Study on motion of an electron in a physical system by using some integral transforms

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Abstract

We present a differential equation of motion of a particle as well as utilizing some integral transform methods for example Mohand transform and Laplace transforms. We begin by showing how the Mohand transformation method applied to a dynamical system. The results are compared with after applying Laplace transform method. The results shows that the both techniques are easy and analyzed with each other.

Keywords: Physical problems, Differential equations, Mohand transform and Laplace transform.

1. Introduction:

The MohandTransform technique was used to solve physical system issues that include boundary value difficulties in virtually every area of science and engineering. The physical differential equations have been solved by a number of integral transforms, including Laplace, Fourier, and others, were utilized.

The "Mohand Transform" is one of the integral transforms that can be utilized in the process of resolving any boundary value problem that manifests itself in the form of a differential equation representing a physical system. It is general known that differential equations may often be computed by applying the Laplace transform technique as part of the solution process. However, the “MohandTransform" is an exception to this rule. In this study, we examined the two different integral transform techniques. To begin, we will explain basic formulae and attributes shared by both transforms.

In addition, LT represents Laplace transform, MT represents Mohand transform whereas D.E represents differential equation.

2. Definition of Mohand transform and Laplace transform:

Mohand transform is defined as for the function f(t) for $f(t) \geq 0$ as $M[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt = p(v),\ k_1 \leq v \leq k_2.$ Where M is the called Mohand transform indicator.	Laplace transform is defined as for the given function f(t) for $f(t) \geq 0$ as $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s),$ Where s is the parameter and L is the called Laplace transform indicator.

Both transform should have the sufficient condition is continuous and exponential.

3.MT and LT of some standard functions

Sl.no.	f(t)	L[f(t)]	M[f(t)]
1	1	$\frac{1}{s}$	v
2	Cosh(at)	$\frac{s}{s^2 - a^2}$	$\frac{v^3}{v^2 - a^2}$
3	Sinh(at)	$\frac{a}{s^2 - a^2}$	$\frac{av^2}{v^2 - a^2}$
4	Cos(at)	$\frac{s}{s^2 + a^2}$	$\frac{v^3}{v^2 + a^2}$
5	e^{at}	$\frac{1}{s - a}$	$\frac{v^2}{v - a}$
6	Sin(at)	$\frac{a}{s^2 + a^2}$	$\frac{av^3}{1 + a^2 v^2}$
7	t	$\frac{1}{s^2}$	1

4. Inverse Mohand and Laplace Transform definitions:

Inverse Mohand transform : If $p(v)$ is the Mohand transform of f(t) is called inverse Mohand transform of $p(v)$ and is represented by $f(t)=M^{-1}[p(v)]$ where M^{-1} is an operator and is called Mohan inverse transform indicator .	Inverse Laplace transform: If f(s) is Laplace transform of f(t) ,then $f(t)=L^{-1}[F(s)]$ is the inverse of f(s), L^{-1} is the inverse Laplace indicator.

5. Inverse Mohand transform and inverse Laplace transform of standard Functions

Sl.no.	f(t)	$L^{-1}[f(t)]$	$M^{-1}[T(v)]$
1	1	$\frac{1}{s}$	v
2	Cosh(at)	$\frac{s}{s^2-a^2}$	$\frac{v^3}{v^2-a^2}$
3	Sinh(at)	$\frac{a}{s^2-a^2}$	$\frac{av^2}{v^2-a^2}$
4	Cos(at)	$\frac{s}{s^2+a^2}$	$\frac{av^2}{v^2-a^2}$
5	e^{at}	$\frac{1}{s-a}$	$\frac{v^2}{v-a}$
6	Sin(at)	$\frac{a}{s^2+a^2}$	$\frac{av^3}{1+a^2v^2}$
7	t	$\frac{1}{s^2}$	1

6**. Mohand and Laplace transform of derivatives of function f(t) :**

6.1 **M.T of** $\frac{d\ f(t)}{dt}$ **for the function f(t)**

If $M[f(t)] = p(v)$ then

$M[f^{'}(t)] = v\,p(v) - v^2\ f(0)$

$M[f^{''}(t)] = v^2\ p(v) - v^3\ f(0) - v^2\ f^{'}(0)$

$M[f^{n}(t)] = v^{n} p(v) - v^{n+1}\ f(0) - v^{n}\ f^{'}(0)\ldots\ldots\ldots v^2\ f^{n-1}(0)$.

6.2 **L .T of** $\frac{d\ f(t)}{dt}$ **for the function f(t) :**

If $L[f(t)] = \bar{f}(s)$ then

$L\left[\frac{d\ f(t)}{dt}\right] = sf(s) - f^{'}(0)$

$L\left[\frac{d^2 f(t)}{dt}\right] = s^2 f(s) - sf(0) - f^{'}(0)$

7**. Main Results:**

Consider motion of an electron is given by the following equations

$$m\frac{d^2x}{dt^2} + eh\frac{dy}{dt} = eE................(1)$$

$$m\frac{d^2y}{dt^2} - eh\frac{dx}{dt} = 0..................(2)$$

Consider the conditions

$$x(0) = 0, \frac{dx}{dt} = 0,\ y(0) = 0, \frac{dy}{dt} = 0.$$ ”

To determine path of an electron ,if it started from the rest.

Both sides, Apply Mohand Transform of equation (1)

$$m\,M\left[\frac{d^2x}{dt^2}\right]+eh\,M\left[\frac{dy}{dt}\right]=M\left[eE\right]$$

$$M\left[\ddot{x}(t)\right]+\omega\,M\left[\dot{y}(t)\right]=\frac{eE}{m} \text{ Where } \omega=\frac{eH}{m}$$

$$\left[v^2M[x(v)]-v^3\,x(0)-v^2\,\dot{x}(0)\right]+\omega\left[v\,M[\,y(v)]-v^2\,y(0)\right]=\frac{eE}{m}$$

Using the initial conditions $x(0)=0,\ y(0)=0\ and\ \dot{x}(0)=0$ we get

$$v^2\ M[x(v)]+\omega\ vM[\,y(v)]=\frac{eE}{m}\ \ldots\ldots\ldots\ldots\ldots..(3)$$

Taking Mohand Transform of equation (2) and applying the conditions on both sides we get

$$m\,M\left[\frac{d^2y}{dt^2}\right]-ehM\left[\frac{dx}{dt}\right]=0.$$

$$m\ v^2\ \dot{y}(v)-eh\,v\,\dot{x}(v)=0.\ \ldots\ldots\ldots\ldots\ (4)$$

Now Solving Equations (3) and (4), we get

$$M\left[y(v)\right]=\frac{eE\omega}{m}\left[\frac{1}{v^3+\omega^2 v}\right]$$

Applying inverse Mohand transform we get ,

$$y(t)=\frac{E}{h\omega}\left[\omega t-\sin\omega t\right].\ \ldots\ldots\ldots\ldots\ldots\ldots..(5)$$

Similarly we get $x(t) = \frac{E}{h\omega}[1 - \cos \omega t]$. ……………………...(6)

Let us consider the equations (1) as well as (2) and both sides apply Laplace transform

$$L\left[\frac{d^2x}{dt^2}\right] + \omega L\left[\frac{dy}{dt}\right] = L\left[\frac{eE}{m}\right]$$

$$\left\{s^2 L[x(t)] - s\,x(0) - \dot{x}(0)\right\} + \omega\left\{sL[y(t)] - y(0)\right\} = L\left[\frac{eE}{m}\right]$$

Since $x(0) = 0, \frac{dx}{dt} = 0,\ y(0) = 0,\ \frac{dy}{dt} = 0.$

$$s^2 L[x(t)] + \omega[sL[y(t)]] = \frac{eE}{\omega s} \text{ …………(7)}$$

Similarly applying Laplace transform for equation (2)

$$L[\frac{d^2y}{dx^2}] - \frac{eh}{m} L\left[\frac{dx}{dt}\right] = 0$$

$$\left\{s^2 L[y(t)] - sy(0) - \dot{y}(0)\right\} - \omega\left\{s\,L[x(t)] - x(0)\right\} = 0$$

Applying boundary conditions

$$\left\{s^2 L[y(t)] - \omega\,sL[x(t)]\right\} = 0. \text{ ………………..(8)}$$

Solving equations (7) and (8)

$$L[x(t)] = \frac{\frac{eE}{\omega}}{s(s^2 + \omega^2)}$$

Applying inverse Laplace transform

$$x(t) = L^{-1}\left[\frac{\frac{eE}{\omega}}{s(s^2 + \omega^2)}\right]$$

Let us consider as $\bar{f}(s) = \frac{1}{s}$ *and* $\bar{g}(s) = \frac{1}{(s^2 + \omega^2)}$

Both sides apply Laplace inverse transform on the functions

We will find $f(t) = 1$ and $g(t) = \frac{\sin \omega t}{\omega}$

Using convolution theorem to get the value of $x(t)$.

$$L^{-1}\left[\bar{f}(s) * \bar{g}(s)\right] = \int_0^t f(u).g(t-u)du$$

Then we get $x(t)$ *and* $y(t)$ value

$$x(t) = \frac{E}{h\omega}[1 - \cos \omega t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

and

$$y(t) = \frac{E}{h\omega}[\omega t - \sin \omega t] \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Comparing equations (5) and (6) and (9) and (10) are similar.

8. Conclusion:

We have effectively generalized the differential equations using the Mohand Transforms and the Laplace Transform approach in this research. It clearly shows that after applying these methods for any physical system problems the solution is similar.

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