\theta_{A\times B}((x_1,y_1),q),\theta_{A\times B}((x_2,y_2),q)\}$

Theorem 3.3:

Let A and B be two homogenous complex Doubt q −fuzzy subsets of rings S_1 and S_2, respectively. If $A\times B$ is a complex Doubt q −fuzzy subring of $S_1\times S_2$, then at least one of the following statements must be hold.
1. $\eta_A(0,q)\le\eta_B(y,q)$ and $\varphi_A(0,q)\le\varphi_B(y,q)$, for all $y\in S_2$ and $q\in Q$
2. $\eta_B(0',q)\le\eta_A(x,q)$ and $\varphi_B(0',q)\le\varphi_A(x,q)$, for all $x\in S_1$ and $q\in Q$

Where 0 and $0'$ are identities of S_1, and S_2 respectively.
Proof: Let $A\times B$ be a complex Doubt q −fuzzy subgroup of $S_1\times S_2$. On contrary, suppose that statements (1) and (2) do not hold. Then there exist $x\in S_1$, $y\in S_2$ and $q\in Q$ such that
1. $\eta_A(0,q)\ge\eta_B(y,q)$ and $\varphi_A(0,q)\ge\varphi_B(y,q)$
2. $\eta_B(0',q)\ge\eta_A(x,q)$ and $\varphi_B(0',q)\ge\varphi_A(x,q)$

Now $\theta_{A\times B}((x,y),q)=\max\{\eta_A(x,q),\eta_B(y,q)\}\,e^{imax\{\varphi_A(x,q),\varphi_B(y,q)\}}$
$\le\max\{\eta_A(0,q),\eta_B(0',q)\}e^{imax\{\varphi_A(0,q),\varphi_B(0,q)\}}=\theta_{A\times B}((0,0'),q)$
But $A\times B$ is fuzzy subgroup. Hence, at least one of the following statements must be hold.
1. $\eta_A(0,q)\le\eta_B(y,q)$ and $\varphi_A(0,q)\le\varphi_B(y,q)$, for all $y\in S_2$ and $q\in Q$
2. $\eta_B(0',q)\le\eta_A(x,q)$ and $\varphi_B(0',q)\le\varphi_A(x,q)$, for all $x\in S_1$ and $q\in Q$

Theorem 3.4:

Let A and B two homogenous complex fuzzy sets of S_1 and S_2 such that $\eta_B(0',q)\le\eta_A(x,q)$ and $\varphi_B(0',q)\le\varphi_A(x,q)$ for all $x\in S_1$, $q\in Q$ and $0'$ is identity of S_2. If $A\times B$ is complex Doubt q −fuzzy subring of$(S_1\times S_2)\times Q$, then A is Doubt q −fuzzy subring.

Proof:

Let A and B be two complex Doubt q −fuzzy subrings of S_1 and S_2. Then $\left(\left((x,0'),(y,0')\right),q\right)\in(S_1\times S_2)\times Q$. By given condition $\eta_B(0',q)\le\eta_A(x,q)$ and $\varphi_B(0',q)\le\varphi_A(x,q)$, for all $x\in S_1$ *and* $q\in Q$. (A and B is homogeneous)

$$\theta_A((x-y),q)=\eta_A((x-y),q)e^{i\varphi_A((x-y),q)}$$
$$=\max\left\{\eta_A((x-y),q)e^{i\varphi_A((x-y),q)},\eta_B((0'-0'),q)e^{i\varphi_B((0'-0'),q)}\right\}$$
$=\max\{\eta_A((x-y),q),\eta_B((0'-0'),q)\}e^{i\max\{\varphi_B((x-y),q),\varphi_B((0'-0'),q)\}}$
$=\max\{max\{\eta_A(x,q),\eta_A(y,q)\},max\{\eta_B(0',q),\eta_B(0',q)\}\}\,e^{i\max\{max\{\varphi_A(x,q),\varphi_A(y,q)\},max\{\varphi_B(0',q),\varphi_B(0',q)\}\}}$
$\le\max\{max\{\eta_A(x,q),\eta_A(y,q)\},max\{\eta_A(x,q),\eta_A(y,q)\}\}\,e^{i\max\{max\{\varphi_A(x,q),\varphi_A(y,q)\},max\{\varphi_A(x,q),\varphi_A(y,q)\}\}}$
$\theta_A((x-y),q)\le max\{\eta_A(x,q),\eta_A(y,q)\}\,e^{i\,max\{\varphi_A(x,q),\varphi_A(y,q)\}}$
$\theta_A((x-y),q)\le max\{\theta_A(x,q),\theta_A(y,q)\}$
Further, $\theta_A(xy,q)=\eta_A(xy,q)e^{i\varphi_A(xy,q)}=\max\left\{\eta_A(xy,q)e^{i\varphi_A(xy,q)},\eta_B((0'0'),q)e^{i\varphi_B((0'0'),q)}\right\}$
$=\max\{\eta_A(xy,q),\eta_B((0'0'),q)\}e^{i\max\{\varphi_B(xy,q),\varphi_B((0'0'),q)\}}$
$=\max\{max\{\eta_A(x,q),\eta_A(y,q)\},max\{\eta_B(0',q),\eta_B(0',q)\}\}\,e^{i\max\{max\{\varphi_A(x,q),\varphi_A(y,q)\},max\{\varphi_B(0',q),\varphi_B(0',q)\}\}}$
$\le\max\{max\{\eta_A(x,q),\eta_A(y,q)\},max\{\eta_A(x,q),\eta_A(y,q)\}\}\,e^{i\max\{max\{\varphi_A(x,q),\varphi_A(y,q)\},max\{\varphi_A(x,q),\varphi_A(y,q)\}\}}$
$=max\{\eta_A(x,q),\eta_A(y,q)\}\,e^{i\,max\{\varphi_A(x,q),\varphi_A(y,q)\}}=max\{\theta_A(x,q),\theta_A(y,q)\}$
$\theta_A(xy,q)\le max\{\theta_A(x,q),\theta_A(y,q)\}$
Hence, proved our claim.

Theorem 3.5:

Let A and B two homogenous complex Doubt q −fuzzy sets of S_1 and S_2 such that $\eta_A(0,q)\le\eta_B(y,q)$ and $\varphi_A(0,q)\le\varphi_B(y,q)$ for all $y\in S_2$ *and* $q\in Q$ and 0 is identity of S_1. If $A\times B$ is complex Doubt q −fuzzy subring of$(S_1\times S_2)\times Q$, then B is Doubt q −fuzzy subring of S_2.
Proof: similar as pervious

Corollary 3.5.1:

Let A and B two homogenous complex Doubt q −fuzzy sets of S_1 and S_2 respectively. If $A \times B$ is complex Doubt q −fuzzy subring of $(S_1 \times S_2) \times Q$, then A is a complex Doubt $q -$ fuzzy subring of S_1 or B is a complex Doubt $q -$ fuzzy subring of S_2.

Definition 3.6:

Let $A \times B$ be Cartesian product of two complex Doubt q −fuzzy subset A and B. Then For $t_1 \in [0,1], and\ t_2 \in [0,2\pi]$ the lower level subset of complex Doubt $q -$ fuzzy set $A \times B$ is defined by

$$(A \times B)_{(t_1,t_2)} = \{((x,y),q) \in ((S_1 \times S_2) \times Q): \eta_{A\times B}((x,y),q) \leq t_1, \varphi_{A\times B}((x,y),q) \leq t_2\}$$

For $t_2 = 0$, we obtain the lower level subset $A_{t_1} = \{((x,y),q) \in P \times Q: \eta_A((x,y),q) \leq t_1\}$ and for $t_1 = 0$, then we obtain the lower level subset $A_{t_2} = \{((x,y),q) \in P \times Q: \varphi_A((x,y),q) \leq t_2\}$.

Theorem 3.7:

Let A and B be two complex Doubt q −fuzzy subsets of rings S_1 and S_2 . Then $(A \times B)_{((t_1,t_2),q)} = A_{((t_1,t_2),q)} \times B_{((t_1,t_2),q)}$.

Proof:

Let $((x,y),q) \in A_{((t_1,t_2),q)} \times B_{((t_1,t_2),q)} \Leftrightarrow (x,q) \in A_{((t_1,t_2),q)}\ and\ (y,q) \in B_{((t_1,t_2),q)}$

$\Leftrightarrow \eta_A(x,q) \leq t_1, \varphi_A(x,q) \leq t_2$ and $\eta_B(x,q) \leq t_1, \varphi_B(x,q) \leq t_2$

$\Leftrightarrow \max\{\eta_A(x,q), \eta_B(y,q)\} \leq t_1$ and $\max\{\varphi_A(x,q), \varphi_B(y,q)\} \leq t_2$

$\Leftrightarrow \eta_{A\times B}((x,y),q) \leq t_1$ and $\varphi_{A\times B}((x,y),q) \leq t_2$

$\Leftrightarrow ((x,y),q) \in (A \times B)_{((t_1,t_2),q)}$

Hence, $(A \times B)_{((t_1,t_2),q)} = A_{((t_1,t_2),q)} \times B_{((t_1,t_2),q)}$

Theorem 3. 8:

Let $A \times B = \{(((x,y),q), \theta_{(A\times B,q)}((x,y),q)): \theta_{(A\times B,q)}((x,y),q) = \eta_{(A\times B,q)}((x,y),q)e^{i\varphi_{(A\times B,q)}((x,y),q)}, ((x,y),q) \in (S_1 \times S_2) \times Q\}$ be homogeneous complex Doubt q −fuzzy set of ring S. Then $A \times B$ is a complex Doubt q −fuzzy subring of $S_1 \times S_2$ if and only $(A \times B)_{((t_1,t_2),q)}$ is a subring of ring $(S_1 \times S_2) \times Q$, for all

Where $\eta_A((e,e'),q) \leq t_1,\ \varphi_A((e,e'),q) \leq t_2$, also (e,e') is identity element of $S_1 \times S_2$.

Proof:

Obviously $(A \times B)_{((t_1,t_2),q)}$ is nonempty, as $(e,e') \in A \times B)_{((t_1,t_2),q)}$

Let $(((x_1,y_1),(x_2,y_2)),q) \in A_{((t_1,t_2),q)}$ be any two elements. Then

$$\eta_A((x_1,y_1),q) \leq t_1, \varphi_A((x_1,y_1),q) \leq t_2 \text{ and } \eta_A((x_2,y_2),q) \leq t_1, \varphi_A((x_2,y_2),q) \leq t_2$$

Now, $\eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right)e^{i\varphi_{A\times B}(((x_1,y_1)-(x_2,y_2)),q)}$

$= \theta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \leq \max\{\theta_{A\times B}((x_1,y_1),q), \theta_{A\times B}((x_2,y_2),q)\}$

$= \max\{\eta_{A\times B}((x_1,y_1),q)e^{i\varphi_{A\times B}((x_1,y_1),q)}, \eta_{A\times B}((x_2,y_2),q)e^{i\varphi_{A\times B}((x_2,y_2),q)}\}$

$= \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}e^{i\max\{\varphi_{A\times B}((x_1,y_1),q),\varphi_{A\times B}((x_2,y_2),q)\}}$

(As A is homogeneous)

$$\eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \leq \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\} = \max\{t_1,t_1\} = t_1$$

$$\varphi_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \leq \max\{\varphi_{A\times B}((x_1,y_1),q), \varphi_{A\times B}((x_2,y_2),q)\} = \max\{t_2,t_2\} = t_2$$

$$\Rightarrow ((x_1,y_1)-(x_2,y_2),q) \in A \times B_{((t_1,t_2),q)}$$

Further, $\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right)e^{i\varphi_{A\times B}(((x_1,y_1)(x_2,y_2)),q)} = \theta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right)$

$\leq \max\{\theta_{A\times B}((x_1,y_1),q), \theta_{A\times B}((x_2,y_2),q)\}$

$= \max\{\eta_{A\times B}((x_1,y_1),q)e^{i\varphi_{A\times B}((x_1,y_1),q)}, \eta_{A\times B}((x_2,y_2),q)e^{i\varphi_{A\times B}((x_2,y_2),q)}\}$

$= \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}e^{i\max\{\varphi_{A\times B}((x_1,y_1),q),\varphi_{A\times B}((x_2,y_2),q)\}}$

(As A is homogeneous)

$$\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \leq \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\} = \max\{t_1,t_1\} = t_1$$

$$\varphi_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \leq \max\{\varphi_{A\times B}((x_1,y_1),q), \varphi_{A\times B}((x_2,y_2),q)\} = \max\{t_2,t_2\} = t_2$$

$$\Rightarrow ((x_1,y_1)(x_2,y_2),q) \in A \times B_{((t_1,t_2),q)}$$

Hence $(A \times B)_{((t_1,t_2),q)}$ is subring

Conversely, let $(A \times B)_{((t_1,t_2),q)}$ is a subring of S and let $\max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\} = t_1$ and $\max\{\varphi_{A\times B}((x_1,y_1),q), \varphi_{A\times B}((x_2,y_2),q)\} = t_2$. Then we have

$\eta_{A\times B}((x_1,y_1),q) \le t_1\ \eta_{A\times B}((x_2,y_2),q) \le t_1,$ and $\varphi_{A\times B}((x_1,y_1),q) \le t_2, \varphi_{A\times B}((x_2,y_2),q) \le t_2$

$\eta_{A\times B}((x_1,y_1),q) \le t_1, \varphi_{A\times B}((x_1,y_1),q) \le t_2$ and $\eta_{A\times B}((x_2,y_2),q) \le t_1, \varphi_{A\times B}((x_2,y_2),q) \le t_2$

This implies that $((x_1,y_1),q) \in (A\times B)_{((t_1,t_2),q)}$ $((x_2,y_2),q) \in (A\times B)_{((t_1,t_2),q)}$

As $(A\times B)_{((t_1,t_2),q)}$ is subring. So, $\left(((x_1,y_1)-(x_2,x_2)),q\right) \in (A\times B)_{((t_1,t_2),q)}$

$\Rightarrow \eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \le t_1$ and $\varphi_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \le t_2$

$\eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \le \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}$

and $\varphi_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \le \max\{\varphi_{A\times B}((x_1,y_1),q), \varphi_{A\times B}((x_2,y_2),q)\}$

$\theta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) = \eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) e^{i\eta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right)}$

$\le \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}\, e^{i\max\{\varphi_{A\times B}((x_1,y_1),q),\varphi_{A\times B}((x_2,y_2),q)\}}$

$= \max\{\eta_{A\times B}((x_1,y_1),q)e^{i\varphi_{A\times B}((x_1,y_1),q)}, \eta_{A\times B}((x_2,y_2),q)e^{i\varphi_{A\times B}((x_2,y_2),q)}\}$

$\theta_{A\times B}\left(((x_1,y_1)-(x_2,y_2)),q\right) \le \max\{\theta_{A\times B}((x_1,y_1),q), \theta_{A\times B}((x_2,y_2),q)\}$

Further, As is $(A\times B)_{((t_1,t_2),q)}$ subring. So $\left(((x_1,y_1)(x_2,x_2)),q\right) \in (A\times B)_{((t_1,t_2),q)}$

$\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le t_1$ and $\varphi_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le t_2$

$\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}$

$\varphi_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le \max\{\varphi_{A\times B}((x_1,y_1),q), \varphi_{A\times B}((x_2,y_2),q)\}$

$\theta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) = \eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) e^{i\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right)}$

$\le \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}\, e^{i\max\{\varphi_{A\times B}((x_1,y_1),q),\varphi_{A\times B}((x_2,y_2),q)\}}$

$= \max\{\eta_{A\times B}((x_1,y_1),q)e^{i\varphi_{A\times B}((x_1,y_1),q)}, \eta_{A\times B}((x_2,y_2),q)e^{i\varphi_{A\times B}((x_2,y_2),q)}\}$

$\theta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le \max\{\theta_{A\times B}((x_1,y_1),q), \theta_{A\times B}((x_2,y_2),q)\}$

$\theta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) = \eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) e^{i\eta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right)}$

$\le \max\{\eta_{A\times B}((x_1,y_1),q), \eta_{A\times B}((x_2,y_2),q)\}\, e^{i\max\{\varphi_{A\times B}((x_1,y_1),q),\varphi_{A\times B}((x_2,y_2),q)\}}$

$= \max\{\eta_{A\times B}((x_1,y_1),q)e^{i\varphi_{A\times B}((x_1,y_1),q)}, \eta_{A\times B}((x_2,y_2),q)e^{i\varphi_{A\times B}((x_2,y_2),q)}\}$

$\theta_{A\times B}\left(((x_1,y_1)(x_2,y_2)),q\right) \le \max\{\theta_{A\times B}((x_1,y_1),q), \theta_{A\times B}((x_2,y_2),q)\}$.

2. Conclusion

We have also defined product of two complex Doubt q −fuzzy subrings and have proved that the product two complex Doubt q −fuzzy subrings is also complex Doubt $q-$ fuzzy subring and discussed various algebraic properties.

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