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Soliton excitation and energy sharing mechanism of a higher dimensional nonlinear Schrödinger equation

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Abstract

By using a lattice model and proposing a model to account for the dynamics of alpha-helical proteins, we examine the dynamics by creating soliton solutions. We investigate graphically how different interaction parameters affect energy transfer across alpha-helical proteins. Under various physical conditions, we determine the soliton's energy, centre of mass, velocity, and stability.

Keywords: Alpha-helical proteins, Nonlinear Schrödinger equation, Soliton.

1. Introduction

One significant form of protein secondary structure is the alpha-helix. In order of double-bonded carbon to an oxygen, nitrogen, and hydrogen that are close to one another (H-N-C=O) experience localized hydrogen bonding which results in the production of alpha-helix proteins. Due to the C=O double bond's (amide-I) vibrational structure in the infrared frequency spectrum, quantum transitions happen in each peptide group. Phonons will spread across the system from one group to the next, resulting in dispersive effects. In addition, on considerably longer time and space scales, completely classical elastic longitudinal waves will move along the chain while the helix acts as a spring. According to Davydov's proposed nonlinear mechanism [1-5], the energy released during the hydrolysis of adenosine triphosphate (ATP) can propagate along the alpha-helix in the form of a soliton. This mechanism is chargeable for the garage and switch of vibrational energy (intrapeptide vibration amide-I) in alpha-helical proteins. In actuality, an amide-I vibration self-traps as a result of the interaction between the high frequency amide-I vibrations (vibrations of the double C=O bond of peptide groups) and the low frequency acoustic vibrations of the protein. Therefore, the interaction of the amide-I mode with longitudinal chain deformation and the effects of dispersion brought on by the resonance interaction of adjacent peptide groups serve as the foundation for the production mechanism of the Davydov soliton. Davydov demonstrated that the N-soliton solutions of the completely integrable nonlinear Schrödinger (NLS) equation govern

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the dynamics of alpha-helical proteins. Many students have studied the dynamics of energy transfer through alpha-helical proteins at the classical and quantum levels after Davydov [6-33].

The nonlinear dynamics of continuous and discontinuous (1+1) dimensional alpha helical protein systems have only been studied in the aforementioned papers. However, the actual 2D and 3D crystal systems are naturally interesting, it is required to generalize lattice models to higher dimensions. Investigating the soliton dynamics in dimensional lattices is very interesting. Magna $etal.$ [34] examined the quasi-soliton states in a three-dimensional discrete model of alpha helical protein in this context by putting forth a Hamiltonian for a square lattice and creating the equations of motion using an appropriate wave function. The solitonic characteristics of energy transmission through alpha-helical proteins in three dimensions, however, have not yet been studied and only the ground state properties in the continuum limit have been described. As a result of the above assumptions, we provide an integrable model of alpha-helical proteins to study soliton dynamics in three dimensions.

Additionally, the mechanism of energy transfer involves inhomogeneous alpha-helical protein systems. Alpha-helical proteins exhibit inhomogeneity as a result of the flaws brought on by the inclusion of extra molecules, such as medicines, in particular locations along the sequence and by the presence of a basic sites, such as nonpolar mimics of thymine [35,36]. It has not yet been documented in the literature how inhomogeneity affects three-dimensional alpha-helical proteins. Non the current research, we additionally investigate the impact of inhomogeneity on the stability of soliton propagation in three dimensions.

The article follows the following outline: In Section 2, a Hamiltonian is presented and the equations of motion are constructed for an alpha-helical protein square lattice model with quadrupole-quadrupole type of interactions. We use the perturbation method to find the soliton solution of the resulting nonlinear equation in section 3. Section 4 discusses the soliton vibration's energy, mass velocity, and center of mass. The stability of the nonlinear system is depicted in Section 5. Section 6 is an overview of the work completed.

2. Model Hamiltonian and Equation of Motion

We use a model that expresses an inhomogeneous system of alpha-helical proteins. Assuming that the identical spine and the nearby chain, where the Hamiltonian is represented by having quadrupole-quadrupole interaction between nearby atoms,

$$H = H_{ex} + H_{ph} + H_{ph-ex}, \tag{1}$$

In Eq. (1), H_{ex} denotes the exchange Hamiltonian representing the internal molecular excitation, H_{ph} the support of the phonon Hamiltonian corresponding to the displacement of the unit cell from its equilibrium position, and H_{ph-ex} represents the Hamiltonian of the bond between internal molecules excitations with the displacements.
The exciton Hamiltonian is given by

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$$H_{ex} = \sum_{l,m,n} \{E_0 B_{l,m,n} B^{\dagger}_{l,m,n} + E_1 B_{l,m,n} B^{\dagger}_{l,m,n} B_{l,m,n} B^{\dagger}_{l,m,n} - J_0 F_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l,m} + B_{n,l,m} B^{\dagger}_{n+1,l,m} + B^{\dagger}_{n,l,m} B_{n,l+1,m} + B_{n,l,m} B^{\dagger}_{n,l+1,m} + B^{\dagger}_{n,l,m} B_{n,l,m+1} + B_{n,l,m} B^{\dagger}_{n,l,m+1}) - J'_0 F_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l+1,m+1} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m+1} + B^{\dagger}_{n,l,m} B_{n+1,l+1,m-1} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m-1} + B^{\dagger}_{n,l,m} B_{n+1,l-1,m-1} + B_{n,l,m} B^{\dagger}_{n+1,l-1,m-1}) - J''_0 F_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l+1,m} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m} + B^{\dagger}_{n,l,m} B_{n+1,l,m+1} + B_{n,l,m} B^{\dagger}_{n+1,l,m+1} + B^{\dagger}_{n,l,m} B_{n,l+1,m+1} + B_{n,l,m} B^{\dagger}_{n,l+1,m+1}) - J_1 G_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l,m} B^{\dagger}_{n,l,m} B_{n+1,l,m} + B_{n,l,m} B^{\dagger}_{n+1,l,m} B_{n,l,m} B^{\dagger}_{n+1,l,m} + B^{\dagger}_{n,l,m} B_{n,l+1,m} B^{\dagger}_{n,l,m} B_{n,l+1,m} + B_{n,l,m} B^{\dagger}_{n,l+1,m} B_{n,l,m} B^{\dagger}_{n,l+1,m} + B^{\dagger}_{n,l,m} B_{n,l,m+1} B^{\dagger}_{n,l,m} B_{n,l,m+1} + B_{n,l,m} B^{\dagger}_{n,l,m+1} B_{n,l,m} B^{\dagger}_{n,l,m+1}) - J'_1 G_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l+1,m+1} B^{\dagger}_{n,l,m} B_{n+1,l+1,m+1} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m+1} B_{n,l,m} B^{\dagger}_{n+1,l+1,m+1} + B^{\dagger}_{n,l,m} B_{n+1,l+1,m-1} B^{\dagger}_{n,l,m} B_{n+1,l+1,m-1} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m-1} B_{n,l,m} B^{\dagger}_{n+1,l+1,m-1} + B^{\dagger}_{n,l,m} B_{n+1,l-1,m-1} B^{\dagger}_{n,l,m} B_{n+1,l-1,m-1} + B_{n,l,m} B^{\dagger}_{n+1,l-1,m-1} B_{n,l,m} B^{\dagger}_{n+1,l-1,m-1}) - J''_1 G_{n,l,m} (B^{\dagger}_{n,l,m} B_{n+1,l+1,m} B^{\dagger}_{n,l,m} B_{n+1,l+1,m} + B_{n,l,m} B^{\dagger}_{n+1,l+1,m} B_{n,l,m} B^{\dagger}_{n+1,l+1,m} + B^{\dagger}_{n,l,m} B_{n+1,l,m+1} B^{\dagger}_{n,l,m} B_{n+1,l,m+1} + B_{n,l,m} B^{\dagger}_{n+1,l,m+1} B_{n,l,m} B^{\dagger}_{n+1,l,m+1} + B^{\dagger}_{n,l,m} B_{n,l+1,m+1} B^{\dagger}_{n,l,m} B_{n,l+1,m+1} + B_{n,l,m} B^{\dagger}_{n,l+1,m+1} B_{n,l,m} B^{\dagger}_{n,l+1,m+1})\}, \quad (2)$$

where the outlines for n, l, and m pass through the unit cell (H-N-C=O) along an infinite hydrogen bond spine $B^{\dagger}_{n,l,m}$ and $B_{n,l,m}$ and forms (annihilates) an excitation at a site (n, l, m). E_0 is the excitation energy. The hopping integrals $J_0, J'_0, J''_0, J_1, J'_1$ and J''_1 are used to represent the in-dispute amide-I bind as well as its neighbours in the X, Y, and Z directions of the hydrogen bonding spine. The quadrupole-quadrupole interaction between a particular amide I bond and its diagonally adjacent bond on the hydrogen-bonding spine is represented by the characters J_1, J'_1 and J''_1.

The phonon is described by

$$H_{ph} = \sum_{n,l,m} \{+\frac{\hat{P}^2_{n,l,m}}{2M} + \frac{K}{2}[(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2\}, \quad (3)$$

The unit cell mass of a peptide and the elastic constant of a helix are denoted by the constants M and K in equation (3) respectively. The operator of the peptide groups longitudinal displacement from its equilibrium position parallel to the helical axis is $U_{n,l,m}$. The momentum $\hat{P}_{n,l,m}$ is conjugate to the displacement $\hat{U}_{n,l,m}$.

$$H_{ph-ex} = \sum_{n,l,m} \{\chi_1 B_{n,l,m} B^{\dagger}_{n,l,m} [(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2] + \chi_2 B_{n,l,m} B_{n,l,m} B^{\dagger}_{n,l,m} B^{\dagger}_{n,l,m} [(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2]\}, \quad (4)$$

where the parameter for the exciton-phonon coupling is χ. The collective excitations of a coherent state or the Hamiltonian are represented as using Eqs. (2), (3), and (4).

$$H = \sum_{l,m,n} \{E_0 B_{l,m,n} B^\dagger_{l,m,n} + E_1 B_{l,m,n} B^\dagger_{l,m,n} B_{l,m,n} B^\dagger_{l,m,n} - J_0 F_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l,m} + B_{n,l,m} B^\dagger_{n+1,l,m} + B^\dagger_{n,l,m} B_{n,l+1,m} + B_{n,l,m} B^\dagger_{n,l+1,m} + B^\dagger_{n,l,m} B_{n,l,m+1} + B_{n,l,m} B^\dagger_{n,l,m+1}) - J_0' F_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l+1,m+1} + B_{n,l,m} B^\dagger_{n+1,l+1,m+1} + B^\dagger_{n,l,m} B_{n+1,l+1,m-1} + B_{n,l,m} B^\dagger_{n+1,l+1,m-1} + B^\dagger_{n,l,m} B_{n+1,l-1,m-1} + B_{n,l,m} B^\dagger_{n+1,l-1,m-1}) - J_0'' F_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l+1,m} + B_{n,l,m} B^\dagger_{n+1,l+1,m} + B^\dagger_{n,l,m} B_{n+1,l,m+1} + B_{n,l,m} B^\dagger_{n+1,l,m+1} + B^\dagger_{n,l,m} B_{n,l+1,m+1} + B_{n,l,m} B^\dagger_{n,l+1,m+1}) - J_1 G_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l,m} B^\dagger_{n,l,m} B_{n+1,l,m} + B_{n,l,m} B^\dagger_{n+1,l,m} B_{n,l,m} B^\dagger_{n+1,l,m} + B^\dagger_{n,l,m} B_{n,l+1,m} B^\dagger_{n,l,m} B_{n,l+1,m} + B_{n,l,m} B^\dagger_{n,l+1,m} B_{n,l,m} B^\dagger_{n,l+1,m} + B^\dagger_{n,l,m} B_{n,l,m+1} B^\dagger_{n,l,m} B_{n,l,m+1} + B_{n,l,m} B^\dagger_{n,l,m+1} B_{n,l,m} B^\dagger_{n,l,m+1}) - J_1' G_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l+1,m+1} B^\dagger_{n,l,m} B_{n+1,l+1,m+1} + B_{n,l,m} B^\dagger_{n+1,l+1,m+1} B_{n,l,m} B^\dagger_{n+1,l+1,m+1} + B^\dagger_{n,l,m} B_{n+1,l+1,m-1} B^\dagger_{n,l,m} B_{n+1,l+1,m-1} + B_{n,l,m} B^\dagger_{n+1,l+1,m-1} B_{n,l,m} B^\dagger_{n+1,l+1,m-1} + B^\dagger_{n,l,m} B_{n+1,l-1,m-1} B^\dagger_{n,l,m} B_{n+1,l-1,m-1} + B_{n,l,m} B^\dagger_{n+1,l-1,m-1} B_{n,l,m} B^\dagger_{n+1,l-1,m-1}) - J_1'' G_{n,l,m} (B^\dagger_{n,l,m} B_{n+1,l+1,m} B^\dagger_{n,l,m} B_{n+1,l+1,m} + B_{n,l,m} B^\dagger_{n+1,l+1,m} B_{n,l,m} B^\dagger_{n+1,l+1,m} + B^\dagger_{n,l,m} B_{n+1,l,m+1} B^\dagger_{n,l,m} B_{n+1,l,m+1} + B_{n,l,m} B^\dagger_{n+1,l,m+1} B_{n,l,m} B^\dagger_{n+1,l,m+1} + B^\dagger_{n,l,m} B_{n,l+1,m+1} B^\dagger_{n,l,m} B_{n,l+1,m+1} + B_{n,l,m} B^\dagger_{n,l+1,m+1} B_{n,l,m} B^\dagger_{n,l+1,m+1}) + \frac{\hat{P}^2_{n,l,m}}{2M} + \frac{K}{2} [(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2] + \chi_1 B_{n,l,m} B^\dagger_{n,l,m} [(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2] + \chi_2 B_{n,l,m} B_{n,l,m} B^\dagger_{n,l,m} B^\dagger_{n,l,m} [(U_{n,l,m} - U_{n-1,l,m})^2 + (U_{n,l,m} - U_{n,l-1,m})^2 + (U_{n,l,m} - U_{n,l,m-1})^2]\}, \quad (5)$$

The site-dependent inhomogeneities $F_{n,l,m}$ and $G_{n,l,m}$ can be included in the Hamiltonian to study the effects of inhomogeneities on nonlinear lattices during energy propagation. The site-dependent inhomogeneous alpha-helical protein's Hamiltonian is described as

$$i\hbar \frac{d\phi_{n,l,m}}{dt} = E_0 \phi_{n,l,m} + 2E_1 \phi^\dagger_{n,l,m} \phi^2_{n,l,m} - J_0 [F_{n,l,m} \phi_{n+1,l,m} + F_{n-1,l,m} \phi_{n-1,l,m} + F_{n,l,m} \phi_{n,l+1,m} + F_{n,l-1,m} \phi_{n,l-1,m} + F_{n,l,m} \phi_{n,l,m+1} + F_{n,l,m-1} \phi_{n,l,m-1}] - J_0' [F_{n,l,m} \phi_{n+1,l+1,m+1} + F_{n-1,l-1,m-1} \phi_{n-1,l-1,m-1} + F_{n,l,m} \phi_{n+1,l+1,m-1} + F_{n-1,l-1,m+1} \phi_{n-1,l-1,m+1} + F_{n,l,m} \phi_{n+1,l-1,m-1} + F_{n-1,l+1,m+1} \phi_{l-1,m+1,n+1}] - J_0'' [F_{n,l,m} \phi_{n+1,l+1,m} + F_{n-1,l-1,m} \phi_{n-1,l-1,m} + F_{n,l,m} \phi_{n+1,l,m-1} + F_{n-1,l,m-1} \phi_{n-1,l,m-1} + F_{n,l,m} \phi_{n,l+1,m+1} + F_{n,l-1,m+1} \phi_{n,l-1,m-1}] - 2J_1 \phi^\dagger_{n,l,m} [G_{n,l,m} \phi^2_{n+1,l,m} + G_{n-1,l,m} \phi^2_{n-1,l,m} + G_{n,l,m} \phi^2_{n,l+1,m} + G_{n,l-1,m} \phi^2_{n,l-1,m} + G_{n,l,m} \phi^2_{n,l,m+1} + G_{n,l,m-1} \phi^2_{n,l,m-1}] - J_1' \phi^\dagger_{n,l,m} [G_{n,l,m} \phi^2_{n+1,l+1,m+1} + G_{n-1,l-1,m-1} \phi^2_{n-1,l-1,m-1} + G_{n,l,m} \phi^2_{n+1,l+1,m-1} + G_{n-1,l-1,m+1} \phi^2_{n-1,l-1,m+1}$$

$$+G_{n,l,m}\phi^2_{n+1,l-1,m-1} + G_{n-1,l+1,m+1}\phi^2_{l-1,m+1,n+1}] - J_1^{"}\phi^{\dagger}_{n,l,m}[G_{n,l,m}\phi^2_{n+1,l+1,m}$$
$$+G_{n-1,l-1,m}\phi^2_{n-1,l-1,m} + G_{n,l,m}\phi^2_{n+1,l,m-1} + G_{n-1,l,m-1}\phi^2_{n-1,l,m-1} + G_{n,l,m}$$
$$\phi^2_{n,l+1,m+1} + G_{n,l-1,m+1}\phi^2_{n,l-1,m-1}] + \chi_1\phi_{n,l,m}[u_{n+1,l,m} - u_{n-1,l,m}$$
$$+u_{n,l+1,m} - u_{n,l-1,m} + u_{n,l,m+1}\phi^2_{n,l-1,m-1}] + \chi_1\phi_{n,l,m}[u_{n+1,l,m} - u_{n-1,l,m}$$
$$-U_{n,l,m-1}] + 2\chi_2\phi^{\dagger}_{n,l,m}\phi^2_{n,l,m}[_{n+1,l,m} - u_{n-1,l,m} + u_{n,l+1,m} - u_{n,l-1,m} +$$
$$u_{n,l,m+1} - U_{n,l,m-1}], \quad (6)$$

$$M\frac{d^2u_{n,l,m}}{dt} = -K[6u_{n,l,m} - u_{n,l,m} + u_{n+1,l,m} - u_{n-1,l,m} + u_{n,l+1,m} - u_{n,l-1,m} +$$
$$u_{n,l,m+1} - u_{n,l,m-1}] + \chi_1[|\phi_{n+1,l,m}|^2 - |\phi_{n-1,l,m}|^2 + |\phi_{n,l+1,m}|^2 -$$
$$|\phi_{n,l-1,m}|^2 + |\phi_{n,l,m+1}|^2 - |\phi_{n,l,m-1}|^2] + +\chi_2[|\phi_{n+1,l,m}|^4 -$$
$$|\phi_{n-1,l,m}|^4 + |\phi_{n,l+1,m}|^4 - |\phi_{n,l-1,m}|^4 + |\phi_{n,l,m+1}|^4 - |\phi_{n,l,m-1}|^4]. \quad (7)$$

The discrete dynamics of alpha-helical proteins are represented by equations (6) and (7). Due to the complexity of these equations, we use the Taylor expansions to vary for the continuum limit, we obtain (38) which are given in Appendix.

$$M_{utt} = K\gamma^2[U_{xx} + U_{yy} + U_{zz}] + 6\delta^2[\chi_1 + 2\chi_2]((|\phi|^2)_x + (|\phi|^2)_y + (|\phi|^2)_z) \quad (8)$$

Introducing the wave variable $\xi = K_1x + K_2y + K_3z - ct$ in Eqs. (38) and (8), and solving Eq. (8) we get $u_{\xi} = 2(\chi_1 + 2\chi_2)A|\phi|^2$ and using it in Eq. (38), we get

$$i\phi_t + a_1\phi + a_2\phi_x + a_3\phi_y + a_4\phi_z + a_5\phi_{xx} + a_6\phi_{yy} + a_7\phi_{zz} + a_8\phi_{xy} + a_9\phi_{xz} + a_{10}$$
$$\phi_{yz} - a_{11}|\phi^2|\phi + a_{12}\phi_{xxx} + a_{13}\phi_{yyy} + a_{14}\phi_{zzz} + a_{15}\phi_{xxy} + a_{16}\phi_{xxz} + a_{17}\phi_{xyy}$$
$$+a_{18}\phi_{xyz} + a_{19}\phi_{xzz} + a_{20}\phi_{yyz} + a_{21}\phi_{yzz} + a_{22}\phi(2|\phi_x|^2 + 2|\phi_y|^2 + 2|\phi_z|^2 + |\phi|^2_{xx}$$
$$+|\phi|^2_{yy} + |\phi|^2_{zz}) + a_{23}(\phi_{xxxx} + \phi_{yyyy} + \phi_{zzzz}) + a_{24}(\phi_{xxxy} + \phi_{xxxz} + \phi_{yyyz} +$$
$$\phi_{xyyy} + \phi_{xzzz}) + a_{25}(\phi_{xxyy} + \phi_{xxzz} + \phi_{yyzz}) + a_{26}(\phi_{xxyz} + \phi_{yyzx} + \phi_{zzxy}) - a_{27}$$
$$|\phi^4|\phi = 0 \quad (9)$$

Where a_1, a_2, a_3... are given Appendix equation (38) Eq. (9) depicts the three-dimensional dynamics of higher-order inhomogeneous alpha-helical proteins. The strong sine-cosine function method supplies an effective way to construct solitary wave solutions. To understand the effect of inhomogeneity on soliton excitations, we solve Eq. (9) using a perturbation technique called the sine-cosine function method.

3. Effect of Inhomogeneity

Sufficient techniques have been put forth in recent years [33-39] for getting explicit travel as well as solitary wave solutions to non-linear evolution equations. Nonlinear equations can be solved by using numerous techniques. To create solitary wave soliton excitation using one of these methods we have chosen the sine-cosine approach.

$$-v_t + a_1 u + a_2 u_x + a_3 u_y + a_4 u_z + a_5 u_{xx} + a_6 u_{yy} + a_7 u_{zz} + a_8 u_{xy} + a_9 u_{xz} + a_{10} u_{yz} - a_{11} u^2 - a_{11} u v^2 + a_{12} u_{xxx} + a_{13} u_{yyy} + a_{14} u_{zzz} + a_{15} u_{xxy} + a_{16} u_{xxz} + a_{17} u_{xyy} + a_{18} u_{xyz} + a_{19} u_{xzz} + a_{20} u_{yyz} + a_{21} u_{yzz} + a_{22} 2uu_x^2 + a_{22} 2uv_x^2 + a_{22} 2uu_y^2 + a_{22} 2uv_y^2 + a_{22} 2uu_z^2 + a_{22} 2uv_z^2 + a_{22} 2u^2 u_{xx} + a_{22} 2u\, vv_{xx} + a_{22} 2u^2 u_{yy} + a_{22} 2uvv_{yy} + a_{22} 2u^2 u_{zz} + a_{22} 2uvv_{zz} + a_{23} u_{xxxx} + a_{23} u_{yyyy} + a_{23} u_{zzzz} + a_{24} u_{xxxy} + a_{24} u_{xxxz} + a_{24} u_{yyyz} + a_{24} u_{xyyy} + a_{24} u_{xzzz} + a_{25} u_{xxyy} + a_{25} u_{xxzz} + a_{25} u_{yyzz} + a_{26} u_{xxyz} + a_{26} u_{yyzx} + a_{26} u_{zzxy} - a_{27} u^5 - a_{27} 2u^3 v^2 - a_{27} u v^4 = 0, \quad (10)$$

$$u_t + a_1 v + a_2 v_x + a_3 v_y + a_4 v_z + a_5 v_{xx} + a_6 v_{yy} + a_7 v_{zz} + a_8 v_{xy} + a_9 v_{xz} + a_{10} v_{yz} - v_{xxz} + a_{17} v_{xyy} + a_{18} v_{xyz} + a_{19} v_{xzz} + a_{20} v_{yyz} + a_{21} v_{yzz} + a_{22} 2vv_x^2 + a_{22} 2vu_x^2 + a_{22} 2vv_y^2 + a_{22} 2vu_y^2 + a_{22} 2vv_z^2 + a_{22} 2vu_z^2 + a_{22} 2v^2 v_{xx} + a_{22} 2uv\, u_{xx} + a_{22} 2v^2 v_{yy} + a_{22} 2uvu_{yy} + a_{22} 2v^2 v_{zz} + a_{22} 2uvu_{zz} + a_{23} v_{xxxx} + a_{23} v_{yyyy} + a_{23} v_{zzzz} + a_{24} v_{xxxy} + a_{24} v_{xxxz} + a_{24} v_{yyyz} + a_{24} v_{xyyy} + a_{24} v_{xzzz} + a_{25} v_{xxyy} + a_{25} v_{xxzz} + a_{25} v_{yyzz} + a_{26} v_{xxyz} + a_{26} v_{yyzx} + a_{26} v_{zzxy} - a_{27} v^5 - a_{27} 2v^3 u^2 - a_{27} v u^4 = 0, \quad (11)$$

Utilizing the wave variable $\xi = x + y + z - ct$ in Eqs. (10) and (11), we get

$$cv_\xi + a_1 u + a_2 u_\xi + a_3 u_\xi + a_4 u_\xi + a_5 u_{\xi\xi} + a_6 u_{\xi\xi} + a_7 u_{\xi\xi} + a_8 u_{\xi\xi} + a_9 u_{\xi\xi} + a_{10} u_{\xi\xi} - a_{11} u^2 - a_{11} u v^2 + a_{12} u_{\xi\xi\xi} + a_{13} u_{\xi\xi\xi} + a_{14} u_{\xi\xi\xi} + a_{15} u_{\xi\xi\xi} + a_{16} u_{\xi\xi\xi} + a_{17} u_{\xi\xi\xi} + a_{18} u_{\xi\xi\xi} + a_{19} u_{\xi\xi\xi} + a_{20} u_{\xi\xi\xi} + a_{21} u_{\xi\xi\xi} + a_{22} 6(uv_\xi^2 + a_{22} 2uv_\xi^2 + a_{22} 2u^2 u_{\xi\xi} + a_{22} 2uvv_{\xi\xi}) + a_{23} 3u_{\xi\xi\xi\xi} + a_{24} 5u_{\xi\xi\xi\xi} + a_{25} 3u_{\xi\xi\xi\xi} + a_{26} 3u_{\xi\xi\xi\xi} - a_{27}(u^5 + 2u^3 v^2 + uv^4) = 0, \quad (12)$$

$$-cu_\xi + a_1 v + a_2 v_\xi + a_3 v_\xi + a_4 v_\xi + a_5 v_{\xi\xi} + a_6 v_{\xi\xi} + a_7 v_{\xi\xi} + a_8 v_{\xi\xi} + a_9 v_{\xi\xi} + a_{10} v_{\xi\xi} - a_{11} v^2 - a_{11} v u^2 + a_{12} v_{\xi\xi\xi} + a_{13} v_{\xi\xi\xi} + a_{14} v_{\xi\xi\xi} + a_{15} v_{\xi\xi\xi} + a_{16} v_{\xi\xi\xi} + a_{17} v_{\xi\xi\xi} + a_{18} v_{\xi\xi\xi} + a_{19} v_{\xi\xi\xi} + a_{20} v_{\xi\xi\xi} + a_{21} v_{\xi\xi\xi} + a_{22} 6(vu_\xi^2 + a_{22} 2vv_\xi^2 + a_{22} 2v^2 v_{\xi\xi} + a_{22} 2uvu_{\xi\xi}) + a_{23} 3v_{\xi\xi\xi} + a_{24} 5v_{\xi\xi\xi\xi} + a_{25} 3v_{\xi\xi\xi\xi} + a_{26} 3v_{\xi\xi\xi\xi} - a_{27}(v^5 - 2v^3 u^2 - vu^4) = 0. \quad (13)$$

Where a_1, a_2, a_3... are given Appendix equation (38) We suppose that Eqs. (12) and (13) provide the following solutions directly:

$$u(x,y,z,t) = \lambda_1 \cos^{\beta_1}(\mu\xi) \quad and \quad v(x,y,z,t) = \lambda_2 \cos^{\beta_2}(\mu\xi), \quad (14)$$

where λ_1 and λ_2 have fixed values. In Eqs. (12) and (13), the nonlinear term is balanced with the linear higher order derivative term to arrive at $\beta_1 = \beta_2 = -1$. Using Eq. (14) and the variables $\beta_1 = \beta_2 = -1$ in Eqs. (12) and (13), we may construct a system of algebraic equations.

$$cos^{-1}(\mu\xi): \quad a_1\lambda_1 - (a_5 + a_6 + a_7 + a_8 + a_9 + a_{10})\lambda_1\mu^2 + (3a_{23} + 5a_{24} + 3a_{25} + 3a_{26})\lambda_1\mu^4, \quad (15)$$

$$cos^{-3}(\mu\xi): \quad 2(a_5 + a_6 + a_7 + a_8 + a_9 + a_{10})\lambda_1\mu^2 - a_{11}\lambda_2^2\lambda_1 - a_{11}\lambda_1^3 -$$

$$6a_{22}\lambda_2^2\lambda_1\mu^2 - 6a_{22}\lambda_1^3\mu^2 - (60a_{23} + 100a_{24} + 60a_{25} + 60a_{26})\lambda_1\mu^4, \quad (16)$$

$$cos^{-5}(\mu\xi): \quad -12a_{22}\lambda_2^2\lambda_1\mu^2 + 12a_{22}\lambda_1^3\mu + (72a_{23} + 12a_{24} + 72a_{25} + 72a_{26})\lambda_1\mu^4 - a_{27}\lambda_1\lambda_2^4 - 2a_{27}\lambda_2^2\lambda_1^3 - a_{27}\lambda_1^5, \quad (17)$$

$$cos^{-2}(\mu\xi)sin(\mu\xi): \quad c\lambda_2\mu + (a_2 + a_3 + a_4)\lambda_1\mu - (a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20} + a_{21})\lambda_1\mu^3, \quad (18)$$

$$cos^{-4}(\mu\xi)sin(\mu\xi): \quad 6(a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20} + a_{21})\lambda_1\mu^3, \quad (19)$$

$$cos^{-5}(\mu\xi)sin^2(\mu\xi): \quad 6a_{22}\lambda_2^2\lambda_1\mu^2 + 6a_{22}\lambda_1^3\mu^2, \quad (20)$$

$$cos^{-1}(\mu\xi): \quad a_1\lambda_1 - (a_5 + a_6 + a_7 + a_8 + a_9 + a_{10})\lambda_2\mu^2 + (3a_{23} + 5a_{24} + 3a_{25} + 3a_{26})\lambda_2\mu^4, \quad (21)$$

$$cos^{-3}(\mu\xi): \quad 2(a_5 + a_6 + a_7 + a_8 + a_9 + a_{10})\lambda_2\mu^2 - a_{11}\lambda_1^2\lambda_1 - a_{11}\lambda_1^3 - 6a_{22}\lambda_1^2\lambda_2\mu^2 - 6a_{22}\lambda_2^3\mu^2 - (60a_{23} + 100a_{24} + 60a_{25} + 60a_{26})\lambda_2\mu^4, \quad (22)$$

$$cos^{-5}(\mu\xi): \quad -12a_{22}\lambda_1^2\lambda_2\mu^2 + 12a_{22}\lambda_2^3\mu + (72a_{23} + 12a_{24} + 72a_{25} + 72a_{26})\lambda_2\mu^4 - a_{27}\lambda_2\lambda_1^4 - 2a_{27}\lambda_1^2\lambda_2^3 - a_{27}\lambda_2^5, \quad (23)$$

$$cos^{-2}(\mu\xi)sin(\mu\xi): \quad c\lambda_1\mu + (a_2 + a_3 + a_4)\lambda_2\mu - (a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20} + a_{21})\lambda_2\mu^3, \quad (24)$$

$$cos^{-4}(\mu\xi)sin(\mu\xi): \quad 6(a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20} + a_{21})\lambda_2\mu^3, \quad (25)$$

$$cos^{-5}(\mu\xi)sin^2(\mu\xi): \quad 6a_{22}\lambda_1^2\lambda_2\mu^2 + 6a_{22}\lambda_2^3\mu^2 \quad (26)$$

We can solve the set of algebraic equations and get the result through symbolic computation.

$$\lambda = \sqrt{\frac{a_{11} - 12a_{22}\mu^2}{2a_{27}}}, \quad (27)$$

$$\mu = \sqrt{\frac{(3a_{23} + 5a_{24} + 3a_{25} + 3a_{26})}{(a_5 + a_6 + a_7 + a_8 + a_9 + a_{10})}}. \quad (28)$$

Hence the solutions of Eq. (9) become

$$u(x, y, z, t) = \lambda_1 sec[\mu(x + y + z - ct)], \quad (29)$$

$$v(x, y, z, t) = \lambda_2 sec[\mu(x + y + z - ct)]. \quad (30)$$

The inhomogeneous perturbed NLS equation is given by the equation (9). Alpha-helical proteins can be affected by nonlinear inhomogeneity in one of the following ways: Because of the additional molecules, alpha-helical proteins contain defects that interfere with their ability to function by resulting in inhomogeneities. The impacts of periodic type inhomogeneity were investigated using cubic and biquadratic initial nonlinear near interactions.

In order to understand how cubic inhomogeneity affects the soliton excitations in the alpha-helical protein chain, we substitute $F(x) = 1 + p_1x^3 + p_2x^2$ and $G(x) = 1 + p_3x^3 + p_4x^2$, where the parameters p_1, p_2, p_3 and p_4. There are $p_1, p_2, p_3, p_4 = 0, E_0 = 1.02$, $E_1 = 1.02$, $J_0 = 0.3$, $J_0' = 0.3$, $J_0'' = 3.1$, $J_1 = 0.3$, $J_1' = 0.2$ and $J_1'' = 3.1$

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respectively. This results in stable soliton propagation in the homogeneous alpha-helical protein chain, as seen in Fig (1a). When the strength of inhomogeneity exceeds this limit, the soliton splits for the values of $p_1 = 0.818$, $p_2 = 1.9$, $p_3 = 6.818$ and $p_4 = 2.9$ as seen in Fig (1b). The splitting deepens and the system is disrupted as inhomogeneity rises interfering with the capacity of proteins to function appropriately. The graphic illustrates how the increase in the nonlinear component of the inhomogeneity fully destroys the solitonic nature. Similar behaviour is displayed by the biquadratic type inhomogeneity of the forms $F(x) = 1 + p_5x^4 + p_6x^2$ and $G(x) = 1 + p_7x^4 + p_8x^2$. The soliton divides and becomes unstable for the values of $p_5 = 0.818$, $p_6 = -0.9$, $p_7 = 6.818$ and $p_8 = 2.9$ if the inhomogeneity increases. Fig. (1c) illustrates the effect of the biquadratic type of inhomogeneity. The periodic inhomogeneity among hydrogen bond stacks is chosen to take the form of $F(x) = 1 + p_9cos(x)$ and $G(x) = 1 + p_{10}cos(x)$ as illustrated in Fig. (1d) introducing periodic fluctuation in the confined region and mild changes at the soliton boundary $p_9 = 0.818$ and $p_{10} = -0.9$. As seen in Fig. (1e) the instability arises in the localised region for the values $p_{11} = 0.918$ and $p_{12} = 0.9$ which also disrupts how the system functions.

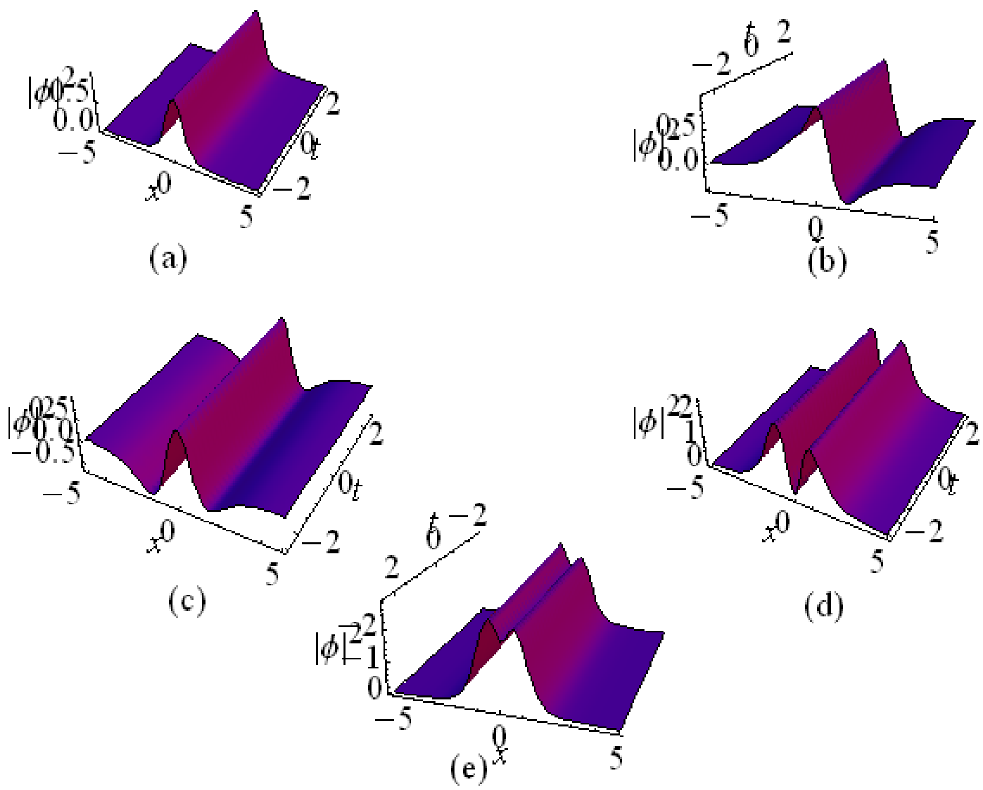

Figure 1: Evolution of $|\phi|^2$ a) without inhomogeneities, with b) cubic, c) biquadratic d) periodic, and e) localized with homogeneities.

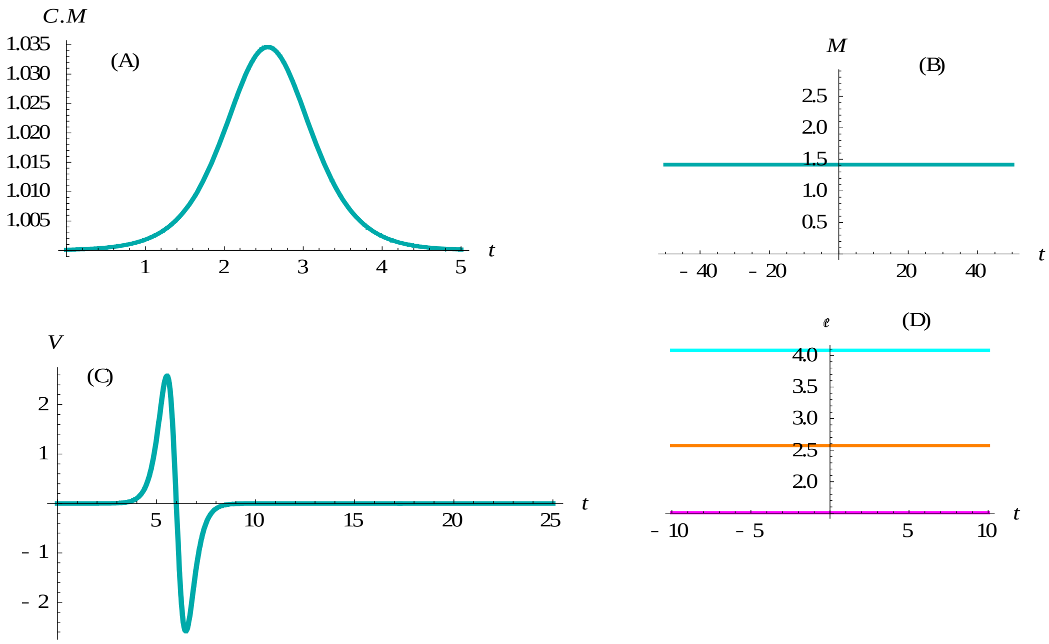

Figure 2: Plot of (A) The soliton's centre of mass, (B) Soliton's mass, (C) Velocity of soliton, (D) Energy of solitonic excitations (a) Potential energy, (b) Kinetic energy and (c) Total energy

Figure (2A) illustrates how the soliton's centre of mass changes over time. The protein lattices mass centre essentially remains in the same location. As a result it is possible that the basic solitons of the protein lattice to stabilize nonlinearly in this parameter range. Fig. (2B) illustrates the soliton mass's temporal progression. The soliton mass is maintained in all circumstances as seen in the diagram. A protein lattices soliton velocity is seen in Fig. (2C). The figure shows an oscillating change in the soliton's velocity before it reaches the uniform velocity. Fig. (2D) The energy of soliton excitation in α-helical protein chains is described by (a) potential energy, (b) kinetic energy, and (c) total energy. This illustration shows how the soliton's energy is conserved. The available parameters are $E_0 = 1.02$, $E_1 = 1.02$, $J_0 = 0.3$, $J_0' = 0.3$, $J_0'' = 3.1$, $J_1 = 0.3$, $J_1' = 0.2$ and $J_1'' = 3.1$ $\chi_1 = -3$ and $\chi_2 = 0.1$.

4. Linear Stability Analysis

For the linear stability study, we start with perturbed Eq. (9). We suppose that a planar wave with constant amplitude.

$$\phi(x, y, z, t) = U_0 exp[i(q_1 x + q_2 y + q_3 z - \omega t)] \tag{31}$$

where ω represents frequency, U_0 represents amplitude and q_1 and q_2 are wave numbers substituting Eq. (31) in to Eq. (10), we obtain the amplitude dependent relationship

$$\omega = -a_1 - ia_2 k_1 - ia_3 k_2 - ia_4 k_3 + a_5 k_1^2 + a_6 k_2^2 + a_7 k_3^2 + a_8 k_1 k_2 + a_9 k_1 k_3 + a_{10}$$

$$k_2k_3 + a_{11}U_0^2 + ia_{12}k_1^3 + ia_{13}k_2^3 + ia_{14}k_3^3 + ia_{15}k_1^2k_2 + ia_{16}k_1^2k_3 + ia_{17}k_2^2k_1 + ia_{18}k_1k_2k_3 + ia_{19}k_2^2k_1 + ia_{20}k_2^2k_3 + ia_{21}k_3^2k_2 - a_{22}U_0^2(k_1^2 + k_2^2 + k_3^2) - a_{23}(k_1^4 + k_2^4 + k_3^4) - a_{24}(k_1^3k_2 + k_1^3k_3 + k_2^3k_3 + k_2^3k_1 + k_3^3k_1) - a_{25}(k_1^2k_2^2 + k_2^2k_3^2 + k_2^2k_3^2) - a_{26}(k_1^2k_2k_3 + k_1k_2^2k_3 + k_1k_2k_3^2) + a_{27}U_0^4. \quad (32)$$

Known as the dispersion relation. We now investigate the linear stability of Eq. (10) Using a perturbed plane wave solutions of the form

$$\phi(x,y,z,t) = (U_0 + \epsilon\phi_1)exp[i(q_1x + q_2y + q_3z - \omega t) + \epsilon\phi_2(x,y,z,t)] \quad (33)$$

where ω is a small parameter and

$$\phi_1(x,y,z,t) = aexp[i\beta(x,y,z,t)], \quad (34)$$

$$\phi_2(x,y,z,t) = bexp[i\beta(x,y,z,t)] \quad (35)$$

Using $\beta(x,y,z,t) = Qx + Qy + Qz - \Omega t$, the dispersion relation between frequency Ω and wave number Q is given by

$$\Omega^2U_0 + \Omega(RU_0 + S) + RS = 0, \quad (36)$$

The dispersion relation can be obtained from the quadratic equation (36).

$$\Omega = \frac{-(RU_0+S)\pm\sqrt{(RU_0+S)^2-4U_0RS}}{2U_0}. \quad (37)$$

The final eigenvalue of ω is lengthy and verbose, so we won't provide it. The component Ω can be used to determine the stability of a nonlinear α-helical protein chain. In accordance with the relationship (35) mentioned above, if $(RU_0 + S) > RS$, Ω becomes complex, the perturbation grows exponentially over time, the stimulated alpha-helical protein system displays MI and soliton production is supported.

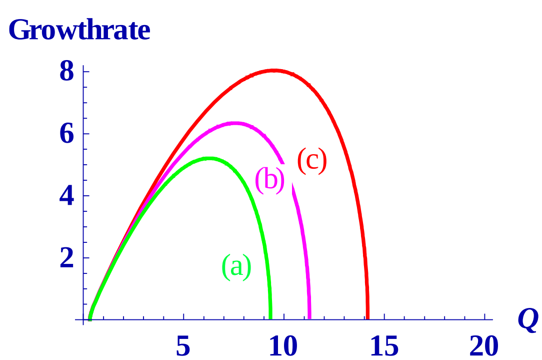

Figure 3: Growth rate Vs wave number Q

Fig. (3) shows growth rate curves when q_2 is fixed and q_1 is varied. The parameters are $U_0 = 0.2$, $J_0 = 0.3$, $J_0' = 0.2$, $J_0'' = 0.1$, $J_1 = 0.1$, $J_1' = 0.2$ and $J_1'' = 0.1$ with (a) $q_1 = -0.04$, $q_2 = -0.2$, $q_3 = -0.2$, $q_4 = -0.05$, $q_5 = -0.2$, $q_6 = -0.2$, $q_7 = -0.06$, $q_8 = -0.2$ and $q_9 = -0.2$, (b) $q_1 = -0.06$, $q_2 = -0.4$, $q_3 = -0.3$, $q_4 = -0.07$, $q_5 = -0.3$, $q_6 = -0.4$, $q_7 = -0.08$, $q_8 = -0.4$ and $q_9 = -0.5$, (c) $q_1 = -0.08$, $q_2 = -0.3$, $q_3 = -0.7$, $q_4 = -0.09$, $q_5 = -0.5$, $q_6 = -0.6$, $q_7 = -0.1$, $q_8 = -0.8$ and $q_9 = -0.9$. The figure shows how the growth rate depends on q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8 and q_9 for constant J_1, J_2 and J_0''. As J_0', q_2, q_5, q_8 increases, the growth rate and the band width shrink and the maximum gain decreases.

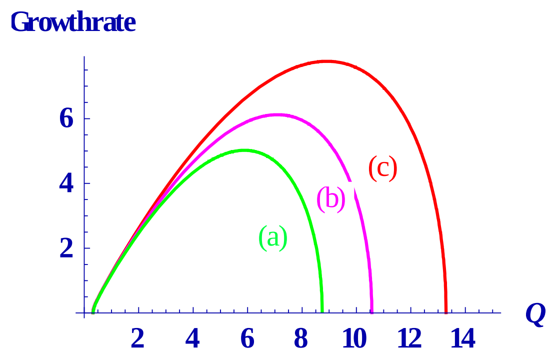

Figure 4: Growth rate Vs wave number Q with periodic inhomogeneity

Soliton formation can be significantly impacted by inhomogeneity. We look into whether certain kinds of nonlinear lattice inhomogeneities affect the stability of solitons. The growth rate of cubic inhomogeneity is predicted in Fig. (4) for various inhomogeneity parameter values. Take as instances the cubic inhomogeneous alpha-helical protein lattices $F(x) = 1 + p_1x^3 + p_2x^2$ and $G(x) = 1 + p_3x^3 + p_2x^2$. The growth rate for cubic inhomogeneities with $p_1 = 0.006$, $p_2 = 0.001$, $p_3 = 0.006$, and $p_4 = 0.001$ is shown in Fig. [4 (a, b, and c)]. As the inhomogeneity increases, the bandwidth becomes smaller and the growth rate shrinks. Above this point, it diminishes and there is less chance of a single wave forming. MI evolves highly at $p_1 = 0.029$. This illustrates the impact of inhomogeneity on modulational instability, which ultimately leads to the creation of soliton. Localized inhomogeneities are given by the equation $(F(x) = 1 + p_6tanh(x))$ and $(G(x) = 1 + p_6tanh(x))$ and cubic $(F(x) = 1 + p_2x^3 + p_3x^2)$ Along with and $(G(x) = 1 + p_2x^3 + p_3x^2)$ biquadratic $(F(x) = 1 + p_4x^4 + p_5x^2)$ and $(g(x) = 1 + p_4x^4 + p_5x^2)$ are the same as the results from the previous paragraph. The threshold values for cubic inhomogeneity are $p_2 = 0.00462$ and $p_3 = 0.005$. Biquadratic inhomogeneity has the values $p_4 = 0.002$ and $p_5 = 1.03$, while localised inhomogeneity has $p_6 = 0.035$. The outcome of cubic, biquadratic and localized are identical to the above results.The threshold values are shown by $p_2 = 0.00462$ and $p_3 = 0.005$ for cubic inhomogeneity. For biquadratic inhomogeneity $p_4 = 0.002$ and $p_5 = 1.03$ and for localized inhomogeneity $p_6 = 0.035$.

5. Conclusion

The paper considers interactions between the quadrupoles in order to analyze the effects of inhomogeneity on a higher order alpha-helical protein system. The behavior of the soliton propagation in the inhomogeneous alpha-helical protein system is examined for a variety of nonlinear inhomogeneities including cubic, biquadratic, and periodic types. Our findings shows that the soliton splits when the level of inhomogeneity rises above the limiting values. A perturbation technique is used to examine the impact of inhomogeneity. Above this limit energy is not shifted with good efficiency. Soliton

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splitting is a sign of instability in soliton propagation and a sudden break in energy transmission both of which have an impact on proteins' capacity to operate normally in the body. Additionally covered were the inhomogeneous NLS equation's modulational instability requirements. The plane wave is modulationally unstable for inhomogeneity below the threshold value and the medium encourages the generation of soliton. Analysis has been done on the solitons centre of mass, interaction energy, mass, velocity, and modulational stability conditions.

6. Appendix

$$
\begin{aligned}
i\hbar\phi_t = {} & [\tfrac{-E_0+6F(J_0+J_0'+J_0'')}{\hbar} + \tfrac{\gamma}{\hbar}[(J_0 + 3J_0' + 2J_0'')F_x + (J_0 + J_0' + 2J_0'')\, F_y + (J_0 - J_0' + 2J_0'') \\
& F_y] - \tfrac{\gamma^2}{\hbar}\tfrac{1}{3}(J_0 + 3J_0' + J_0'')F_{xx} + (\tfrac{J_0}{2} + \tfrac{3J_0'}{2} + J_0'')\,(F_{yy} + F_{zz}) + (3J_0' + 2J_0'')\,\,(F_{xy} \\
& +F_{xz} + F_{yz})] + \tfrac{\gamma^3}{\hbar}[(\tfrac{J_0}{6} + \tfrac{J_0'}{2} + \tfrac{J_0''}{3})F_{xxx} + (\tfrac{J_0}{6} + \tfrac{J_0'}{6} + \tfrac{J_0''}{3})F_{yyy} + (\tfrac{J_0}{6} - \tfrac{J_0'}{6} + \tfrac{J_0''}{3})F_{zzz} \\
& +\tfrac{1}{2}(J_0' + J_0'')(F_{yzz} + F_{xxy}) + \tfrac{1}{2}(-J_0' + J_0'')(F_{xxz} + F_{yyz}) + \tfrac{1}{2}(3J_0' + J_0'')(F_{xyy} \\
& +F_{xzz}) + J_0'F_{xyz}] - \tfrac{\gamma^4}{\hbar}[(\tfrac{J_0}{24} + \tfrac{J_0'}{8} + \tfrac{J_0''}{12})(F_{xxxx} + F_{yyyy} + F_{zzzz}) + \tfrac{1}{2}(J_0' + \tfrac{J_0''}{3}) \\
& (F_{xxxy} + F_{xxxz} + F_{xzzz} + F_{xyyy} + F_{yyyz} + F_{yzzz}) + \tfrac{1}{4}(3J_0' + J_0'')\,(F_{xxyy} + F_{xxzz} \\
& +F_{yyzz}) + \tfrac{3J_0'}{2}(F_{xxyz} + F_{yyzx} + F_{zzxy})]]\phi - [\tfrac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F_x + (J_0' + J_0'') \\
& F_y + (J_0' - J_0'')F_z] - \tfrac{\gamma^3}{\hbar}(J_0 + 3J_0' + J_0'')F_{xx} + (3J_0' + J_0'')\,(F_{xy} + F_{xz}) + \tfrac{1}{2}(3J_0' + \\
& J_0'')(F_{yyy} + F_{zzz}) + 3J_0'F_{yz}] + \tfrac{\gamma^4}{\hbar}[(\tfrac{J_0}{24} + \tfrac{J_0'}{8} + \tfrac{J_0''}{12})F_{xxx} + \tfrac{1}{2}(J_0' + J_0'')F_{xxy} - \tfrac{1}{2}(J_0' - \\
& J_0'')(F_{xxz} + F_{yzz}) + \tfrac{1}{2}(3J_0' + J_0'')(F_{xyy} + F_{xzz}) + \tfrac{1}{6}(J_0' + J_0'')F_{yyy} - \tfrac{1}{6}(J_0' - J_0'') \\
& F_{zzz}]]\phi_x - [\tfrac{\gamma^2}{\hbar}(J_0' + J_0'')(F_x + F_z) + (J_0 + 3J_0' + 2J_0'')F_y] - \tfrac{\gamma^3}{\hbar}[\tfrac{1}{2}(J_0' + J_0'')(F_{xx} \\
& +F_{zz}) + (J_0' + J_0'')(F_{xy} + F_{yz}) + (\tfrac{J_0}{2} + \tfrac{J_0'}{2} + J_0'')F_{yy} + J_0'F_{xz}] + \tfrac{\gamma^4}{\hbar}[\tfrac{1}{6}(J_0' + J_0'')(F_{xxx} \\
& +F_{zzz}) + \tfrac{1}{2}(3J_0' + J_0'')(F_{xxy} + F_{yzz}) + \tfrac{J_0'}{2}(F_{xxz} + F_{xzz}) - \tfrac{1}{2}(F_{xyy} + F_{yyz}) - J_0'F_{xyz} \\
& +(\tfrac{J_0}{6} + \tfrac{J_0'}{2} + \tfrac{J_0''}{3}) + F_{yyy}]]\phi_y - [\tfrac{\gamma^2}{\hbar}(J_0'' - J_0')F_x + (J_0' + J_0'')F_y + (J_0 + 3J_0' + 2J_0'')F_z] \\
& - \tfrac{\gamma^3}{\hbar}[\tfrac{1}{2}(J_0'' - J_0')(F_{xx} + F_{yy}) + J_0'F_{xy} + (J_0'' - J_0')(F_{xz} + F_{yz}) + (\tfrac{J_0}{2} + \tfrac{J_0'}{2} + \tfrac{J_0''}{2})F_{zz}] \\
& + \tfrac{\gamma^4}{\hbar}[-\tfrac{1}{6}(J_0' + J_0'')\,F_{xxx} + \tfrac{J_0'}{2}F_{xxy} + \tfrac{1}{2}(3J_0' + J_0'')(F_{xxz} + F_{yyz}) - \tfrac{J_0'}{2}F_{xyy} + J_0'F_{xyz} \\
& - \tfrac{1}{2}(J_0' + J_0'')F_{xzz} + \tfrac{1}{6}(J_0' + J_0'')F_{yyy} + \tfrac{1}{2}(J_0' + J_0'')F_{yzz} + (\tfrac{J_0}{6} + \tfrac{J_0'}{2} + \tfrac{J_0''}{3})F_{zzz}]]\phi_z \\
& -[\tfrac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F] - \tfrac{\gamma^3}{\hbar}(\tfrac{J_0}{2} + \tfrac{3J_0'}{2} + J_0'')F_x + \tfrac{1}{2}(J_0' + J_0'')F_y + \tfrac{1}{2}(J_0'' - J_0')F_z] \\
& + \tfrac{\gamma^4}{\hbar}[(\tfrac{J_0}{4} + \tfrac{3J_0'}{4} + \tfrac{J_0''}{2})F_{xx} + \tfrac{1}{2}(3J_0' + J_0'')F_{xy} + \tfrac{1}{2}(3J_0' + J_0'')F_{xz} - \tfrac{1}{4}\tfrac{1}{2}(3J_0' + J_0'') \\
& F_{yy} + \tfrac{1}{4}(3J_0' + J_0'')F_{zz} + \tfrac{3J_0'}{2}F_{yz}]]\phi_{xx} - [\tfrac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F] + \tfrac{\gamma^3}{\hbar}[\tfrac{1}{2}\,(3J_0' + \\
& J_0'')F_x + (-J_0' + J_0'')F_z + (\tfrac{J_0}{2} + \tfrac{J_0'}{2} + J_0'')F_y] + \tfrac{\gamma^4}{\hbar}[\tfrac{1}{4}(3J_0' + J_0'')(F_{xx} + F_{zz}) + \tfrac{1}{2}
\end{aligned}
$$

$$(3J_0' + J_0'')\,(F_{xy} + F_{yz}) + \frac{J_0'}{2}F_{xz} + (\frac{J_0}{4} + \frac{J_0'}{4} + \frac{J_0''}{2}) + F_{yy}]]\phi_{yy} - [\frac{\gamma^2}{\hbar}(J_0 + 3J_0' 2J_0'') F] - \frac{\gamma^3}{\hbar}[-\frac{1}{2}(3J_0' + J_0'')F_x + \frac{1}{2}(J_0' + J_0'')F_y + (\frac{J_0}{2} - \frac{J_0'}{2} + J_0'')F_z] + \frac{\gamma^4}{\hbar}(F_{xx} + F_{yy}) + \frac{3J_0'}{2}F_{xy} + \frac{1}{2}(3J_0' + J_0'')\,(F_{xz} + F_{yz}) + (\frac{J_0}{4} + \frac{J_0'}{4} + \frac{J_0''}{2})F_{zz}]]\phi_{zz} - [\frac{\gamma^2}{\hbar}(3J_0' + J_0'') F] - \frac{\gamma^3}{\hbar}\,[(3J_0' + J_0'')F_x + (J_0' + J_0'')F_y + J_0'F_z] + \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')(F_{xx} + F_{yy}) + (3J_0' + J_0'')F_{xy} + 3J_0'(F_{xz} + F_{yz}) + \frac{3J_0'}{2}F_{zz}]]\phi_{xy} - [\frac{\gamma^2}{\hbar}2(3J_0' + J_0'')F] - \frac{\gamma^3}{\hbar}(3J_0' + J_0'')F_x + J_0'F_y + (-J_0' + J_0'')F_z] + \frac{\gamma^4}{\hbar}(F_{xx} + F_z) + 3J_0'(F_{xy} + F_{yz}) + \frac{3J_0'}{2}F_{yy} + (3J_0' + J_0'')F_{xz}]]\phi_{xz} - [\frac{\gamma^3}{\hbar}2(3J_0' + J_0'')F] - \frac{\gamma^3}{\hbar}[3J_0'F_x + (J_0' + J_0'')F_y + (J_0'' - J_0')F_z] + \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')(F_{yy} + F_{zz}) + 3J_0'(F_{xy} + F_{xz}) + \frac{3J_0'}{2}F_{xx} + (3J_0' + J_0'')F_{yz}]]\,\phi_{yz} + [\frac{4\chi\chi_1 A\gamma^2}{\hbar} + \frac{2\gamma^2[E_1 - 6G(J_0 + J_0' + J_0'')]}{\hbar} + \frac{2\gamma^3(G_x + G_y + G_z)}{\hbar}(J_0 + J_0' + 2J_0'')]\,|\phi|^2)\phi - \frac{\gamma^4}{\hbar}[(\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_x + \frac{1}{6}(J_0' + J_0'')F_y + (J_0' - J_0'')F_z]\phi_{xxx} - \frac{\gamma^4}{\hbar}[(\frac{J_0}{6} + (J_0' + J_0'')(F_x + F_y) + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_z]\phi_{yyy} - \frac{\gamma^4}{\hbar}[-\frac{1}{6}(J_0' - J_0'')F_x + \frac{1}{6}(J_0' + J_0'')F_y + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_z]\phi_{zzz} - \frac{\gamma^4}{\hbar}[\frac{1}{2}(J_0' + J_0'')F_x + \frac{1}{2}(3J_0' + J_0'')F_y + \frac{J_0''}{2}F_z]\phi_{xxy} - \frac{\gamma^4}{\hbar}[-\frac{1}{2}(J_0' - J_0'')(F_x + F_z) + \frac{J_0''}{2}F_y]\phi_{xxz} - \frac{\gamma^4}{\hbar}[\frac{1}{2}(J_0 + 3J_0' + 2J_0'')F_x + \frac{1}{2}(J_0' + J_0'')F_y - \frac{J_0''}{2}F_z]\phi_{xyy} - \frac{\gamma^4}{\hbar}[J_0'(F_x + F_z) - J_0''F_y]\phi_{xyz} - \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')F_x + \frac{J_0'}{2}F_y - \frac{1}{2}(J_0' - J_0'')F_z]\phi_{xzz} - \frac{\gamma^4}{\hbar}[-\frac{1}{2}J_0'F_x + \frac{1}{2}(J_0' + J_0'')F_y + \frac{1}{2}(3J_0' + J_0'')F_z]\phi_{yyz} - \frac{\gamma^4}{\hbar}[\frac{J_0'}{2}F_x + \frac{1}{2}(3J_0' + J_0'')F_y + \frac{1}{2}(J_0' + J_0'')F_z]\phi_{yzz} + \frac{2\frac{\gamma^4}{\chi}\chi_1 A}{3\hbar}\phi(2(|\phi_x|^2) + (|\phi_y|^2) + (|\phi_z|^2) + (|\phi|^2)_{xx} + (|\phi|^2)_{yy} + (|\phi|^2)_{zz}) - \frac{\gamma^4}{\hbar}[(\frac{J_0}{12} + \frac{J_0'}{4} + \frac{J_0''}{6})F](\phi_{xxxx} + \phi_{yyyy} + \phi_{zzzz}) - \frac{\gamma^4}{\hbar}[(J_0' + \frac{J_0''}{3})F](\phi_{xxxy} + \phi_{xxxz} + \phi_{yyyz} + +\phi_{xyyy} + \phi_{xzzz}) - \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')F](\phi_{xxyy} + \phi_{xxzz} + \phi_{yyzz}) - \frac{\gamma^4}{\hbar}[3J_0'F](\phi_{xxyz} + \phi_{yyzx} + \phi_{zzxy}) - \frac{8\chi_2\chi A\gamma^4}{\hbar}|\phi|^4\phi, \quad (38)$$

a values of equation (10) is given by:

where $a_1 = [\frac{-E_0 + 6F(J_0 + J_0' + J_0'')}{\hbar} + \frac{\gamma}{\hbar}[(J_0 + 3J_0' + 2J_0'')F_x + (J_0 + J_0' + 2J_0'')F_y + (J_0 - J_0' + 2J_0'')F_y] - \frac{\gamma^2}{\hbar}\frac{1}{3}(J_0 + 3J_0' + J_0'')F_{xx} + (\frac{J_0}{2} + \frac{3J_0'}{2} + J_0'')(F_{yy} + F_{zz}) + (3J_0' + 2J_0'')(F_{xy} + F_{xz} + F_{yz})] + \frac{\gamma^3}{\hbar}[(\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_{xxx} + (\frac{J_0}{6} + \frac{J_0'}{6} + \frac{J_0''}{3})F_{yyy} + (\frac{J_0}{6} - \frac{J_0'}{6} + \frac{J_0''}{3})F_{zzz} + \frac{1}{2}(J_0' + J_0'')(F_{yzz} + F_{xxy}) + \frac{1}{2}(-J_0' + J_0'')(F_{xxz} + F_{yyz}) + \frac{1}{2}(3J_0' + J_0'')(F_{xyy} + F_{xzz}) + J_0'F_{xyz}] - \frac{\gamma^4}{\hbar}[(\frac{J_0}{24} + \frac{J_0'}{8} + \frac{J_0''}{12})(F_{xxxx} + F_{yyyy} + F_{zzzz}) + \frac{1}{2}(J_0' + \frac{J_0''}{3})(F_{xxxy} + F_{xxxz} + F_{xzzz} + F_{xyyy} + F_{yyyz} + F_{yzzz}) + \frac{1}{4}(3J_0' + J_0'')(F_{xxyy} + F_{xxzz} + F_{yyzz}) + \frac{3J_0'}{2}(F_{xxyz} + F_{yyzx} + F_{zzxy})]]$,

$a_2 = -[\frac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F_x + (J_0' + J_0'')F_y + (J_0' - J_0'')F_z] - \frac{\gamma^3}{\hbar}(J_0 + 3J_0' + J_0'')F_{xx} + (3J_0' + J_0'')(F_{xy} + F_{xz}) + \frac{1}{2}(3J_0' + J_0'')(F_{yyy} + F_{zzz}) + 3J_0'F_{yz}] + \frac{\gamma^4}{\hbar}[(\frac{J_0}{24} + \frac{J_0'}{8} + \frac{J_0''}{12})F_{xxx} + \frac{1}{2}(J_0' + J_0'')F_{xxy} - \frac{1}{2}(J_0' - J_0'')(F_{xxz} + F_{yzz}) + \frac{1}{2}(3J_0' + J_0'')(F_{xyy} + F_{xzz}) + \frac{1}{6}(J_0' + J_0'')F_{yyy} - \frac{1}{6}(J_0' - J_0'')F_{zzz}]]$, $a_3 = -[\frac{\gamma^2}{\hbar}(J_0' + J_0'')(F_x + F_z) + (J_0 + 3J_0' + 2J_0'')F_y] - \frac{\gamma^3}{\hbar}[\frac{1}{2}(J_0' + J_0'')(F_{xx} + F_{zz}) + (J_0' + J_0'')(F_{xy} + F_{yz}) + (\frac{J_0}{2} + \frac{J_0'}{2} + J_0'')F_{yy} + J_0'F_{xz}] + \frac{\gamma^4}{\hbar}[\frac{1}{6}(J_0' + J_0'')(F_{xxx} + F_{zzz}) + \frac{1}{2}(3J_0' + J_0'')(F_{xxy} + F_{yzz}) + \frac{J_0'}{2}(F_{xxz} + F_{xzz}) - \frac{1}{2}(F_{xyy} + F_{yyz}) - J_0'F_{xyz} + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3}) + F_{yyy}]]$, $a_4 = -[\frac{\gamma^2}{\hbar}(J_0'' - J_0')F_x + (J_0' + J_0'')F_y + (J_0 + 3J_0' + 2J_0'')F_z] - \frac{\gamma^3}{\hbar}[\frac{1}{2}(J_0'' - J_0')(F_{xx} + F_{yy}) + J_0'F_{xy} + (J_0'' - J_0')(F_{xz} + F_{yz}) + (\frac{J_0}{2} + \frac{J_0'}{2} + \frac{J_0''}{2})F_{zz}] + \frac{\gamma^4}{\hbar}[-\frac{1}{6}(J_0' + J_0'')F_{xxx} + \frac{J_0'}{2}F_{xxy} + \frac{1}{2}(3J_0' + J_0'')(F_{xxz} + F_{yyz}) - \frac{J_0'}{2}F_{xyy} + J_0'F_{xyz} - \frac{1}{2}(J_0' + J_0'')F_{xzz} + \frac{1}{6}(J_0' + J_0'')F_{yyy} + \frac{1}{2}(J_0' + J_0'')F_{yzz} + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_{zzz}]]$, $a_5 = -[\frac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F] - \frac{\gamma^3}{\hbar}(\frac{J_0}{2} + \frac{3J_0'}{2} + J_0'')F_x + \frac{1}{2}(J_0' + J_0'')F_y + \frac{1}{2}(J_0'' - J_0')F_z] + \frac{\gamma^4}{\hbar}[(\frac{J_0}{4} + \frac{3J_0'}{4} + \frac{J_0''}{2})F_{xx} + \frac{1}{2}(3J_0' + J_0'')F_{xy} + \frac{1}{2}(3J_0' + J_0'')F_{xz} - \frac{1}{4}\frac{1}{2}(3J_0' + J_0'')F_{yy} + \frac{1}{4}\frac{1}{2}(3J_0' + J_0'')F_{zz} + \frac{3J_0'}{2}F_{yz}]]$, $a_6 = -[\frac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F] + \frac{\gamma^3}{\hbar}[\frac{1}{2}(3J_0' + J_0'')F_x + (-J_0' + J_0'')F_z + (\frac{J_0}{2} + \frac{J_0'}{2} + J_0'')F_y] + \frac{\gamma^4}{\hbar}[\frac{1}{4}(3J_0' + J_0'')(F_{xx} + F_{zz}) + \frac{1}{2}(3J_0' + J_0'')(F_{xy} + F_{yz}) + \frac{J_0'}{2}F_{xz} + (\frac{J_0}{4} + \frac{J_0'}{4} + \frac{J_0''}{2})F_{yy}]]$, $a_7 = -[\frac{\gamma^2}{\hbar}(J_0 + 3J_0' + 2J_0'')F] - \frac{\gamma^3}{\hbar}[-\frac{1}{2}(3J_0' + J_0'')F_x + \frac{1}{2}(J_0' + J_0'')F_y + (\frac{J_0}{2} - \frac{J_0'}{2} + J_0'')F_z] + \frac{\gamma^4}{\hbar}(F_{xx} + F_{yy}) + \frac{3J_0'}{2}F_{xy} + \frac{1}{2}(3J_0' + J_0'')(F_{xz} + F_{yz}) + (\frac{J_0}{4} + \frac{J_0'}{4} + \frac{J_0''}{2})F_{zz}]]$, $a_8 = -[\frac{\gamma^2}{\hbar}(3J_0' + J_0'')F] - \frac{\gamma^3}{\hbar}[(3J_0' + J_0'')F_x + (J_0' + J_0'')F_y + J_0'F_z] + \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')(F_{xx} + F_{yy}) + (3J_0' + J_0'')F_{xy} + 3J_0'(F_{xz} + F_{yz}) + \frac{3J_0'}{2}F_{zz}]]$, $a_9 = -[\frac{\gamma^2}{\hbar}2(3J_0' + J_0'')F] - \frac{\gamma^3}{\hbar}(3J_0' + J_0'')F_x + J_0'F_y + (-J_0' + J_0'')F_z] + \frac{\gamma^4}{\hbar}(F_{xx} + F_z) + 3J_0'(F_{xy} + F_{yz}) + \frac{3J_0'}{2}F_{yy} + (3J_0' + J_0'')F_{xz}]]$, $a_{10} = -[\frac{\gamma^3}{\hbar}2(3J_0' + J_0'')F] - \frac{\gamma^3}{\hbar}[3J_0'F_x + (J_0' + J_0'')F_y + (J_0'' - J_0')F_z] + \frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')(F_{yy} + F_{zz}) + 3J_0'(F_{xy} + F_{xz}) + \frac{3J_0'}{2}F_{xx} + (3J_0' + J_0'')F_{yz}]]$, $a_{11} = [\frac{4\chi\chi_1 A\gamma^2}{\hbar} + \frac{2\gamma^2[E_1 - 6G(J_0 + J_0' + J_0'')]}{\hbar} + \frac{2\gamma^3(J_0 + J_0' + 2J_0'')(G_x + G_y + G_z)}{\hbar}]$, $a_{12} = -\frac{\gamma^4}{\hbar}[(\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_x + \frac{1}{6}(J_0' + J_0'')F_y + (J_0' - J_0'')F_z]$, $a_{13} = -\frac{\gamma^4}{\hbar}[(\frac{J_0}{6} + (J_0' + J_0'')(F_x + F_y) + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_z]$, $a_{14} = -\frac{\gamma^4}{\hbar}[-\frac{1}{6}(J_0' - J_0'')F_x + \frac{1}{6}(J_0' + J_0'')F_y + (\frac{J_0}{6} + \frac{J_0'}{2} + \frac{J_0''}{3})F_z]$, $a_{15} = -\frac{\gamma^4}{\hbar}[\frac{1}{2}(J_0' + J_0'')F_x + \frac{1}{2}(3J_0' + J_0'')F_y + \frac{J_0''}{2}F_z]$, $a_{16} = -\frac{\gamma^4}{\hbar}[-\frac{1}{2}(J_0' - J_0'')(F_x + F_z) + \frac{J_0''}{2}F_y]$, $a_{17} = -\frac{\gamma^4}{\hbar}[\frac{1}{2}(J_0 + 3J_0' + 2J_0'')F_x + \frac{1}{2}(J_0' + J_0'')F_y - \frac{J_0''}{2}F_z]$, $a_{18} = -\frac{\gamma^4}{\hbar}[J_0'(F_x + F_z) - J_0''F_y]$, $a_{19} = -\frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')F_x + \frac{J_0'}{2}F_y -$

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$\frac{1}{2}(J_0' - J_0'')F_z]$, $a_{20} = -\frac{\gamma^4}{\hbar}[-\frac{1}{2}J_0'F_x + \frac{1}{2}(J_0' + J_0'')F_y + \frac{1}{2}(3J_0' + J_0'')F_z]$, $a_{21} = -\frac{\gamma^4}{\hbar}[\frac{J_0'}{2}F_x + \frac{1}{2}(3J_0' + J_0'')F_y + \frac{1}{2}(J_0' + J_0'')F_z]$, $a_{22} = \frac{2\frac{\gamma^4}{\chi}\chi_1 A}{3\hbar}$, $a_{23} = -\frac{\gamma^4}{\hbar}[(\frac{J_0}{12} + \frac{J_0'}{4} + \frac{J_0''}{6})F]$, $a_{24} = -\frac{\gamma^4}{\hbar}[(J_0' + \frac{J_0''}{3})F]$, $a_{25} = -\frac{\gamma^4}{\hbar}[\frac{1}{2}(3J_0' + J_0'')F]$, $a_{26} = -\frac{\gamma^4}{\hbar}[3J_0'F]$, $a_{27} = \frac{8\chi_2\chi A\gamma^4}{}$ and $A = [\frac{\gamma^2(K_1+K_2+K_3)}{Mc^2 - k\gamma^2(K_1^2+K_2^2+K_2^3)}]$.

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