Non-Bondage Strongly Equitable Split Domination Number of Some Graphs

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Abstract

A finite, undirected graph G= (V, E) consists of a finite non – empty set of vertices V = V(G) together with a set E= E(G) of unordered pairs of distinct vertices called edges. A subset D of $V(G)$ is called a strongly equitable dominating set of G if for every $v \in V - D$, there exists atleast one $u \in D$ such that u and v are adjacent also $deg(u) \geq deg(v)$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of minimal strongly equitable dominating set is called as strongly equitable domination number. It is denoted by $\gamma_{se}(G)$ *and this D is a split domination set if* $< V - D >$ is disconnected. The non bondage number of a graph, is the cardinality of a maximum number of edges, whose removal $(G - E)$, results in a graph with domination number is equal to a domination number of G and it is denoted by $b_{nses}(G)$. That is $\gamma_{ses}(G - E) = \gamma_{ses}(G)$.

Keywords: *Domination number, strongly domination number, equitable domination number, strongly equitable domination number, non-bondage number, split domination number and non-bondage strongly equitable split domination number.*

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1. Introduction

Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Domination in graphs is one of the major research areas in graph theory. It has the potential to solve many real-life problems involved in social network, Design and analysis of Communication network, defense surveillance etc.

The concept of strong weak domination and domination balance in a graph was introduced by Sampath Kumar *et al.* [10]. Let $u, v \in V(G)$ then u strongly dominates v if (i) $uv \in E(G)$ and (ii) $deg(u) \geq deg(v)$. A non-empty subset $D \subseteq V(G)$ is a strongly dominating set of G if every vertex is $V - D$ is strongly dominated by at least one vertex in D. The minimum cardinality of strongly dominating set is called *strongly* dominating number denoted by $\gamma_s(G)$. Gayla S. Domke *et al.* [6] initiated the study of bondage and reinforcement associated with fractional domination. The literature on domination and its parameters clearly explained in two books authored by Haynes *et al.* [7, 8].

The domination number of a graph γ(G) is the minimum size of a dominating set of vertices in G. Swaminathan *et al.* [11] introduced the concept of degree equitable domination on graphs. A non-empty subset D of $V(G)$ is called an equitable dominating set if for every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of equitable dominating set is called *equitable* dominating number denoted by $\gamma_e(G)$. Hemalatha G *et al.* [9] established domination number and the bondage number for some families of graphs. Ameenal Bibi K. *et al*. [2] investigated split domination, inverse domination and equitable domination of the middle graphs and central graphs of cycles and paths, where the P_n and C_n of those middle graphs didn't satisfy the equitable domination condition. Vaidya S. K. *et al.* [12] investigated strong domination number of some classes of graphs and studies their related parameter. For this, they considered simple, finite, connected and undirected graph G with vertex set V and edge set E. Strong equitable domination number and the inverse Strong equitable domination number of some special classes are investigated by Ameenal Bibi K. *et al.* [1] and also attained the eccentricity, radius and diameter of those graphs. The concept of split domination has been studied by V.R. Kulli [11].Kalavathi and Palani [10] was initiate of (G,D) – non – bondage number of a graph denoted by b_n(G). In this paper, we examined strongly equitable bondage number for some families of graphs.

2. Preliminaries

Definition 2.1
The Soifer graph is a planar graph on 9 vertices and 20 edges that tangles the Kempe chains in Kempe's algorithm and is smaller than the Kittel graph and Errera graph.

Definition 2.2
The Chvatal graph is an undirected graph with 12 vertices and 24 edges, discovered by Vaclav Chvatal. It is triangle-free its girth is four. It is 4-regular; each vertex has exactly four neighbours.

Definition 2.3
The Fritsch graph is a planar graph with 9 vertices and 21 edges. It was obtained by Fritsch as a minimal sized counter example to the Alfred Kempe's attempt to prove the four-color theorem

Definition 2.4
The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges, the smallest non-Hamiltonian Polyhedral graph. It is named after British astronomer Alexander Stewart Herschel.

Definition 2.5
The Moser Spindle graph (also called the Moser graph) is an undirected graph, named after mathematicians Leo Moser and his brother William with 7 vertices and 11 edges.

Definition 2.6
The Franklin graph is a 3-regular graph with 12 vertices and 18 edges. The Franklin graph is named after Philip Franklin. It is a 3-vertex connected and 3-edge connected perfect graph.

3. Non bondage strongly equitable split domination number of some graphs

Definition 3.1
A non-empty subset D of $V(G)$ is a dominating set of G if every vertex in $V - D$ is adjacent to

atleast one vertex in D. The dominating set D is called Strongly equitable split dominating set of a graph G if for every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and (i) $deg(u) \geq deg(v)$ (ii) $|deg(u) - deg(v)| \leq 1$ and $< V - D >$ $is\ disconnected.$ The strongly equitable domination non bondage number of graphs G is the cardinality of a maximum number of edges, whose removal $(G - E)$, results in a graph with domination number is equal to a domination number of G and it is denoted by $b_{nses}(G)$. That is $\gamma_{ses}\ (G - E) = \gamma_{ses}\ (G)$. In this section, we obtained the strongly equitable domination non bondage number of some special classes of graphs.

Theorem 3.2

Let G be a Soifer graph, then $b_{nses}(G)$ $=[n - 7]$ if $n = 9$

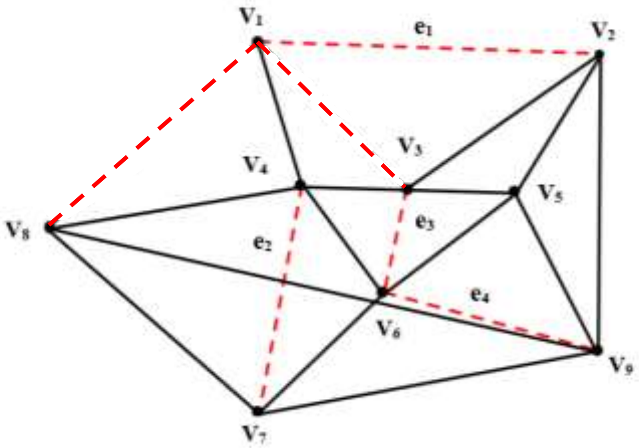

Figure 1. Soifer graph $(G - E)$

Proof:

Let G be a Soifer graph with 9 vertices and 20 edges. $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ are the vertices in G. Let $D = \{ v_4, v_6 \}$ is a strongly equitable dominating split set of Soifer graph and $< V - D > is\ disconnected, that\ is$ $\gamma_{ses}\ (G) = 2$. Let $(G - E)$ be obtained from G by removal of an edge v_1v_2. Now there is no change in domination number $\gamma_{ses}\ (G - E) = \gamma_{ses}\ (G)$. Let G be obtained from G by removal of the edges v_3v_6, $v_4v_7\ , v_8v_1\ , v_3v_1\ and\ v_6v_9$. *Figure 1* shows the removal edge set $\{e_1, e_2, e_3, e_4\}$. Therefore $D = \{v_4, v_6\}$. Thus $\gamma_{ses}\ (G - E) = 2$. The vertex v_3 is strongly dominates the vertices v_1, v_2, v_4, v_5. The vertex v_4 is strongly dominates the vertices v_3, v_6. The vertex v_8 is strongly dominates the vertices v_1, v_4, v_9 where $v_3, v_4,$ are dominating vertices in $(G - E)$. Hence the above dominating set D = $\{ v_3, v_4,\}$ satisfies the strongly domination condition $deg(u) \geq deg(v)$ where $u \in D$, $v \in v - D$, $uv \in E(G - E)$ and this strongly domination sets satisfying the equitable condition $|deg(u) - deg(v)| \leq 1\ and\ < V - D >$ $is\ disconnected.$ Thus $\gamma_{se}\ (G - E) = 2$. *Figure 1* shows that the strongly equitable domination number of $(G - E)$ is equal to the strongly equitable split domination number of G. Therefore $\gamma_{ses}\ (G - E) = \gamma_{ses}\ (G)$.Hence $b_{nses}(G) = n - 7$.

Theorem 3.3

Let G be a Chvatal graph, then $b_{nses}(G) = \left\lceil \frac{n}{3} \right\rceil$ if $n = 12$

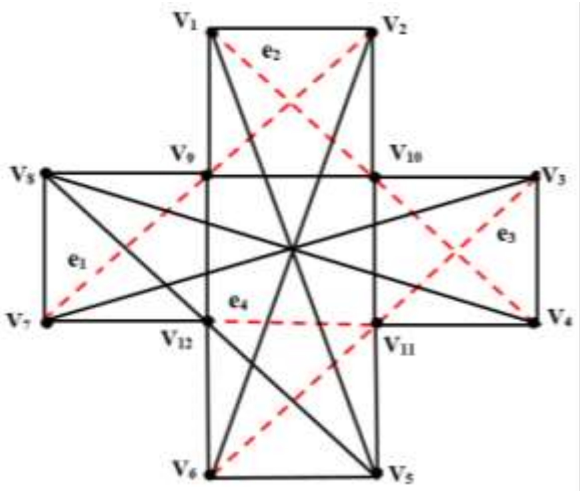

Figure 2. Chvatal graph (G-E)

Proof:
Let G be a 4-regular Chvatal graph with 12 vertices and 24 edges. $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ are the vertices of Chvatal graph G. Let $D = \{v_1, v_3, v_6, v_{10}\}$ and $< V - D >$ $is\ disconnected$ .Therefore D is a strongly equitable split dominating set of Chvatal graph and $\gamma_{ses}\ (G) = 4$. Let $(G - E)$ be obtained from G by removal of an edge v_2v_7. Now there is no change in domination number $\gamma_{se}\ (G - E) = \gamma_{se}\ (G)$. Let $(G - E)$ be obtained from G by removal of the edges v_1v_4, $v_3v_6 and v_{11}v_{12}$. *Figure 2* shows the removal edge subset $\{e_1, e_2, e_3, e_4\}$. Therefore $D = \{\ v_5, v_7, v_8, v_{10}\}$. Thus $\gamma_{se}\ (G - E) = 4$. The vertices of D are adjacent to the vertices of $V - D\ is\ disconnected$ and $u \in D$, where v_3, v_4, v_5, v_6 and v_{10} are dominating vertices in $(G - E)$. Hence the above dominating set $D = \{\ v_5, v_7, v_8, v_{10}\}$ satisfies the strongly domination condition $deg(u) \geq deg(v)$. We also satisfying the equitable condition $|deg(u)\ -\ deg(v)\ |\ \leq 1$. Thus $\gamma_{se}\ (G - E) = 5$. *Figure 2* shows that the strongly equitable split domination number of $(G - E)$ is equal to the strongly equitable domination number of G. Therefore $\gamma_{ses}\ (G - E) = \gamma_{ses}\ (G)$. Hence then $b_{nses}(G) = \left[\frac{n}{3}\right]$.

Theorem 3.4
If the graph G is a Fritsch graph, the $b_{nses}(G) =\ [n - 7]$ if $n = 9$

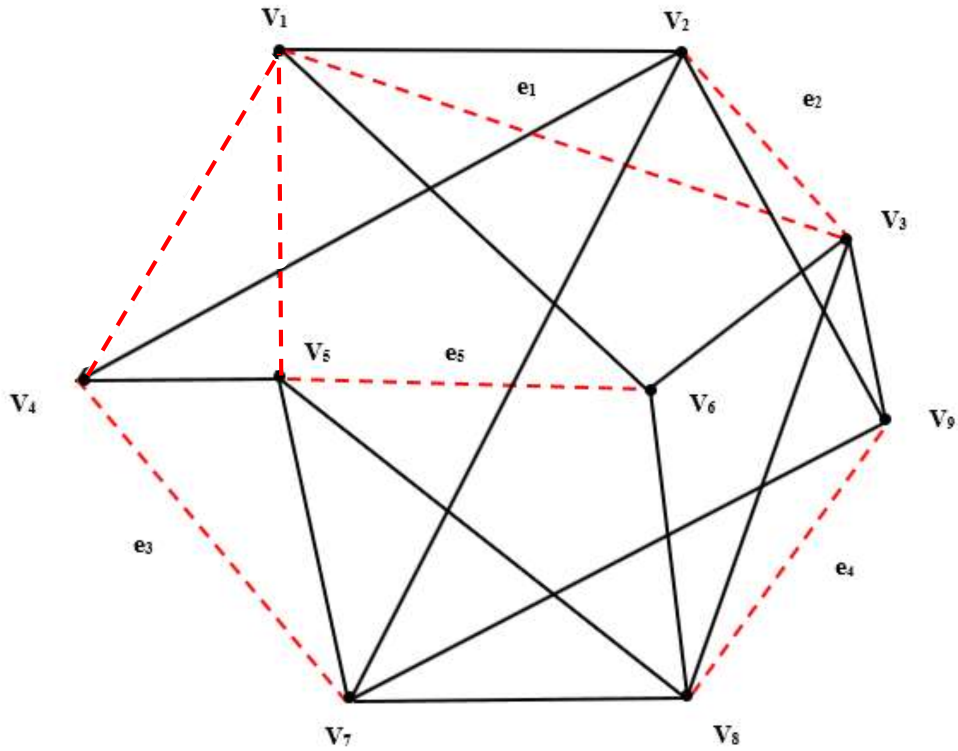

Figure 3 Fritsch graph $(G - E)$

Proof:
Let Fritsch graph G is a planar graph on 9 vertices and 21 edges. $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ are the vertices of Fritsch graph G. The set $D = \{v_1, v_7\}$ is the strongly equitable split dominating set of Fritsch graph and $< V - D >$ is a disconnected, that is $\gamma_{ses}(G) = 2$. Let $(G - E)$ be obtained from G by removal of an edge $v_1 v_3$. Now there is no change in domination number $\gamma_{se}(G - E) = \gamma_{se}(G)$. Let $(G - E)$ be obtained from G by removal of the edges $v_2 v_3$, $v_9 v_8$, $v_5 v_{6,} v_5 v_1, v_4 v_1$ and $v_4 v_7$. *Figure 3* shows the removal edges subset $\{e_1, e_2, e_3, e_4, e_5\}$. . Therefore $D = \{v_2, v_8\}$. Thus $\gamma_{ses}(G - E) = 3$. The vertex v_1 is strongly dominates the vertices v_2, v_4, v_5, v_6. The vertex v_7 is strongly dominates the vertices $v_2, v_{9,} v_8, v_5$. The vertex v_8 is strongly dominates the vertices v_3, v_7, v_5, v_6 where v_1, v_7, v_8 are dominating vertices in $(G - E)$. Hence these three vertices strongly dominate all the other vertices of $(G - E)$. Therefore, $deg(u) \geq deg(v)$. For every $v \in V - D = \{v_2, v_3, v_4, v_5, v_6, v_9\}$ *is disconnected* there exists $u \in D$ such that $|deg(u) - deg(v)| \leq 1$. Thus the minimum strongly equitable split domination number of $(G - E)$ is 2 ie. $\gamma_{ses}(G - E) = 2$. *Figure 3* shows that the strongly equitable split domination number of $(G - E)$ is equal to the Strongly equitable split domination number of G. Therefore $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Hence $b_{nses}(G) = [n - 7]$.

Theorem 3.5
Let G be a Herschel graph, then $b_{nses}(G) = [n - 8]$ if $n = 11$

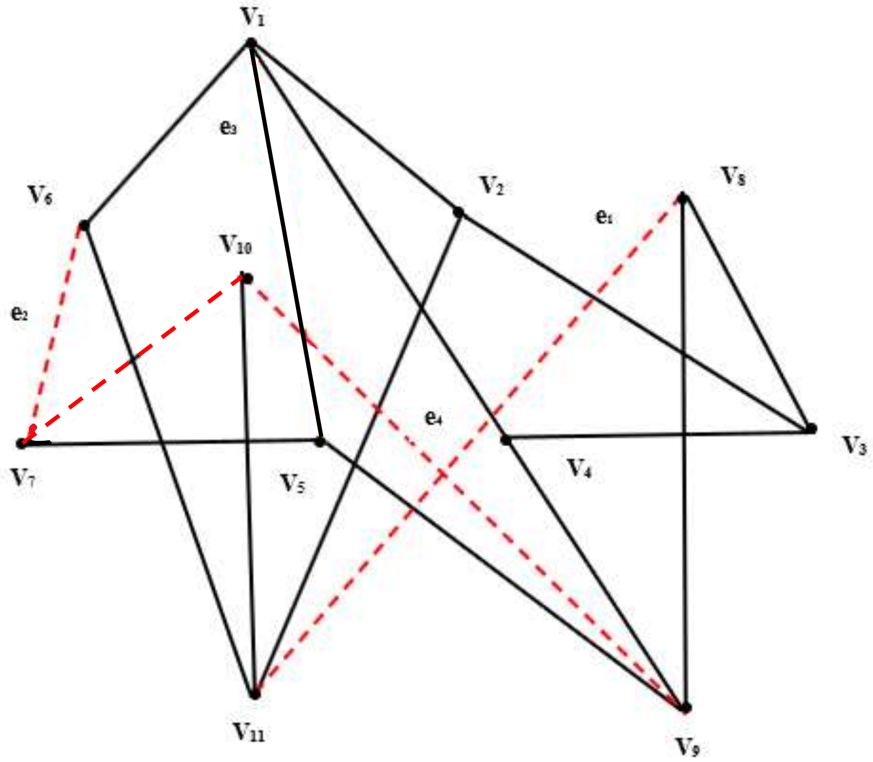

Figure 4 Herschel graph $(G - E)$

Proof:
Let Herschel graph G is a bipartite undirected graph with 11 vertices and 18 edges. Suppose $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$ are the vertices of Herschel graph G. The set $D = \{v_1, v_3, v_{11}\}$ is the strongly equitable split dominating set of Herschel graph and $\gamma_{ses}(G) = 3$. Let $(G - E)$ be obtained from G by removal of an edge $v_{11}v_8$. Now there is no change in domination number $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Let $(G - E)$ be obtained from G by removal of the edges v_6v_7, $v_1v_5, v_7v_{10}\, and v_9v_{10}$. *Figure 3* shows the removal of edge subset $\{e_1, e_2, e_3, e_4\}$. Therefore $D = \{v_3, v_5, v_{11},\}$. Thus $\gamma(G - E) = 3$. Here the vertex v_1 is strongly dominates the vertices v_6, v_2, v_4. The vertex v_2 is strongly dominates the vertices v_1, v_{11}, v_3. The vertex v_7 is strongly dominate the vertices v_{10}, v_5. Where v_1, v_2, v_7 are dominating vertices in $(G - E)$. Hence these four vertices strongly dominate all the other vertices of $(G - E)$. Therefore $deg(u) \geq deg(v)$. For every $v \in V - D = \{v_3, v_4, v_5, v_6, v_8, v_{10}, v_{11}\}\, is\ disconnected,$ there exists $u \in D$ such that $|deg(u) - deg(v)| \leq 1$. Thus the minimum strongly equitable split domination number of $(G - E)$ is 3 ie $\gamma_{ses}(G - E) = 3$. *Figure* shows that the strongly equitable split domination number of $(G - E)$ is equal to the strongly equitable split domination number of G. Therefore $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Hence $b_{nses}(G) = [\,n - 8]$.

Theorem 3.6
If the graph G be Moser Spindle graph, then $b_{nses}(G) = [\,n - 5]$ if $n = 7$

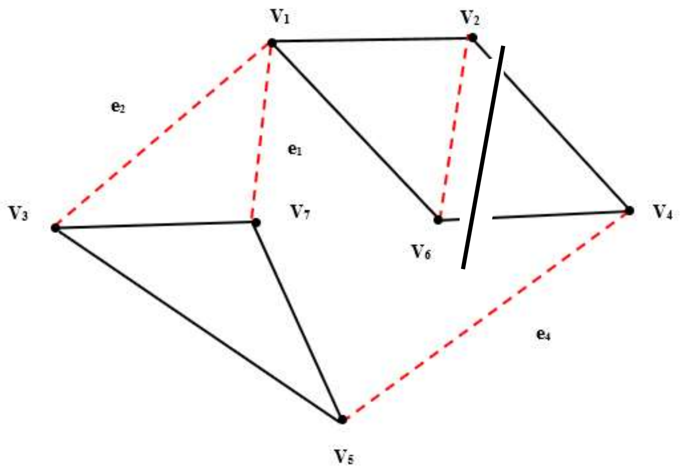

Figure 5 Moser Spindle graph $(G - E)$

Proof:
By the definition, the Moser Spindle graph is an undirected graph with 7 vertices and 11 edges. $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$ are the vertices of Moser Spindle graph G. The set $D = \{v_1, v_3\}$ is the strongly equitable split dominating set of Moser Spindle graph and $\gamma_{ses}(G) = 2$. Let $(G - E)$ be obtained from G by removal of an edge v_1v_7. Now there is no change in domination number $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Let $(G - E)$ be obtained from G by removal of the edges $v_1v_3,\ v_2v_6\ and\ v_4v_5$. *Figure 6* shows the removal of edge subset $\{e_1, e_2, e_3, e_4\}$. Therefore $D = \{v_6, v_7\}$. Thus $\gamma_{ses}(G - E) = 2$. Here out of these 7 vertices, we need only 2 vertices to dominate all the other vertices of $(G - E)$. Hence $deg(u) \geq deg(v)$. For $u \in D$, $v \in V - D = \{v_2, v_4, v_5, v_7\} is\ disconnected$ such that $|deg(u) - deg(v)| \leq 1$. Thus, the minimum strongly equitable split domination number of $(G - E)$ is 2. i.e. $\gamma_{ses}(G - E) = |D| = 2$. Figure shows that the strongly equitable domination number of $(G - E)$ is equal to the strongly equitable split domination number of G. Therefore $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Hence $b_{nses}(G) = [n - 5]$.

Theorem 3.7
For a Franklin graph G such that $b_{nses}(G) =$ n−8 if $n = 12$

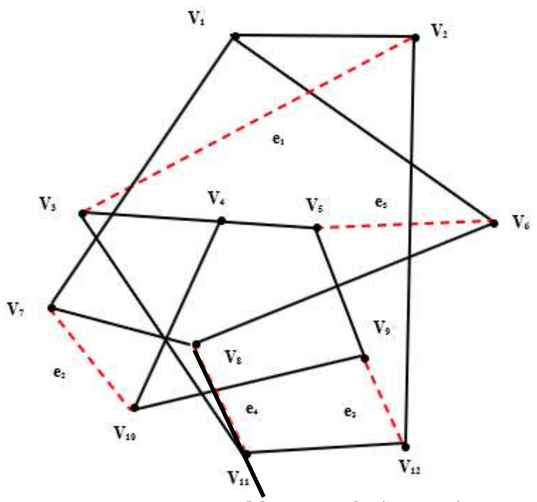

Figure 6 Franklin graph $(G - E)$

Proof:
Let G be a Franklin graph with 12 vertices and 18 edges. Suppose $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ are the vertices of Franklin graph G. The set $D = \{v_4, v_7, v_8, v_{12}\}$ is a strongly equitable

split dominating set of Franklin graph and $\gamma_{ses}(G) = 4$. Let $(G - E)$ be obtained from G by removal of an edge $v_2 v_3$. Now there is no change in domination number $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Let $(G - E)$ be obtained from G by removal of the edges $v_7 v_{10}$, $v_{12} v_9$, $v_8 v_{11}$ and $v_6 v_5$. *Figure 6* shows the removal edge subset $\{e_1, e_2, e_3, e_4, e_5\}$. Therefore $D = \{v_1, v_4, v_{10,}, v_{11}\}$. Thus $\gamma_{ses}(G - E) = 4$. The vertices of D are adjacent to the vertices of $V - D = \{v_2, v_3, v_4, v_5, v_7, v_8, v_{12}\} is\ disconnected$ and $u \in D = \{v_1, v_6, v_9, v_{10}, v_{11}\}$ strongly dominates $v \in V - D$ such that $deg(u) \geq deg(v)$. We also found that $|\,deg(u) - deg(v)\,| \leq 1$. Thus the strongly equitable split domination number of $(G - E)$ is 4. i.e. $\gamma_{ses}(G - E) = 4$. *Figure* 7 shows that the Strongly equitable split domination number of $(G - E)$ is equal to the strongly equitable domination number of G. Therefore $\gamma_{ses}(G - E) = \gamma_{ses}(G)$. Hence $b_{nses}(G)$ = n−8.

3. Conclusion

In this paper, we established non -bondage strongly equitable split number as parameter for some families of graphs. The non- bondage number of a graph, is the cardinality of a maximum subset of edges, whose removal $(G - E)$, results in a graph with domination number is equal to the domination number of G and it is denoted by non- bondage strongly equitable split domination number. We also proved that $\gamma_{ses}(G - E) = \gamma_{ses}(G)$.

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