A Review of Two-dimensional Offline Rectangular Strip Packing Problem Heuristics

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Abstract

Packing and cutting problems have been considered a subdiscipline of operation research for more than half a century. These problems can arise in numerous settings, including pallet loading, fabric, paper, wood, metal, glass cutting, textile production, and multiprocessing scheduling. This article focuses on the 2D strip packing problem, which belongs to the NP-hard class of problems. There are three categories of rectangular strip packing problems (offline, online, and almost). The main objective of this paper is to provide a concise overview of the effective heuristics used to solve the 2D offline rectangular strip packing problem over the past twelve years. Based on a search methodology, 31 papers are found and analyzed. The statistical analysis indicates that 62% of researchers tend to implement heuristic methods. In addition, the most optimal solutions for the most recent updates of the 2DSPPs, which account for less than 15% of collected papers, are still necessary to cope with changes and assess the quality of the proposed solution approaches. Some recommendations for future research are presented in the conclusion.

Keywords: Packing problems, 2D strip packing, and heuristics.

1. Introduction

The old classification of 2D cutting & packing problems, depicted in Fig 1, divides packing problems into two categories; spatial bin packing problems and non-spatial packing problems. In general, spatial packing consists of the 2D bin and strip packing problems. 2D bin packing problems include single bin packing problems, multiple bin packing problems, and their variations (offline, online, almost online). In contrast, non-spatial packing problems, such as capital budgeting problems, in which projects represent small objects and share capital, a large object, must be assigned or allocated to these projects. Projects include new machines or machine replacements, new plants or products, and other research development projects.

Dyckhoff [1] presented the first typology of cutting and packing based on four characteristics, dimensionality, assignment type, large objects assortment, and small items assortment. Based on Dyckhoff's typology, Wäscher [2] modified some criteria for incorporating other problem types (such as the 2D strip packing problem). Ntene [3] extended these criteria by introducing a typology with six fields, as shown in the following array format:

$$\alpha \mid \beta \mid \chi \mid \Upsilon \mid \lambda \mid \tau\ (\tau_o, \tau_p, \tau_m, \tau_g)$$

Where:

α: dimensionality, β: shapes of items packed, χ: area where items will be packed, Υ: level of information, λ: objective of packing, τ: constraints of packing which are denoted in binary variables 0 or 1. One indicates this type of constraint is considered while zero is not. τ_o: rotation of rectangles during packing can be by 0 or 90 degrees where each side of all rectangles is parallel to the two sides of the strip. τ_p: constraints imposed for placing items. τ_m: constraints concerned with modifications in resources required (time, height of shape, or width) to complete the packing mission. τ_g: guillotine cutting constraint. Guillotine cutting means edge-to-edge cutting.

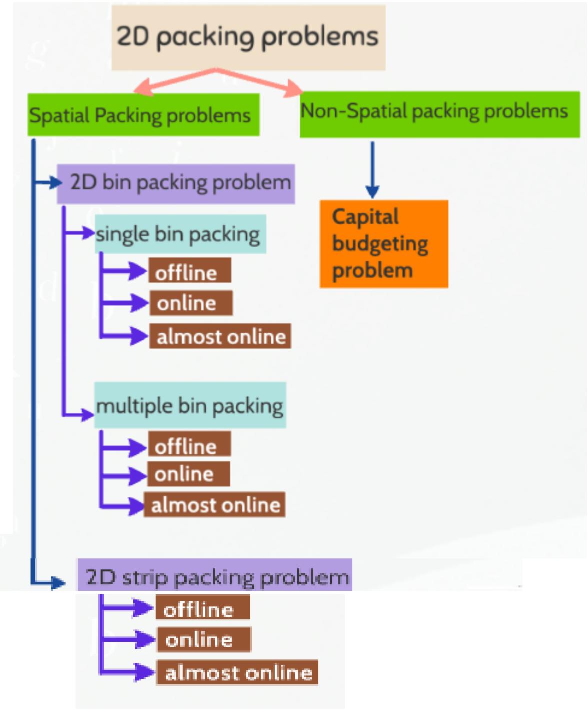

Fig. 1. Different classes of 2D packing problems (Old Classification)… N. Ntene [3]

In 2016, Oliveira et al. [4] introduced new classification of heuristics algorithms that used to solve strip packing problems.This classification consists of three types of constructive heuristics : fitness-based (e.g., best fit), positioning-based (e.g., bottom left), and profile-based (e.g., skyline). While the heuristics algorithms that used before were based on the packing type are classified into three categories: level, shelf, and plane heuristics.

In level algorithms, the strip is divided into levels or sections. Level height is determined by the height of the tallest rectangle packed in the level. Shelf algorithms compute the height of each newly generated shelf by some specific ways. The shelf height is not necessarily equal the height of tallest rectangle. On the other hand, plane algorithms does not divide strip plane. Consequently, rectangles can be cut or packed in any position within the strip plane.

Prior to explicitly naming the 2D strip packing problem, which is a branch of the 2D stock cutting problem, the initial developments for the 2D strip packing problem are examined. In 1939, Gilmore & Gomory [5] published the first paper to address the second dimension of this problem, but it was not translated into English until 1960 that was mentioned by Dowsland & Dowsland [6]. However, other papers handled similar problems by AE Paul and JR Walter 1954, Metzger 1958, Eilon 1960. That is what Dowsland & Dowsland [6] said in his sevey paper. Adding to this, all these cases were for small-scale problems. Their success in solving the one-dimensional stock-cutting problem motivated them to investigate the two-dimensional case. In 1961, Gilmore & Gomory introduced the use of column generation to solve the problem of 2D bin packing and cutting. The formulation of dynamic programming was proposed and presented. The problem was then determined to be a programming issue involving binary integers. Therefore, the formulation was too complex for the Branch and Bound algorithm to solve. Dowsland & Dowsland [6] refered that , Decani found the first optimal solution of 2D and 3D bin packing and cutting problems in 1978. He suggested that a simple tree search algorithm for packing identical items in 2D and 3D required 50 hours for 10 pieces. According to Dowsland & Dowsland [6], Beasly in 1985 demonstrated that using Lagrangian relaxation can be suitable for moderately sized problems. Smith H 1985 was the first to apply a metaheuristic “genetic” algorithm to obtain economically viable solutions.

This research focuses on 2D Orthognal strip packing problems (2DSPPs), which are primarily concerned with packing small shapes (typically assumed to be rectangles) orthogonally(or parallel to width or height of strip) without overlap within a strip with a fixed width and a preassumed infinite height. The objective is to minimize the packing’s total height. The problem of our research can be summarized using the following format [3]:

2D | R | SP | Off | MiS | *, *, *, *

The star symbol in the final field indicates that constraints in two cases (yes or no) are considered. After correctly defining the problem using one of these typologies, researchers examine the problem to identify compromise solutions. Because these solutions unquestionably result in reasonable cost savings, which is what every organization typically seeks. The primary objective of this paper is to examine the solution methods used to solve 2DSPPs. The paper is organized as follows: in section 2, the search methodology is introduced. In section 3, the results of the search methodology are displayed and analyzed. Section 4 contains common plane heuristics, meta-heuristics, and exact methods. Finally, the last section introduces the research conclusions.

2. Search Methodology

We restricted the reviewed literature as follows: relevant papers or articles were identified using the Elsevier and springer databases, with "2D strip packing" as the title word under the Engineering field. Only articles written in English within the impacted journal are only considered. Therefore, no textbooks, conference papers, or survey papers related to our selected type of strip packing problem are taken into consideration. The most significant literature from the past twelve years (from 2010 to 2022) was reviewed, along with the most effective and relevant papers.

3. Results of search Methodology

After applying the above-mentioned search methodology, 31 papers are found and shown in Table 1, which displays all details of papers from 2010 to 2022 related to our topic. The publication year of each article is listed in ascending order in Table 1. R, G, and NG mean 90°-rotational, guillotine constraint and non-guillotine constraint. The number 1 in each cell indicates that the feature or the constraint is considered, while 0 indicates that it is not considered.

Table 1. Summary of literature review related to 2DSPP in the period between 2010-2022.

Serial	Citation	Problem characteristics													Objective				Solution method		
		2D	3D	SP	Bin	Shelf	Level	Plane	R	G	NG	With- Unloading	Load balancing	Multidrop	MIS	MVP	MAP	MINS	Heuristic	Meta Heuristic	Exact
1	S. Imahori and M. Yagiura [7]	1	0	1	0			1	1	0	1	0	0	0	1				1		
	S. Imahori and M. Yagiura [7]	1	0	1	0			1	0	0	1	0	0	0	1				1		
2	F. G. Ortmann, N. Ntene, and J. H. van Vuuren [8]	1	0	1	0			1	1	1	0	0	0	0	1				1		
3	L. Wei, W. C. Oon, W. Zhu, and A. Lim [9]	1	0	1	0			1	1	0	1	0	0	0	1				1		
4	S. C. H. Leung, D. Zhang, and K. M. Sim[10]	1	0	1	0			1	0	0	1	0	0	0	1				1		
5	E. K. Burke, M. R. Hyde, and G. Kendall[11]	1	0	1	0			1	0	0	1	0	0	0	1					1	
6	Y. Arahori, T. Imamichi, and H. Nagamochi [12]	1	0	1	0			1	1	0	1	0	0	0	1						1
	Y. Arahori, T. Imamichi, and H. Nagamochi [12]	1	0	1	0			1	0	0	1	0	0	0	1						1
7	P. M. Castro and I. E. Grossmann [13]	1	0	1	0			1	0	0	1	0	0	0	1						1
8	M. Mesyagutov, G. Scheithauer, and G. Belov [14]	1	0	1	0			1	0	0	1	0	0	0	1						1
9	T. A. De Queiroz and F. K. Miyazawa [15]	1	0	1	0	0	1	0	1	0	1	1	1	1	1				1		
10	K. T. Park, H. Kim, S. Lee, H. K. Lee, J. H. Ryu, and I. B. Lee,[16]	1	0	1	0			1	0	1	0	0	0	0	1						1
11	Ö. B. Aşik and E. Özcan[17]	1	0	1	0			1	1	0	1	0	0	0	1				1		
12	T. Wauters, J. Verstichel, and G. Vanden Berghe [18]	1	0	1	0			1	1	0	1	0	0	0	1				1		
	T. Wauters, J.	0	1	1	0			1	1	0	1	0	0	0	1				1		

	Verstichel, and G. Vanden Berghe [18]																				
13	K. He, Y. Jin, and W. Huang [19]	1	0	1	0			1	1	0	1	0	0	0	1				1		
15	Y. Cui, L. Yang, and Q. Chen [20]	1	0	1	0			1	1	1	0	0	0	0	1				1		
16	J. L. M. Da Silveira, F. K. Miyazawa, and E. C. Xavier [21]	1	0	1	0			1	1	0	1	1	0	0	1					1	
	J. L. M. Da Silveira, F. K. Miyazawa, and E. C. Xavier [21]	1	0	1	0	0	1	0	0	0	1	1	0	0	1				1		
17	S. Yang, S. Han, and W. Ye [22]	1	0	1	0			1	0	0	1	0	0	0	1				1		
18	J. L. M. Da Silveira, E. C. Xavier, and F. K. Miyazawa,[23]	1	0	1	0			1	1	0	1	1	0	0	1				1		
19	J. L. M. Da Silveira, E. C. Xavier, and F. K. Miyazawa [23]	1	0	0	1	1		0	0	0	1	1	0	0	1				1		
20	R. Harren, K. Jansen, L. Prädel, and R. Van Stee[24]	1	0	1	0			1	0	0	1	0	0	0	1				1		
21	J. Thomas and N. S. Chaudhari [25]	1	0	1	0			1	1	0	1	0	0	0	1					1	
22	L. Wei, T. Tian, W. Zhu, and A. Lim [26]	1	0	1	0			1	1	1	0	0	0	0	1				1		
	L. Wei, T. Tian, W. Zhu, and A. Lim [26]	1	0	1	0			1	0	1	0	0	0	0	1				1		
23	D. Zhang, L. Shi, S. C. H. Leung, and T. Wu [27]	1	0	1	0			1	0	0	1	0	0	0	1					1	
24	I. Babaoğlu [28]	1	0	1	0			1	0	0	1	0	0	0	1					1	
25	L. Wei, Q. Hu, S. C. H. Leung, and N. Zhang [29]	1	0	1	0			1	0	0	1	0	0	0	1				1		
26	L. Wei, Y. Wang, H. Cheng, and J. Huang [30]	1	0	1	0			1	0	0	1	1	0	0	1				1		
27	K. Jansen and M. Rau [31]	1	0	1	0			1	0	0	1	0	0	0	1				1		
28	H. Fırat and N. Alpaslan [32]	1	0	1	0			1	1	1	0	0	0	0	1				1	1	
29	K. Zhu, N. H. Ji, and X. D. Li [33]	1	0	1	0			1	0	0	1	0	0	0	1				1		
30	R. G. Rakotonirainy and J. H. van Vuuren [34]	1	0	1	0			1	0	0	1	0	0	0	1				1		
31	R. G. Rakotonirainy and J. H. van Vuuren [34]	1	0	1	0			1	0	0	1	0	0	0	1					1	
32	H. Becker, O. Araujo, and L. S. Buriol [35]	1	0	1	1			1	0	1	0	0	0	0	0		1	1			1
33	E. Oviedo-Salas et al.[36]	1	0	1	0			1	0	0	1	0	0	0	1					1	
34	J. Verstichel, P. De Causmaecker, and G. Vanden Berghe [37]	1	0	1	0			1	1	0	1	0	0	0	1				1		

MIS: Minimize Strip height,**MVP**: Minimize the volume of packing,**MINS**: Minimize the number of stocks,**MAP**: Maximize Profit

In Tables 2 and 3, all solution methods are categorized as follows: plane, level, and shelf heuristics, metaheuristics, and exact methods. The citations for Level, shelf heuristics, metaheuristics, and exact methods, as well as their acronyms, are presented in Table 2. Table 3 displays the plane heuristics description, its acronyms, and its citation. Following this, Table 4 displays the journals of all papers, their most recent impact factor, and the total number of papers within the scope of our research from 2010 to 2022. The impact factor for each paper'journal is between 0.455 and 11.251.

Table 2. List of level, shelf heuristics, metaheuristics, exact methods, citations, and their acronyms.

Authors	Heuristic Acronym	Algorithm
[7]	SASm, BFS	Modified size-alternating stack algorithm, best fit with stacking algorithm
[10]	SWP	Squeaky wheel optimization packing methodology
[11]	BB-2SPFH, RS, BB-1CBPFH	Branch and bound for 2Dstrip packing with fixed height, Restricted stair, one-dimensional continuous bin packing problem with fixed height
[12]	NDCS, NCS	New hybrid discrete-space approach, new continuous-space approach (new mixed-integer linear programming approaches for2DSP
[13]	LPPR	LP-based pruning rules
[15]	MINLP	A non-convex mixed-integer linear programming model
[20]	GRASP	Greedy Randomized Adaptive Search Procedure
[25]	GPA	Genetic algorithm-based placement approach
[28]	FOA	Fruit fly optimization algorithm
[32]	N-FP, SA, BLF	No-Fit Polygons, simulated annealing algorithm, Bottom-Left Fill
[34]	IA, SPGAL	Intelligent search algorithm, Genetic algorithm for strip packing
[36]	MILP	Mixed Integer Linear Programming
[36]	GRASP	Greedy Randomized Adaptive Search Procedure

Fig. 2 depicts the distribution of all solution methods used between 2010 to 2022 based on our research methodology. According to our pie graph details, plane heuristics represent 58% of the literature review gathered solution methods. Shelf or level heuristics for 2DSPPs account for 4% of total research. In the past twelve years, meta-heuristics have comprised 23% of solution methods, while exact techniques have comprised approximately 15% of solution methods. The total number of tracked research over the past twelve years is depicted in Figure 3. The highest number of articles or published papers (9) was recorded in 2013, while no articles were found in 2015 and 2018. The distribution of all problem characteristics during this time period is depicted in Figure 4. Only one article considered the third dimension in addition to two dimensions for the solution method. Moreover, only 3% of papers addressed load balancing, load bearing, and multidrop constraints. In 42% of studies, rotational constraints are taken into account. In 18% of papers, guillotine constraints on solution methods were considered. In 16% of all papers, unloading constraints were found as well.

Approximately two-thirds, or 62%, of researchers, are interested in using the heuristic approach distributed between 2010 and 2020, as shown in Fig. 2, whereas 38% of researchers use meta-heuristics or exact approaches distributed over the same time period. Fig.3 also displays the number of articles published each year.

Table 3. List of plane heuristics, citations, and their acronyms.

Authors	Acronym	Algorithm
[7]	BF	Best Fit
[8]	IDBS, 2DRPH	Iterative Doubling Binary Search algorithm, Greedy Heuristic for 2D Rectangular single large object Problem
[10]	ISA	Intelligent Search Algorithm
[15]	B-BF	Balanced Best Fit
[17]	BBF	Bidirectional Best Fit Heuristic
[37]	T-w BF	Three Way best fit heuristic
[19]	DHA	Deterministic heuristic algorithm
[20]	SGVCP	(Sequential Grouping and Value Correction Procedure)
[17]	SP	Shaking Procedure
[22]	SRA	Simple Randomized Algorithm
[23]	MNFDH	Modified Next Fit Decreasing Height
[24]	AASA	Approximation Algorithm based on Steinberg's Algorithm
[26]	BLBA	Block-based Layer-Building Algorithm

[27]	PH	Priority Heuristic
[29]	ISH	Improved Skyline heuristic
[30]	OSFFH	Open Space-based First-Fit Heuristic
[31]	PPAA	Pseudo-Polynomial Approximation Algorithm
[33]	RSRA	Reinforcement Learning-based Simple Random Algorithm

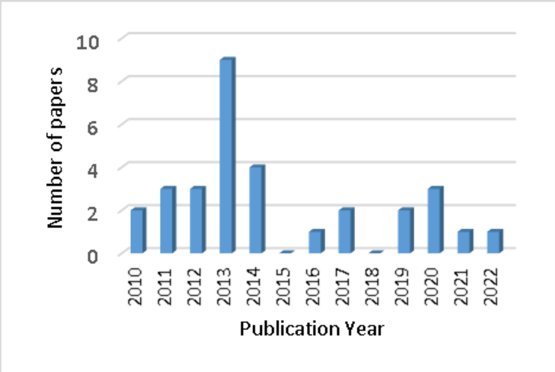

Fig. 2. Number of reviewed articles / Publication year.

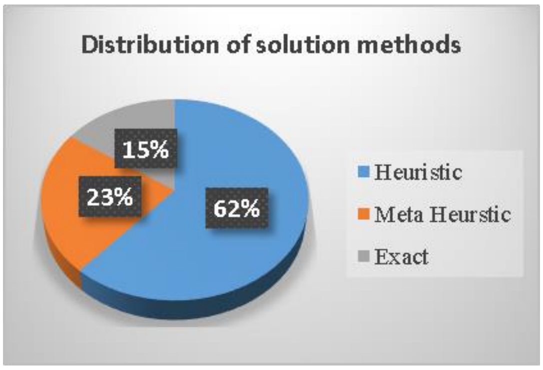

Fig. 3. Distribution of all solution methods found for 2DSPPs.

Table 4. Journal names and impact factor of cited papers 'journals:

Journal	IF	# Of papers 2010-2022
Computers & Operations Research	5.16	11
European Journal of Operational Research	6.363	5
International Journal of Production Economics	11.251	1
Expert Systems with Applications	8.665	2
International transactions In Operational Research	3.61	1
Discrete Applied Mathematics	1.254	2
Computational Geometry: Theory and Applications	0.455	1
Journal of Industrial Engineering International	3.366	1
Applied Mathematical Modelling	5.336	1
Theoretical Computer Science	1.002	1
IEEE Open Access Journal	3.467	1
Procedia Computer Science	2.267	2
Applied Soft Computing Journal	9.028	1
Computers & Industrial Engineering	7.811	1
Applied sciences (Switzerland)MDPI	3.143	1
Computers and Chemical Engineering	4.13	1

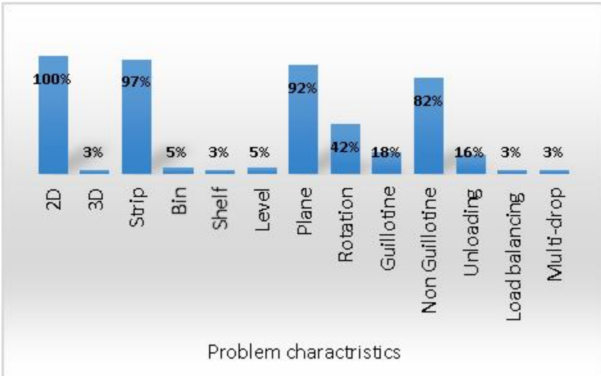

Fig. 4. The distribution of all problem characteristics on our tracked papers

4. Summary of Literature review

This section summarizes well-known (plane, level, shelf) heuristics, metaheuristics, and exact 2DSPP heuristics and metaheuristics discovered based on our search criteria. In the first subsection, we summarize the plane heuristics discovered by our search methodology between 2010 and 2022. In the second subsection, metaheuristics algorithms and exact methods are presented.

4.1 Heuristics

4.1.1 Plane Heuristics

Imahori & Yagiura [7] computed the worst-case approximation ratio of the burke algorithm made in 2004, suggesting an efficient implementation of this algorithm. In order to retain the current skyline, they stored the other rectangles to be packed and efficiently found the best-fit rectangle for every step. They used an effective data structure, which improved the best-fit heuristic's efficiency from O(n^2) to O (n.log n). Wei et al. [9] developed a skyline heuristic for 2D rectangular strip packing. The algorithm was applicable for both oriented and unoriented, non-guillotine situations. They utilized the tabu search procedure as a subroutine to aid their 2DRP heuristic in achieving optimal performance. On benchmark test problems, this algorithm outperformed the best existing approach for the 2D rectangle packing problem (2DRP). Furthermore, there are other excellent heuristic algorithms available in 2011, such as the two-stage intelligent search method (ISA) made up of local search (LS) and simulated annealing proposed by Leung et al. [10]. Rotation of items ad guillotine cuts was not considered. LS merely swaps two places of each item in a specific order. Finally, simulated annealing is used to achieve a better result.

Cui et al., [20]. presented a heuristic algorithm for the 2D guillotinable non-oriented rectangular strip packing problem. The proposed method is based on a sequential grouping and value correction procedure that evaluates multiple potential solutions. It generates the subsequent section from a subset of the remaining items and then modifies the values of the included items. The algorithm was used to resolve 13 benchmark instance groups. It could improve the quality of solutions for all groups. Yang et al. [22] proposed a straightforward randomized approach (SRA) for packing 2D strips without considering rotations of items and guillotine cuts. Leung et al. [10] improved the algorithm by replacing the simulated annealing algorithm with SRA without any parameters. The randomized algorithm was described with an updated scoring rule and a strategy for prioritizing waste minimization. The computational results demonstrated that the method was fundamental and outperformed previously reported metaheuristics.

Wauters et al. [18] reported a shacking procedure for 2DSP and 3DSP, in which the guillotine cut constraint is not considered, and the rotation of items is allowed. Improving the common bottom-left-fill (BLF) method for 2DSP and the deepest-bottom-left-fill (DBLF) technique for 3DSP is the primary contribution of this paper. The heuristic procedure begins with an ordered list of items and then alternates between the forward and reverse construction phases. Applying the bottom-left-fill algorithm is the next step. In conclusion, the authors suggested integrating their proposed procedure with any metaheuristic approach. Verstichel et al. [37] improved the best-fit heuristic for not oriented, not-guillotine cases. The addition of item placement and ordering strategies resulted in the three-way best-fit heuristic. Three strategies are most suitable (the best-fitting rectangle is put at the right-hand side of the gap). Max Different: (the best fitting rectangle is put where the top-level difference with its neighbor is maximal). Min Different: (the best fitting rectangle is put where the difference in top level with its neighbor is minimal).

In 2013, Ender ozcan et al. [38] released a paper titled “Bidirectional best-fit heuristic considering compound placement for two-dimensional orthogonal rectangular strip packing,” 2013. No guillotine cutting is needed, and all rectangular shapes can be rotated by 90 degrees. They changed the original bidirectional best-fit Heuristic of Asik and Zcan [17] to consider combinations of pairs of rectangles in the bidirectional best-fit heuristic. In contrast to other heuristics, the authors claim that the performance of adjustments to this method was comparable to state-of-the-art metaheuristics. However, the price of this improvement was undoubtedly an increase in runtime.

He et al. [19] conducted research about 2DSP, where rotation of items is allowed, and no guillotine packing constraint was considered. Their contribution was developing a scoring rule for candidate gaps where the selected item will be placed. The proposed deterministic heuristic algorithm (DHA) contains two phases: the rapid constructive phase (greedy strategy for placing all rectangles) and the partial tree search (depth-first search for finding the most promising placement). Experiments on four groups of benchmarks stated that their proposed (DHA) achieved highly competitive results compared with the latest algorithms from the literature.

By April 2014, A block-based layer-building approach for the 2D guillotine strip packing was introduced by Wei et al. [19]. They investigate two variants of the 2D strip packing problem: The Guillotine constraint is mandatory, and 2DSP rectangles can be rotated. Additionally, they examined the fixed orientation of rectangles. In brief, they combined three of the most effective solutions for rectangular packing problems into a single algorithm. They conclude that the block-based layer algorithm reduces search space, combines the benefits of two techniques, and that “simple heuristics such as best-fit remain the most effective tool for dealing with large-scale instances” but leave waste space at the end of the packing. Zhang et al. [19] presented a priority heuristic for the non-oriented guillotine rectangular packing problem. It was the first heuristic to use a prioritization strategy to select an item for a predefined position. The remaining space is then partitioned into two rectangles and packed recursively. Its worst-case time complexity was $T(n) = O(n^2)$. Overall, it is suitable for a variety of packing problems and is particularly effective for large-scale issues because it can select the item with the highest priority from a pool of available items. In addition, they suggested using it in conjunction with metaheuristics or exact methods in the future. Wei et al.[29] developed an improved skyline-based heuristic (ISH) for 2DSP without a guillotine cut constraint and without item rotation. The authors improved the best-fit Heuristic of Burke et al. [11] for 2DSP. They used the fitness number to choose the best-fitting rectangle for the gap rather than the rectangle with the greatest width. They tried a variety of sequences to improve the results by using a straightforward random local search. The trials on the benchmark test sets demonstrated the effectiveness of their strategy. The ISH, having a complexity of O (n log(n)), has been found to be superior to most heuristic algorithms.

Wei et al. [30] developed an open space-based heuristic for 2DSP with unloading, non-guillotine-orientated cutting constraints. The objective is to pack all the items into the strip while minimizing the actual length used. In addition, the resulting packaging must comply with unloading restrictions. The open space technique can effectively represent all possible positions and satisfy the unloading constraint. Also proposed was a First-Fit heuristic approach based on the open space. A randomized local search without parameters was recommended to improve the solution. According to computational findings on well-known instances, the open space is appropriate for solving the 2DSPU. This strategy is also evaluated using a 2D orthogonal packing problem with unloading constraints. The outcomes were favorable, and in the future, they recommended using open space for three-dimensional packing problems with unloading constraints (3DSPU) or other packing problems with unloading constraints, as well as exact algorithm-based open space for 2DSPU. Fırat & Alpaslan [30] published an article about a practical solution to the manufacturing industry's two-dimensional rectangular packing problem. This paper addresses the 2D rectangle packing area minimization problem (2D-RPAMP) using a hybrid solution that combines heuristic and meta-heuristic techniques. In order to develop a more effective packing method, the strip packing problem was transformed from a 2D rectangle packing area minimization problem (2D-RPAMP) into a number of 2D strip packing problems (2DSPPs). They fixed the width of the container and expected to reduce its height. To determine the optimal rectangle, they presented a scoring criterion based on area. In addition, a bottom left fill-based packing technique is combined with a simulated annealing-based meta-heuristic approach with no control parameters in order to solve (2D-RPAMP) more efficiently. Following this, the no-fit polygon heuristic method is used to prevent rectangle items from overlapping. Lastly, experimental results on benchmark test sets demonstrated that their strategy consistently outperforms the vast majority of currently employed methods. Moreover, it was observed that the performance of the proposed method remains impeccable as the number of rectangles increases.

To solve the not-oriented 2DSPP with non-guillotine cuts, Zhu et al. [33] published a hybrid heuristic algorithm that relies on improved rules and reinforcement learning. The reinforcement learning (RL) method is an artificial intelligence approach that can directly conclude helpful information from the data. RL has been extensively applied to problems involving combination optimization, such as the traveling salesman problem, the vehicle routing problem, and the packing problem. In this hybrid heuristic, the scoring rules based on the skyline algorithm are strengthened to minimize space waste. The skyline algorithm served as the framework for defining the rectangle arrangement rules. The Deep Q-Network (DQN) was developed to obtain the initial rectangle sequence and was an essential supplement for placement rules. In conclusion, researchers have discovered that reinforcement learning can achieve outstanding performance on eight data sets for combinatorial optimization problems. Conversely, RL is more expensive and time-consuming. In the future, the authors plan to focus on neural network structure to improve RL's learning ability.

4.1.2 level heuristics:

De Queiroz & Miyazawa [21] addressed 2DSPP in two practical situations. In the first, load balancing constraints and multidrop requirements are combined while the second is responsible for load distribution and load-bearing constraints. Models and heuristics based on zero-one integer (such as balanced best fit) Both situations were presented with heuristics. In order to obtain a solution in a realistic scenario, they proposed approximate procedures or heuristics. Da Silveira et al. [21] developed two heuristic algorithms for the Strip Packing Problem with Unloading Constraints (SPU). The issue was whether rotating items could be achieved with non-guillotine cuts, and the unloading restriction was taken into account in its entirety. In this work, the SPU problem was investigated for the first time practically. The first proposed heuristic is based on a strategy of bin packing. While the second algorithm is based on the well-known algorithm First Fit Decreasing Height (FFDH). The well-known first fit decreasing height (FFDH) algorithm is the basis of another approximation algorithm. Overall, these all-proposed heuristics performed well, and excellent results were obtained.

4.3 Shelf heuristics:

Da Silveira et al. [21] published an approximation algorithm for Strip Packing with Unloading constraints. (SPU). Constraints such as rotation of items, which was permitted, guillotine cutting, which was optional, and unloading, which was present). da Silveira et al. considered a special case of the two-dimensional strip packing problem in which the removal of items from the strip is restricted. The version (designated by SPUvh) allows the removal of an item with a single horizontal and vertical motion. They created an algorithm whose asymptotic performance limit for SPUvh is 3. They devised a bin packing-based algorithm with an asymptotic approximation ratio of 5.745 for the case in which only vertical movements are permitted, and unloading constraints are present. In addition, they developed approximate algorithms for restricted cases. They only presented two algorithms for parametric cases and some theorems with their proofs. No benchmark problems were used, they only presented two algorithms for parametric cases, and some theorem and their proof were reached.

4.2 Metaheuristics and exact methods:

Burke et al. [11] published a method for optimizing 2DSP. The contribution of the paper was the presentation of a significantly more straightforward iterative packing procedure based on squeaky wheel optimization. The squeaky wheel optimization packing methodology (SWP) produced better results than a previously published simulated annealing methodology (BF+SA). Castro & Grossmann [13] published a paper about finding an exact solution for strip packing from time representation in scheduling. Two mixed-integer linear programming-based strategies for the 2D orthogonal strip packing problem have been proposed. Several literature-based problems were utilized to evaluate the performance of the models. Using a branch-and-bound technique, Arahori et al. [12] designed an algorithm that resolves subproblems represented by placements of g-staircases to 2SP with fixed height. By extending the problem of dividing an integer set into two subsets with the same total sum to 2SP, they determined a new lower bound for the ideal value (Partition). In addition, they developed a new algorithm for the one-dimensional contiguous bin packing problem with fixed height (1CBPFH). In order to reduce the search space, they added a few novel concepts to the branch-and-bound metaheuristics. For example, canonical forms of practical solutions to 2SP and 1CBPFH were identified to reduce the search space. In addition, they discovered that the optimal values of instances “gcut02” and other algorithms or authors have not achieved “cgcut02” (without rotations). Mesyagutov et al. focused on the 2D orthogonal feasibility problem (OPP-2) as well as the 2D strip packing problem (SPP-2). They examined and enhanced constraint programming (CP) techniques for orthogonal packing problems and incorporated LP-based pruning rules of various types into the constraint propagation or generation process of CP. As previously mentioned in the level heuristics subsection, De Queiroz & Miyazawa [14] presented both heuristics and integer models. To validate these models, many computational experiments were done. Their introduced models are consistent to express these practical situations. However, they only apply when the number of grid or bin positions is limited. In order to solve two-stage and three-stage two-dimensional strip packing problems in LCD mother glass manufacture, various formulas were proposed by Yang et al. [22]. The numerical results demonstrated that other linear models cannot solve the problem for large item types due to the increased size of the integer space from the original model, whereas the two-step formulation solved this problem within a short computation time. For the Strip Packing Problem with Unloading Constraints. da Silveira et al. [21] introduced two additional approximation techniques and a new GRASP heuristic. GRASP was developed based on a well-known GRASP heuristic for the Strip packing problem. They made various changes to it to account for the unique characteristics of the SPU problem. Their GRASP heuristics produced better results could be a valid alternative. Thomas & Chaudhari [25] presented a hybrid strategy that incorporated a proposed genetic encoding and evolution scheme with a placement strategy. This combination produced enhanced population evolution and accelerated convergence to the optimal solution. A comprehensive test of the strategy was conducted using benchmark instances, which was the first GA-based article providing the optimal solution for such a huge dataset. Babaoğlu [28] implemented fruit fly algorithm on 2DSPP. To determine the order of the rectangles in the dataset, FOA is employed. Then, the rectangles are packed using the BLF technique in accordance with the FOA-derived sequence. Two enhanced strip packing metaheuristics are suggested by Rakotonirainy & van Vuuren [34]. While the second technique applies the method of simulated annealing directly in the space of fully defined packing layouts without an encoding of solutions, the first algorithm uses a hybrid approach in which the method of simulated annealing is combined with a heuristic construction procedure. Oviedo-Salas et al. [36] developed a greedy randomized adaptive search method (GRASP) with flags as a novel approach to the 2DSP problem. After accommodating an object, these flags show the remaining area; they also save the available width and height for the items that will come after. They also propose three waste functions as substitute goal functions for the GRASP candidate list, as well as they used enhanced selection for the restricted candidate list, which limits the object alternatives to better elements. To verify that the object fits in the flag, they employed overlapping functions because there were some instances where a flag's width can be incorrect due to the insertion of new objects.

5. Conclusions :

This paper's primary objective is to provide a concise overview of the effective heuristics used to solve the 2D offline rectangular strip packing problem over the past twelve years. Based on the proposed search methodology, 31 papers are found and analyzed.

The research findings based on the statistical analysis of the results can be summarized in the following four points. (1) there are three constraints that fewer researchers considered them such as : load balancing (appears only in 4 papers) , multidrop and load bearing constraint (were only found in 1 paper) and unloading constraints. Therefore, it is suggested that researchers modify previously proposed methods to accommodate these constraints. Thus, it is possible to achieve the integration of the vehicle routing problem and the packing problem, which guarantees the practicability of the loading and unloading process. According to Oliveira et al., the most common heuristics employed by researchers are the distribution of heuristics, which consist of constructive fitness-based and positioning-based heuristics, with a small contribution from profile-based constructive heuristics. Recent researchers have utilized the improvement heuristics technique based on initial solutions such as (GA,GRASP). Based on the old classification of heuristics, 92% of all approximated heuristics tracked articles were plane heuristics, while shelf and level heuristics had the lowest proportion. In conclusion, we can say that more than sixty percent of researchers recommend relying on known approximation solution methods. (3) Finding more precise solutions for the most recent updates of the 2DSPPs, which represent less than 15% of the collected papers, is still necessary to keep up with changes and evaluate the quality of newly proposed solution methods. The majority of researchers do not employ Van Vuuren's typology. Consequently, it is necessary to modify or enhance this typology to distinguish all proposed constraints without confusion.

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