# AN INNOVATIVE ANALYTICAL APPROACH FOR SOLVING FPDES ARISING IN ION ACOUSTIC WAVES IN PLASMA

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**Abstract:**
This manuscript uses Aboodh Transform Method (ATHPM) as a homotopic perturbation to solve linear and non-linear time FDE's such as regularized long wave (RLW) equations. This technique (ATHPM) is a very good founded, effective and simple path to find correct and approximate solutions for long wave (RLW) linear and nonlinear time fractions, combined with the Aboodh transforming method. The result demonstrates the precision of the Aboodh Transform combined homotopy disruption process. We find that it can be used extensively to analyze other issues in the planet.

**Keywords: -** Aboodh transform, Homotopy decomposition Method, RLW equations, and Water waves in oceans, Probability density Function (PDF), Fractional differential equations (FDE's).

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## Introduction

In the earlier studies, partial fractional differential equations (PFDE) have been examined in field of biology, biomedical sciences, viscoelastic polymers, electrochemistry, In electromagnetics and plasma physics, component discrete fractional equations are separated. There is considerable interest in computational study of FDE's. In this analysis, we have the Variational Iteration Method (VIM), Adomain Decomposition Method (ADM) and Differential Transforming Method (DTM) which are common for the solution of both FODE and FPDE [1-6].

Around solutions to fractional difference equations and physical problems called disturbance approaches are given by several tools. These approaches have some drawbacks, since limited parameters are a first prerequisite for estimated solutions which are often complicated [6-13]. There is a non-small parameter theoretical method. Homotopy Perturbation Aboodh transform (HPATM) [14-19] is a mixture of Ordinary Differential Equations (ODE) and the Partial Differential Equations (PDEs) [21] and the Aboodh Transformation System (ATM). In this paper Aboodh Transform and Homotopy perturbation Method together to solve Nonlinear fractional partial differential equations [20] occurring in Randomized Long Wave in Plasma.

This review covers the following partial differential non-linear equations:

i. $D_m^{\gamma}\varphi+\left(\frac{\varphi^2}{2}\right)-\varphi_{xx}=0, m>0, x\in R, 0<\gamma\le 1$ , (1)

ii. $D_m^{\gamma}\varphi+\varphi_x+6\varphi^2\varphi_x-\mu\varphi_{xx}=0,\ m>0, x\in R, 0<\gamma\le 1,$ with I.C.

$$\varphi(x,0)=\sqrt{a}\sec h(b(x-x_0)).b=\sqrt{\frac{a}{\mu(a+1)}},$$ (2)

iii. $D_m^{\gamma}\varphi + \varphi_x+\varphi\varphi_x+\varphi_{xxm} = 0,\ m > 0,\ x\in R,\ 0<\gamma \le 1$, (3)

iv. $D_m^{\gamma}\varphi + \varphi_x = \varphi_{xxm},\ m >0,\ x\in R,\ 0 < \gamma \le 1$, (4)

v. $D_m^\gamma + \varphi_{xxxx} = 0, m > 0, x \in R, 0 < \gamma \le 1,$ (5)

Then latter (1) applies to a nonlinear regularised time fractional wave equation, (2) to the non-linear regularised long wave fractional equation, (3) to time – the fractional linear regularised long wave equations (GRLW) and (4) and (5) to the equations of a regularised fraction (RLW). In equations (1-5) where the order of the derivative of the fraction. The derivative is understood as Caputo. The function the co-ordinates are PDF, x is geographical, m is time-based, and a and b are consistencies, is the positive parameter and the location is first based. This vocabulary has a fractional derivative order parameter that can be used to find the different answers. For The partial equations translate into standard equations.

**Basic concepts of Fractional Calculus:**
Any of the features and meanings that can be included in this work in the following section.

**Definition 1:**
Intrinsically, the gamma function is bound into a partial calculus. It is best to expand the fraction over all real numbers to illustrate the gamma function. The definition of the gamma function is described.

$$\Gamma(\mu) = \int_0^\infty e^{-t} t^{\mu-1} dt, where\, \mu > 0, \qquad (6)$$

**Definition 2:**
A real function f(x), x > 0, in space is stated to be $C_\mu$ , $\mu \in R$ if a real number occurs $p > \mu$, such that f(x) = $x^p g(x)$, where $g(x) \in [0, \infty)$ and in space it's claimed to be $C_\mu^m$ if $f^{(m)} \in C_m, m \in N.$

**Definition 3:**
Fractional integral order of the Riemann-Liouville of $\alpha \ge 0$, of a function $f \in C_m, \mu \ge -1,$ is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-1)^{\alpha-1} f(t) dt, \alpha > 0, t > 0$$

, (7)

$$J^\alpha f(t) = f(t).$$

Some properties of the operator:
For $f \in C_m, \mu \ge -1, \alpha, \beta \ge 0\ and \gamma > -1,$

$$J^\alpha J^\beta f(t) = J^{\alpha+\beta} f(t), J^\alpha J^\beta f(t) = J^\beta J^\alpha f(t)$$

$$J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\alpha+\gamma}.$$

**Lemma1:**
If $m-1 < \alpha \le m, m \in N\ and\ f \in C_m, \mu \ge -1, then\, D^\alpha J^\alpha f(t) = f(t) and,$

$$J^\alpha D_0^\alpha f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0) \frac{t^k}{k!}, k > 0,$$

(8)

**Definition 4**: (Fractional order partial derivatives)
Assume f(t) is a n - vector variables $t_i$, i = 1, 2, 3…, n also of class C and $D \in R_n$. We describe f(t) for $t_i$ a partial derivative as an extension of definition 2.

$$a\partial_t^\alpha f = \frac{1}{\Gamma(m-\alpha)} \int_0^{t_i} (t_i - 1)^{m-\alpha-1} \partial_{t_i}^\alpha f(t_j)|_{x_{j=k}} dt$$

(9)

**3. Formulae for Aboodh Transformation Method**
The Aboodh transform is a new transformation that is described by exponential order functions and is considered by Set A functions:
A={f(x): ∃ M, $k_1, k_2$>0, |f(t)|< M $e^{-vt}$}, (10)

The set M must have a small number for a specified purpose, $k_1, k_2$ perhaps limitless or finite. The integral equation determines the transformation of Aboodh.
A[f(t)] = k(v) = $\frac{1}{v}\int_0^\infty f(t)e^{-vt}dt$, t≥0, $k_1 \le v \le k_2$, (11)

The descriptions and basic equations will obtain the following effects
1) $A[t^n] = \frac{n!}{v^{n+2}}$
2) $A[f'(t)] = vK(v) - \frac{f(0)}{v}$
3) $A[f''(t)] = v^2K(v) - \frac{f'(0)}{v} - f(0)$
4) $A[f^{(n)}(t)] = v^nK(v) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2-n+k}}$

**Theorem 1:**
If f (v) is Aboodh transform of g(k). We identify that Aboodh transformation is given to observe from derivatives with integral order:
$A[g'(k)] = vf(v) - \frac{g(0)}{v}$.

**Proof:**

Let's shift the Aboodh $g^{-\prime}(k) = \frac{d}{dk}f(k)$, Using the following sections integration

$A\left[\frac{d}{dk}g(k)\right] = \frac{1}{v}\int_0^\infty \frac{d}{dk} g(k)\, e^{-vk}dk = \lim_{p\to\infty}\frac{1}{v}\int_0^p \frac{d}{dt} g(k)\, e^{-vk}dk$

$= \lim_{p\to\infty}\left\{\left[\frac{1}{v}g(k)e^{-vk}\right]_0^p + \frac{1}{v}\int_0^p g(k)e^{-vk}dk\right\}$

$= vK(v) - \frac{f(0)}{v},$

(12)

Theorem 1 proof gives us equation (5). If we do the same, Aboodh transforms the second order derivative

$A\left[\frac{d^2}{dk^2}g(k)\right] = A\left[\frac{d}{dk}\left(\frac{d}{dk}g(k)\right)\right] = vA\left[\frac{d}{dk}g(k)\right] - \left|\frac{\frac{d}{dk}g(k)}{v}\right|_{k=0}$

$= vA\left[vK(v) - \frac{f(0)}{v}\right] - \left|\frac{\frac{d}{dk}g(k)}{v}\right|_{k=0}$

$= v^2K(v) - \frac{g'(0)}{v} - f(0).$

If we pursue the same path, we obtain the following Aboodh transformation of the nth order:

$A\left[g^{(n)}(t)\right] = v^nK(v) - \sum_{k=0}^{n-1}\frac{g(0)}{v^{2-n+k}}$ for $n\geq 1$, (13)

$A\left[g^{(n)}(t)\right] = v^n\left[K(v) - \sum_{k=0}^{n-1}\frac{g^{(k)}(0)}{v^{2+k}}\right]$, (14)

**Theorem 2:**

If K(v) is the f(t) Aboodh transform, the Aboodh transform of the Liouville derivative may be considered:

$A[D^\alpha f(t)] = v^a\left|K(v) - \sum_{k=1}^{n}\frac{D^{\alpha-k}f(0)}{v^{\alpha-k+2}}\right|$; $-1 < n - 1 \leq \alpha < n$, (15).

**Proof:**

$A[D^\alpha f(t)] = v^\alpha K(v) - \sum_{k=0}^{n-1} v^{k[D^{\alpha-k-1}f(0)]}$

$= v^\alpha K(v) - \sum_{k=0}^{n} v^{k-1}\left[D^{\alpha-k}f(0)\right] = v^kK(v) - \sum_{k=1}^{n} v^{k-2}\left[D^{\alpha-k}f(0)\right]$

$= v^\alpha K(v) - \frac{1}{v^{-k+2}}\sum_{k=1}^{n}\left[D^{\alpha-k}f(0)\right] = v^kK(v) - \sum_{k=0}^{n}\frac{1}{v^{\alpha-k+2-\alpha}}\left[D^{\alpha-k}f(0)\right]$

$= v^\alpha K(v) - \sum_{k=1}^{n} v^\alpha\frac{1}{v^{\alpha-k+2}}\left[D^{\alpha-k}f(0)\right].$

Thus, the fractional order of f(t) Aboodh is modified as follows:

$A[D^\alpha f(t)] = v^\alpha\left[k(v) - \sum_{k=1}^{n}\left(\frac{1}{v}\right)^{\alpha-k+2}\left[D^{\alpha-k}f(0)\right]\right].$

(16)

**Definition: 4**

Caputo fractional derivative transition of Aboodh is described as follows by using theorem 2:

$A[D_t^\alpha g(t)] = v^\alpha A[g(t)] - \sum_{k=0}^{m-1} v^{k-\alpha-2}\, g^{(k)}(0)$, $m-1 < \alpha < m$, (17)

**4. Aboodh Transform Homotopy Perturbation Method (ATHPM)**

We pursue an overall form of partial differential non-linear equation, to show the fundamental principle of this approach:

$D_m^\lambda u(x,m) = L(u(x,m)) + N(u(x,m)) + f(x,m)$, $\lambda > 0$, (18)

Under the following terms

$D_0^k u(x,0) = g_k$, $k = 0,\ldots, n-1$, $D_0^n u(x,0) = 0$ and $n = [\lambda]$

Where $D_m^\lambda$ Denotes Caputo derivative operator without lack of generality, f is defined, N is a nonlinear general divide, and L is a linear division operator. f is defined.

Aboodh is converted to accomplish on both sides of equations.

$A\left[D_m^\lambda u(x,m)\right] = A[L(u(x,m))] + A[N(u(x,m))] + A[f(x,m)],$

(19)

With Aboodh's distinction property and beyond first criteria, we have:

$A[u(x,m)] = v^{-\lambda}A[L(u(x,m))] + v^{-\lambda}A[N(u(x,m))] + g(x,m),$

(20)

Working on both sides of equation with the Aboodh inverse (20) gives:

$u(x,m) = G(x,m) + A^{-1}\left[v^{-\lambda}A[L(u(x,t))]\right] + A^{-1}\left[v^{-\lambda}A[N(u(x,t))]\right],$ (21)

Where G(x,t) The function f(x, t) and its initial state are defined by the established function.

Now we use the form of homotopy disturbance.

$u(x,m) = \sum_{n=0}^{\infty} p^n u_n(x,m),$ (22)

And the nonlinear definition can be separated.

$Nu(x,m) = \sum_{n=0}^{\infty} p^n H_n(u)$ (23)

Wherever $H_n(u)$ It's polynomial and given by

$H_n(u_0, u_1, u_2, \ldots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i u_i(x,m)\right)\right]_{p=0},$ n=0,1,2

(24)

We are willing to substitute the equations (24) and (23) of equation (22):

$\sum_{n=0}^{\infty} p^n u_n(x, m) = G(x,m) + p\left[A^{-1}\left[v^{-\lambda}A[L(\sum_{n=0}^{\infty} p^n u_n(x,m))\right]\right] + v^{-\lambda}A[N(\sum_{n=0}^{\infty} p^n u_n(x,m))]]],$
(25)

Which is the Aboodh transformation relation and homotopic disruption method with the aid of polynomials which compare the coefficient of similar forces with p

$p^0: u_0(x,m) = G(x,m),$
$p^1: u_1(x,m) = A^{-1}[v^{-\lambda}A[L(u_0(x,m)) + H_0(u)]],$
$p^2: u_2(x,m) = A^{-1}[v^{-\lambda}A [L(u_1(x,m)) + H_1(u)]],$
$p^3: u_3(x,m) = A^{-1}[v^{-\lambda}A[L(u_2(x,m)) + H_2(u)]],$
$p^n = u_n(x, m) = A^{-1}[v^{-\lambda}A[L(u_{n-1}(x,m)) + H_{n-1}(u)]],$ (26)

Then the solution is;
$u(x, m) = \lim_{p\to\infty} u_n(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + \ldots$ (27).

The solution above converges in general very easily.

**Main Idea of (ATHPM)**
Let us consider a fractional nonlinear, non – homogeneous PDE of the form.

$D_t^{\delta}\varphi(x,m) = L(\varphi(x,m) + N(\varphi(x,m)) + g(x,m), \delta > 0,$ (28)

With the initial conditions listed below.

$D_0^{\delta}\varphi(x,0) = f_x, x = 0,1,2,\ldots\ldots, n-1$
(29)
$D_0^{n}\varphi(x,0) = 0$ and $n = [\delta]$
Where $D_m^{\delta}$ denotes without loss

**Applications**

**Ex:-1** $D_m^{\delta}\varphi + \left(\frac{\varphi^2}{2}\right) - \varphi_{xxm} = 0, m > 0, x \in R, 0 < \delta \leq 1,$
(30)
For the first criterion
$\varphi(x,0) = x,$
(31)

We use Aboodh transform on (30) with first condition (31)

$A[\varphi(x,m)] = x + V^{-\alpha} A\left[\varphi_{xxt} - \left(\frac{\varphi^2}{2}\right)_x\right]$
(32)

Using (32) transform the inverse Aboodh

$\varphi(x,m) = x + A^{-1}\left[V^{-\alpha} A\left[\varphi_{xxm} - \left(\frac{\varphi^2}{2}\right)_x\right]\right]$
(33)

Now, we use the form of homotopic disturbance we obtain

$\sum_{n=0}^{\infty} P^n \varphi_n(x,m) = x + p\left(A^{-1}\left[V^{-\alpha} A[(\sum_{n=0}^{\infty} P^n \varphi_n(x,t)_{xxm}) - (\sum_{n=0}^{\infty} P^n H_n(\varphi))]\right]\right),$ (34)

Non-linear word defined by polynoms $H_n(\varphi)$
So, it is given Polynomial.

$\sum_{n=0}^{\infty} P^n H_n(\varphi) = \left(\frac{\varphi^2}{2}\right)_x$
(35)

He is described as some of the factors.

$H_0(\varphi) = \left(\frac{\varphi_0^2}{2}\right)_x$
$H_1(\varphi) = (\varphi_0\varphi_1)_x \vdots$

We determine the P line power coefficients

$P^0: \varphi_0(x,m) = x$
$P^1: \varphi_1(x,m) = A^{-1}[V^{-\gamma}A(\varphi_0)_{xxm} - H_0(\varphi)] = \frac{-x\, m^{\gamma}}{\Gamma(1+\gamma)}$
$P^2: \varphi_2(x,m) = A^{-1}\left[V^{-\gamma} A[(\varphi_1)_{xxm} - H_1(\varphi)]\right] = \frac{x\, m^{2\gamma}}{\Gamma(1+2\gamma)}$ .

Hence series solution is given by as $\varphi(x, t) = \sum_{k=0}^{\infty} \varphi_k(x,m)$ or

$\varphi(x, m) = x - \frac{x\, m^{\gamma}}{\Gamma(1+\gamma)} + x\frac{m^{2\gamma}}{\Gamma(1+2\gamma)} - \ldots$
$= x\left(1 - \frac{m^{\gamma}}{\Gamma(1+\gamma)} + \frac{m^{2\gamma}}{\Gamma(1+2\gamma)} + \cdots + \frac{(-m)^{n\gamma}}{\Gamma(1+n\gamma)}\right)$
(36)

For the special care when $\gamma = 1$, we can get solution

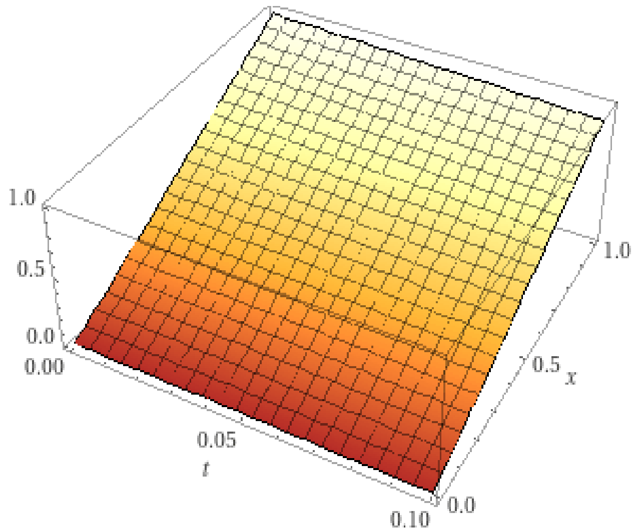
$= x\left(1 + \frac{(-m^{\gamma})}{\Gamma(1+\gamma)} + \frac{(-m^{\gamma})^2}{\Gamma(1+2\gamma)} + \cdots + \frac{(-m)^{n\gamma}}{\Gamma(1+n\gamma)} + \cdots\right)$
$= x\left(E_{\gamma}(-m^{\gamma})\right)$
(37)

Where $E_{\gamma}(-m^{\gamma})$ is a for the special care when $\gamma = 1$, we can get solution
$x\,E(-t) = x.e^{-m}$ , (38)

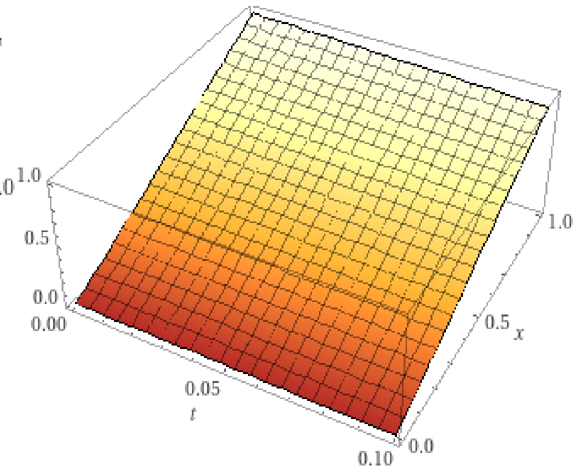
This is an exact solution and findings above are identical to the exact closed value.

**Table 1:** Evaluation of exact value with ATHPM value at $\gamma = 1$ and m = 0.1 for Example 1

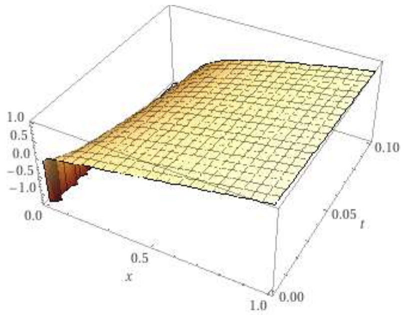
| X | Exact Value | ATHPM Value | Absolute error |
|---|---|---|---|
| 0.0 | 0.000000 | 0.000000 | 0.000000 |
| 0.1 | 0.090909 | 0.090483 | 0.000425 |
| 0.2 | 0.181818 | 0.180967 | 0.000850 |
| 0.3 | 0.272727 | 0.271451 | 0.001276 |
| 0.4 | 0.363636 | 0.361934 | 0.001701 |
| 0.5 | 0.454545 | 0.452418 | 0.002126 |
| 0.6 | 0.545454 | 0.542902 | 0.002551 |
| 0.7 | 0.636361 | 0.63338 | 0.002974 |
| 0.8 | 0.727272 | 0.723869 | 0.003402 |
| 0.9 | 0.818181 | 0.814353 | 0.003838 |
| 1.0 | 0.909090 | 0.904837 | 0.004253 |




**(a) ATHPM value**




**(b) Exact value**



**(c) Absolute Error**

**Figure 1** displays the result of $\varphi(x,t))$ for Eq. (30) when $\gamma$= 1
(a) Exact Value, (b) ATHPM Value (c) Absolute error = | Exact solution – ATHPM **Solution|**

**Ex:2** MRLW equation

$D_m^\gamma \varphi + \varphi_x + 6\varphi^2\varphi_x - \mu\varphi_{xxt} = 0$, m>0, x∈R, 0<$\gamma \leq 1$, (39)

With the initial conditions listed below

$\varphi(x,0) = \sqrt{a}\ \text{sech}\ (b(x-x_0))$, $b = \sqrt{\frac{a}{\lambda(a+1)}}$ (40)

Where λ the parameter positive and $x_0$ is is a place initially based.

Transform Aboodh on (39) with initial state (40) Transform, we have

$A\ [\varphi(x,m)] = \sqrt{a}\ \text{sech}\ (b(x-x_0)) + v^{-\gamma}A[\lambda\varphi_{xxm} - \varphi_x - 6\varphi^2\varphi_x]$, (41)

We have reverse transition with Aboodh

$\varphi(x,m) = \sqrt{a}\ \text{Sech}\ (b(x-x_0)) + A^{-1}\left[v^{-\gamma}A[\lambda\varphi_{xxm} - \varphi_x - 6\varphi^2\varphi_x]\right]$, (42)

Now applying HPT, we get

$\sum_{n=0}^{\infty} p^n\varphi_n(x,m) = \sqrt{a}\ \text{Sech}\ (b(x-x_0)) + p\ (A^{-1}[v^{-\gamma}A[\lambda(\sum_{n=0}^{\infty} p^n\varphi_n\ (x,m)) +$

$(\sum_{n=0}^{\infty} p^n \varphi_n(x,m))_{xxm} -$
$(\sum_{n=0}^{\infty} p^n H_n(\varphi))]])_{xxm}$ (43)

Non-linear word defined by polynomials $H_n(\varphi)$
So, it is given Polynomial
$\sum_{n=0}^{\infty} p^n H_n(\varphi) = \varphi\varphi_x$
(44)

The factors of He's polynomial is given by
$H_o(\varphi) = (\varphi_0)(\varphi_0)$
$H_1(\varphi) = (\varphi_0)(\varphi_1)_x + \varphi_1(\varphi_0)_x$
(46)

We have determined the coefficients of identical powers of p.
$p^0: \varphi(x,t) = \sqrt{a}\,\mathrm{Sech}(b(x-x_0))$
$p^1: \varphi(x,m) = A^{-1}[v^{-\gamma}A[\lambda(\varphi_0)_{xxt} - (\varphi_0)_x - H_0(\varphi)]]$
$= \sqrt{a}b\,[1 + 6a\mathrm{sech}^2(b(x-x_0))]$
$\mathrm{sech}(b(x-x_0))\tan(b(x-x_0))\frac{m^\gamma}{\Gamma(\gamma+1)}$,⁝

This indicates the sequence as
$\varphi(x,m) = \sum_{k=0}^{\infty} \varphi_k(x,m)$
$\varphi(x,m) = \sqrt{a}\mathrm{Sech}\,(b(x-x_0)) + \sqrt{a}b\,[1 + 6a\mathrm{Sech}^2 - (6(x-x_0))]$
$\mathrm{Sech}(b(x-x_0))\tanh(b(x-x_0))\frac{m^\gamma}{\Gamma(\gamma+1)} + \ldots..,$ (47)

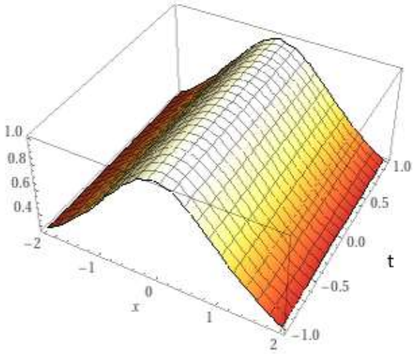
Putting $\gamma = 1$ in (47), we get integer order problem as (45)

$$\varphi(x,m) = \sqrt{a}\sec h(b(x-x_0)) + \sqrt{a}\,bm[1+6a\sec h^2(b(x-x_0))]$$
$$\sec h(b(x-x_0))\tan(b(x-x_0))\frac{m}{\Gamma(\gamma+1)} + \ldots.$$
(48)

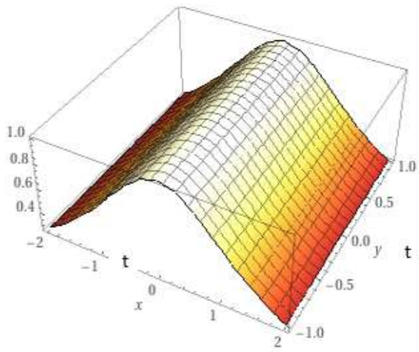
The findings above are identical to the exact closed solution
$$\varphi(x,m) = \sqrt{a}\sec h[b(x-(a+1)m-x_0)],$$
(49)

**Table 2:** Evaluation of exact Value with ATHPM Value at $\gamma = 1$ and m = 0.01 for Example 2

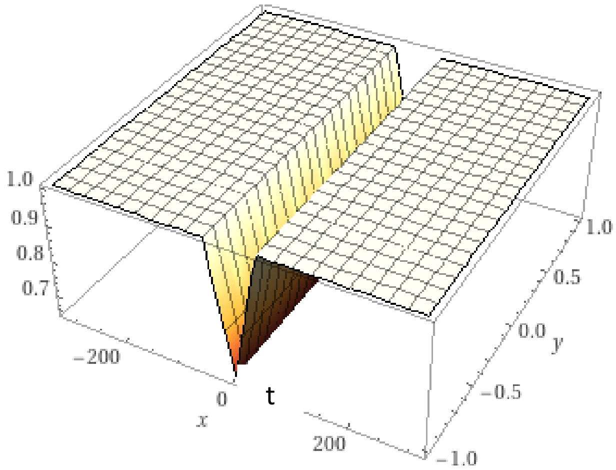
| X | Exact value | ATHPM value | Absolute error |
|---|---|---|---|
| 0.0 | 0.030103 | 0.030103 | 2.17E−8 |
| 0.1 | 0.030132 | 0.030132 | 1.61E−9 |
| 0.2 | 0.030161 | 0.030161 | 1.153E−8 |
| 0.3 | 0.030161 | 0.030189 | 1.143E−8 |
| 0.4 | 0.030218 | 0.030218 | 1.134E−8 |
| 0.5 | 0.030244 | 0.030246 | 1.123E−8 |
| 0.6 | 0.0302738 | 0.030273 | 1.116E−8 |



**(b)Exact Value**



**(b) ATHPM Value**




**(c) Absolute Error**

**Figure 2** displays the result of $\varphi(x, m)$ for Eq. (40) when $\gamma = 1$, $\lambda = 1$, $a = 0.001$ and $x_0 = 10$: (a) Exact Value, (b) ATHPM value and (c) Absolute error = | Exact Value – ATHPM **Value**|

**Ex:3** GRLW equation

$D_m^\gamma \varphi + \varphi_x + \varphi\varphi_x + \varphi_{xxt} = 0$, m > 0, x∈ R, 0<γ ≤ 1, (50)

For the first criterion

$\varphi(x,v) = 3a\,\text{sech}^2(bx)$, a>0, $b = \frac{1}{2}\sqrt{\frac{a}{1+a}}$, (51)

Applied on (50) with initial condition (51) Aboodh transform, we have

$A[\varphi(x, m)] = 3a\,\text{sech}^2(bx) - u^\gamma\, A[\varphi_x + \varphi_{xxt} + \varphi\varphi_x]$

Using reverse transformation of Aboodh now we have $\varphi_x$

$\varphi(x, m) = 3a\,\text{sech}^2(bx) - A^{-1}\left[u^\gamma A[\varphi_x + \varphi_{xxt} + \varphi_x]\right]$

Now incorporating He's polynomial method, we have

$\sum_{n=0}^{\infty} p^n \varphi_n(x, m) = 3a\,\text{sech}^2(bx) - P[A^{-1}[u^\gamma A(\sum_{n=0}^{\infty} p^n \varphi_n(x, m)) + (\sum_{n=0}^{\infty} p^n \varphi_n(x, m))_{xxm} + (\sum_{n=0}^{\infty} p^n H_n(\varphi))]$, (52)

Non-linear word defined by polynomials $H_n(\varphi)$

So, It is given Polynomial

$\sum_{n=0}^{\infty} P^n H_n(\varphi) = \varphi\varphi_x$, (53)

Any of his words were symbolized as polynomials.

$H_0(\varphi) = \varphi_0(\varphi_0)_x$

$H_1(\varphi) = \varphi_0(\varphi_1)_x + \varphi_1(\varphi_0)_x$ ⋮

We have determined the coefficients of identical powers of p. We get

$P^0 : \varphi_0(x, m) = 3a\,\text{sech}^2(bx)$

$P^1 : \varphi_1(x, m) = -A^{-1}[u^\gamma A[(\varphi_0)_x] + (\varphi_0)_{xxm} + H_0(\varphi)]$

$$= 3ab\left[1 + 6a + \cosh(2bx)\left[\text{sech}^4(bx)\tanh(bx)\frac{m^\gamma}{\Gamma(\gamma+1)}\right]\right],$$ (54)

⋮

So, the solution in series is given $\varphi(x, m) = \sum_{n=0}^{\infty} \varphi_i(x, m)$ (or)

$\varphi(x, m) = 3a\,\text{sech}^2(bx) + ab[1 + 6a + \cosh(2bx)]\text{sech}^4(bx)\tanh(bx)\frac{m^\gamma}{\Gamma(\gamma+1)}] + \ldots,$ (55)

Putting $\gamma = 1$ in (55), Then the problem of integer order has been settled as follows:

$\varphi(x, t) = 3a\,\text{sech}^2(bx) + ab[1 + 6a + \cosh(2bx)]\text{sech}^4(bx)\tanh(bx)\frac{m}{\Gamma(\gamma+1)}] + \ldots,$ (56)

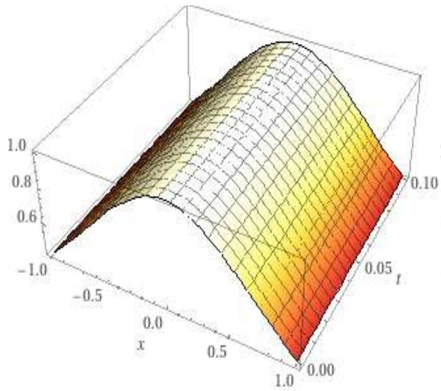
It is the same solution as the same closed solution.

$\varphi(x, m) = 3a\,\text{sech}^2[b(x - (1 + a)m)]$, (57)

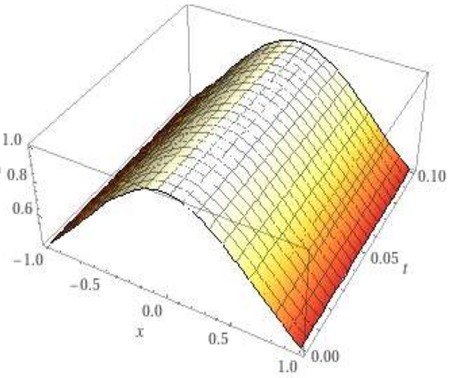
**Table 3** Evaluation of exact value with ATHPM Value at γ = 1 and m = 0.1 for Example 3

| X | Exact Solution | ATHPM | Absolute Error |
|---|---|---|---|
| 0.0 | 0.002999 | 0.002999 | 3.053E-11 |
| 0.1 | 0.003000 | 0.003000 | 4.656E-13 |

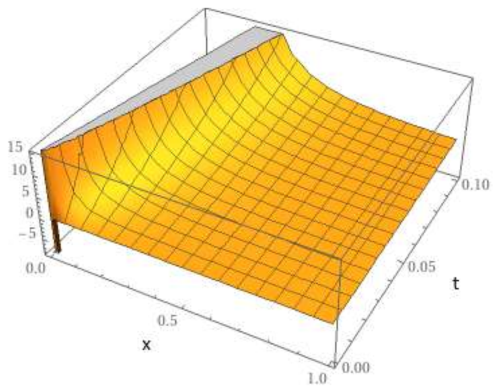
| | | | |
|---|---|---|---|
| 0.2 | 0.002999 | 0.002999 | 3.031E-11 |
| 0.3 | 0.002999 | 0.002999 | 5.971E-11 |
| 0.4 | 0.002999 | 0.002999 | 9.103E-13 |
| 0.5 | 0.002996 | 0.002996 | 1.199E-10 |
| 0.6 | 0.002998 | 0.002998 | 1.502E-10 |
| 0.7 | 0.0029997 | 0.002999 | 1.798E-11 |
| 0.8 | 0.002999 | 0.002999 | 2.094E-10 |
| 0.9 | 0.0029995 | 0.0029995 | 2.408E-10 |
| 1.0 | 0.0029993 | 0.002999 | 2.706E-10 |




**(a) Exact Value**




**(b)ATHPM Value**




**(c) Absolute Error**

**Figure 3** displays the result of $\varphi(x,t)$forEq. (50)when $\gamma$= 1 and a= 0.001:
(a) Exact Value, (b) ATHPM Value (c) Absolute error| Exact Value – ATHPM **Value**

**Ex: 4** RLW equation

$D_m^{\gamma}\varphi + \varphi_x = \varphi_{xxm}$, m >0, x∈R, 0 < γ ≤1, (58)

With initial condition $\varphi(x, 0) = e^{-x}$, (59)

Aboodh transforms on the first condition of (58) (59), we have

$A[\varphi(x,m)] = e^{-x} + u^{\gamma}A[2\varphi_{xxm} - \varphi_x]$, (60)

Using reverse transformation of Aboodh now, we have

$\varphi(x,m) = e^{-x} + A^{-1}\left[u^{\gamma}A[2\varphi_{xxm} - \varphi_x]\right]$, (61)

Now using HPT, we have

$\sum_{n=0}^{\infty} p^n\varphi_n(x,t) = e^{-x} + p\left(A^{-1}\left[u^{\gamma}A\left[2\left(\sum_{n=0}^{\infty} p^n\varphi_n(x,m)\right)_{xxm} - \left(\sum_{n=0}^{\infty} p^n\varphi_n(x,m)_x\right)\right]\right]\right)$, (62)

Computing coefficient of the powers of $p_x$ we get

$p^0$ : $\varphi_0(x,m) = e^{-x}$

$p^1$ : $\varphi_1(x,m) = A^{-1}\left[u^{\gamma}A[2 - (\varphi_0)_{xxm} - (\varphi_0)_x]\right] = e^{-x}\frac{m^{\gamma}}{\Gamma(\gamma+1)}$ (63):

The series solution is then represented $\varphi(x,t) = \sum_{i=0}^{\infty}\varphi_i(x,m)$

or

$$\varphi(x,m) = e^{-x} + e^{-x}\frac{m^{\gamma}}{\Gamma(\gamma+1)} + \ldots, \quad (64)$$

Putting γ =1 in (64), we're seeing a classic dilemma answer

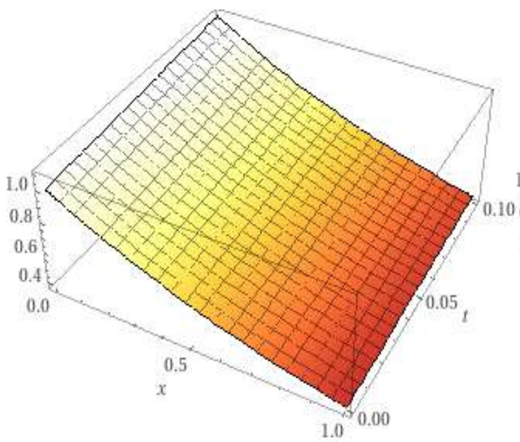
$\varphi(x, m) = e^{-x} + m\, e^{-x} + \ldots,$ (65)

It is the same solution as the same closed solution.

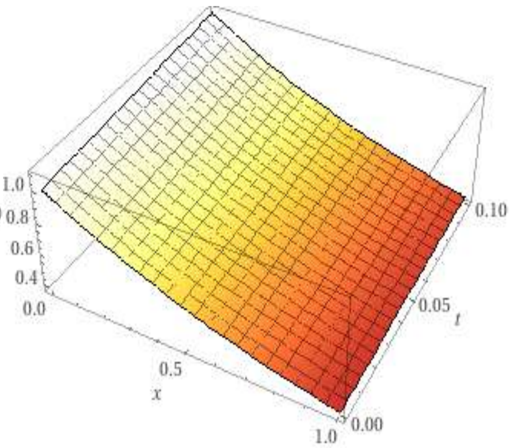
$\varphi(x, m) = e^{m-x},$ (66)

**Table 4:** Evaluation of exact solution with ATHPM solution at γ =1 and t = 0.0000001

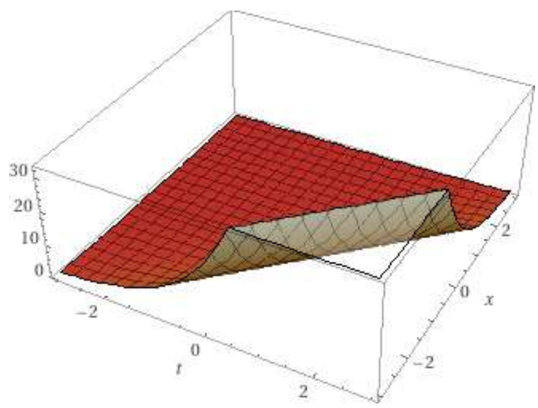
| X | Exact Solution | ATHPM Solution | Absolute Error |
|---|---|---|---|
| 0.0 | 3.00000 | 3.00000 | 0.000000000 |
| 0.1 | 2.970198 | 2.97019 | 8.396E-9 |
| 0.2 | 2.883129 | 2.88312 | 2.343E-8 |
| 0.3 | 2.74541 | 2.74541 | 1.098E-7 |
| 0.4 | 2.566916 | 2.566916 | 2.435E-07 |
| 0.5 | 2.359343 | 2.3593 | 3.925E-07 |
| 0.6 | 2.134733 | 2.134734 | 5.223E-07 |
| 0.7 | 1.904219 | 1.90421 | 6.062E-07 |
| 0.8 | 1.677165 | 1.67716 | 6.352E-07 |
| 0.9 | 1.460752 | 1.46075 | 6.160E-07 |
| 1.0 | 1.259923 | 1.25992 | 5.613E-07 |




**(a) ATHPM Value**




**(b) Exact Value**




**(c) Absolute Error**

**Figure 4** displays the result of $\varphi(x, t)$for Eq. (58)when $\gamma$= 1:
(a) ATHPM Value, (b) Exact Value (c) Absolute error| Exact solution –ATHPM **Solution|**

**Ex: 5** Let us analyze the next linear RLW factional period.

$D_m^{\gamma} + \varphi_{xxxx} = 0, m > 0, x \in R, 0 < \gamma \leq 1,$ (67)

For the first condition.

$\varphi(x, o) = \sin x,$ (68)

Transform Aboodh on (67) with original state (68)

$A[\varphi(x, m)] = \sin x - u^{\gamma} A[\varphi_{xxxx}],$ (69)

Now applying inverse Aboodh transform, we have
$\varphi(x,m) = \sin x - A^{-1}[u^{\gamma}A[\varphi_{xxxx}]]$, (70)

Now employing HPT, we have
$\sum_{n=0}^{\infty} p^n \varphi_n(x,m) =$
$\sin x - p(A^{-1}[(\sum_{n=0}^{\infty} p^n \varphi_n(x,t))_{xxxx}])$, (71)

We determine the coefficients of the same p powers.
$p^0: \varphi_0(w,m) = \sin x$,
$p^1: \varphi_1(x,m) = -A^{-1}[u^{\gamma}[(\varphi_0)_{xxxx}]]$
$= -\sin x \dfrac{m^{\gamma}}{\Gamma_{(\gamma+1)}}$ ⁝⁝

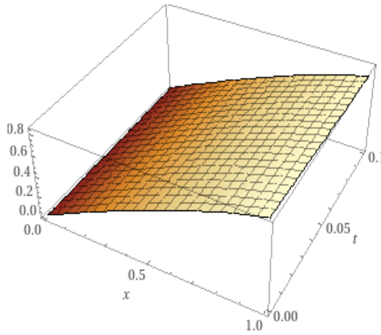
So the solution in series is given
$\varphi(x,m) = \sum_{n=0}^{\infty} p^n \varphi_n(x,m)$ or
$\varphi(x,m) = \sin x - \sin x \dfrac{m^{\gamma}}{\Gamma_{(\gamma+1)}} + \cdots$, (72)

Putting $\gamma = 1$ in (72), we're having the classical dilemma answer
$\varphi(x,m) = \sin x - t\sin x + \cdots$, (73)

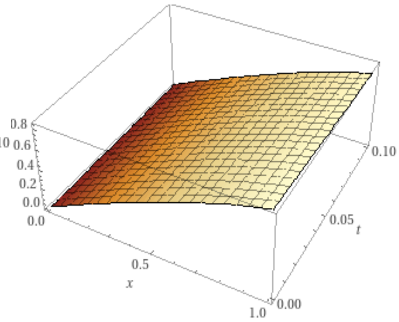
It is the same solution as the same closed solution.
$\varphi(x,m) = e^{-m}\sin x$, (74)

**Table 5 :** The exact solution is compared to the ATHPM solution at $\gamma = 1$ and $m = 0.0000001$

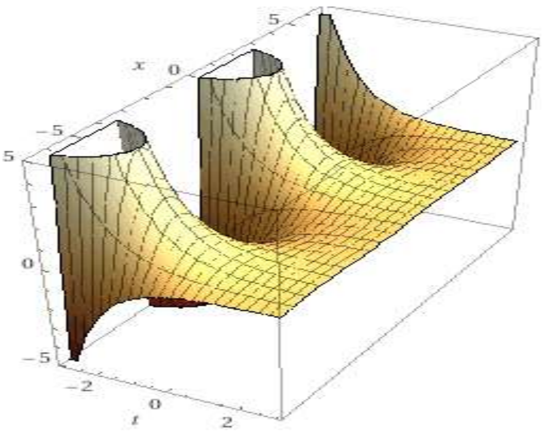
| X | Exact Value | ATHPM Value | Absolute Error |
|---|---|---|---|
| 0.0 | 0.300000 | 0.300000 | 0.0000000000 |
| 0.1 | 0.299318 | 0.299931 | 2.721E-10 |
| 0.2 | 0.299727 | 0.299727 | 5.404E-10 |
| 0.3 | 0.299387 | 0.299387 | 8.013E-10 |
| 0.4 | 0.298911 | 0.298911 | 1.051E-9 |
| 0.5 | 0.298301 | 0.298301 | 1.285E-9 |
| 0.6 | 0.297558 | 0.297558 | 1.601E-9 |
| 0.7 | 0.296683 | 0.296683 | 1.896E-9 |
| 0.8 | 0.295678 | 0.295678 | 2.164E-9 |
| 0.9 | 0.294544 | 0.294544 | 2.404E-9 |
| 1.0 | 0.293283 | 0.293284 | 2.5112E-9 |




(a) **ATHPM Value**




**(b) Exact Value**




**(c) Absolute Error**

**Figure 5** displays the result of $\varphi(x,m)$ for Eq. (67) when $\gamma$= 1:

(a) ATHPM value, (b) Exact Value
(c) Absolute error| Exact Value – ATHPM **Value|**

**Conclusions and Observations**

The proposed analytical approach offers a systematic and efficient method for solving FPDEs. By introducing fractional derivatives and utilizing powerful mathematical techniques, the researchers were able to obtain exact values to the governing equations of ion acoustic waves. This breakthrough enables a deeper understanding of the underlying dynamics and behavior of plasma waves. In this paper, we all explore the ATHPM for obtaining accurate and approximative values of nonlinear time-fractional RLW, nonlinear time-fractional MRLW, and nonlinear time-fractional GRLW using Mathematica software. This work shown that the ATHPM is relatively straight forward, efficient, suitable for a variety of nonlinear equations, and appropriate. The ATHPM has creative methods for reducing the size of mathematical computations. In addition to the fact that to solve nonlinear equations without the need of Adomian's polynomials, the ATHPM has great suitability for the numerical results. The ATHPM has a distinct advantage regarding the decomposition algorithm. Furthermore, the ATHPM is well suited for researching additional nonlinear equations that appear in nonlinear science and engineering. In the final stage, we draw the conclusion that the ATHPM can give good development in all numerical methods, has a variety of novel applications in science and engineering. Furthermore, the obtained solutions provide valuable insights into the physical properties of ion acoustic waves in plasma. The researchers were able to analyze the dispersion relation, phase velocity, and other relevant parameters, shedding light on the nature of these waves and their interactions within the plasma medium. This information is crucial for developing accurate models and predicting plasma behavior in various applications. This research opens new avenues for further investigation and application, driving progress in plasma physics and related disciplines.

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