Mathematical Based Performance Comparison of TUS and Triple Connected Dominator Coloring Sets for MANET

R. Jothiraj[1] and B. Chandralekha[2]

[1]*Department of Mathematics, Tagore Engineering College, Rathinamangalam 600127, Chennai, Tamilnadu, India.*
[2]*Department of Physics, Tagore Engineering College, Rathinamangalam 600127, Chennai, Tamilnadu, India.*

Email:jothirajlvp@gmail.com

Abstract

A mobile ad hoc network is a self-created, self-organized and self-administering set of nodes connected via wireless links without the aid of any fixed infrastructure or administrator. The concept of triple umpiring system (TUS) was introduced in [1] by considering the existence of path containing any three vertices (node) of a graph G using triple connected dominator coloring sets. In our system each node's behavior from source to destination is closely monitored by a set of three umpires. If any misbehavior is noticed umpires flag off the guilty node from the circuit. We have proposed triple connected dominator chromatic number to exemplify the basic TUS for salvaging, the circuit during disruptions in route reply and data forwarding phases. Let $G(V,E)$ be a graph. A triple connected dominator coloring of a graph G is a proper three coloring of G in which every vertex dominates every vertex of atleast one color class. The minimum number of colors required for a triple connected dominator coloring set of G is called the triple connected dominator chromatic number of G and is denoted by $\chi_{tcc}(G)$. In this paper, we obtain bounds for general graphs and characterize the corresponding extremal graphs.

Keywords**:** TUS, domination number, triple connected graph, Backtracking,

1. Introduction

Graph theory is one of the most developing branches of mathematics with wide applications to computer science. Graph Theory is applied in diverse areas such as social sciences, linguistics, physical sciences, communication engineering and others. Wireless Ad Hoc Networks (WAN) are becoming increasingly attractive for a variety of application areas, including security, hurricanes and a broad range of military scenarios. Detection mechanism is based on promiscuous hearing. Promiscuous hearing means listening to communications that is not meant for oneself. This is made possible by the wireless nature of the medium. Thus when node B sends packets to C, A along with B's neighbors observes the event. The behavioral deviation, if any, on the part of B, in the retransmission of the messages it received from A, can be readily observed by all of the listening nodes. In the data forwarding B's behavior will be supervised by A, while during route reply process node C will be the supervisor. In TUS, three umpires decide the fate of B collectively. Two of the umpires are neighbors specifically commissioned for the role; the third umpire will be immediate predecessor node A, in its umpiring role. In our system each node's behavior from source to destination is closely monitored by a set of three umpires. If any misbehavior is noticed umpires flag off the guilty node from the circuit [2-3]. We have proposed strong connected domination set to exemplify the basic TUS for salvaging, the circuit during disruptions in route reply and data forwarding phases.

The concept of triple connected graphs and triple connected dominator coloring was introduced [4-6]. Considering the path of the three vertices of G. They have read three attributes of attached drawings, which have made many decisions. In this paper, we use this idea to develop the concept of TUS with strong triple connected dominating set and strong triple connected domination number of a graph.

The rest of the paper is organized as follows: Section 2 discusses Network Model and Mathematical Assumptions. Section 3 discusses Backtracking solution using Triple connected dominator coloring problem. Section 4 provides details about Triple connected dominator coloring set. Section 5 Regular and 3- Regular Graphs using Triple connected dominator coloring set. Section 6 discusses the relating with other graph theoretical parameters and Section 7 gives the conclusions.

2. Network Model and Mathematical Assumptions

In this section, we formulate the wireless mobile ad hoc network and strong triple connected assumptions. A Mathematical Based Performance Comparison of TUS and Strong Triple Connected Domination Sets for MANET, refer to [1] and as shown in the Fig. 1.

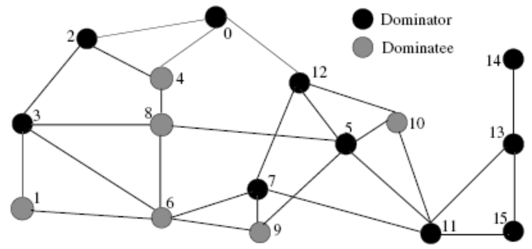

Fig. 1. Relation between dominator and dominate of network

3. Backtracking solution using Triple connected dominator coloring problem

Problems with coloring the vertices of a general graph are subject to the condition that no two adjacent vertices have the same color. This problem is called the graph coloring problem [7]. For the graph-coloring problem, we are interested in assigning colors to the vertices of an undirected graph, with the constraint that no two adjacent vertices are assigned the same color. The optimized version colors the map using minimal colors. A version of the decision, called k coloring, asks whether a graph is colorable using most k colors. The 3-color problem is a special case of the k-color problem $k = 3$. The 3-coloring problem is known to be $NP-$complete, meaning that there is no known polynomial time algorithm to solve it. Using backtracking does not provide an efficient algorithm either, but this is a good example to illustrate the technique.

Observation 3.1 The backtracking-based algorithm (Listing 1) for generating the search tree can be encoded as a recursive method Graph Color(), or the equivalent iterative method, Graph ColorIT(), with the rest of the program code (Listing 2).Note that both of these methods do not end when the first solution is found; rather, the search continues to find all possible solutions. We should note that generating all possible solutions allows for solving the optimization version of a problem. Every triple connected dominator coloring set is a TUS. During data forwarding phase if node N_i misbehaves i.e. it drops packets, it is observed by nodes N_{i-1} , U_{i-1}, and U_{i+1} and they send M-Flag messages, and convict the guilty node; further they unicast messages among themselves to establish an alternative path via N_{i-1} , U_i, U_{i+1} and N_{i+1}. In this segment Self_USS will operational as shown in the Fig. 2. On the other hand if node N_i goes out of communication link but the umpiring nodes do not receive the hello messages from N_i and simply switch over to alternative path, thus booking of innocent node is avoided. Thirdly umpire U_i may go out of communication link with U_{i-1}, N_{i-1}, N_i, and U_{i+1} .In such a case N_{i-1} monitors N_i and N_i monitors N_{i+1} under Self_USS.

Observation 3.2. Every triple connected dominator coloring set is a TUS. If node N_i misbehaves, and say announces a wrong sequence number. This is immediately observed by the node N_{i+1} which sets the status bit of N_i to red and starts RREP salvaging operation as shown in the Fig. 2. On the other hand if N_i simply loses communication link with N_{i+1}, N_i is not booked; only route reply salvaging is undertaken.

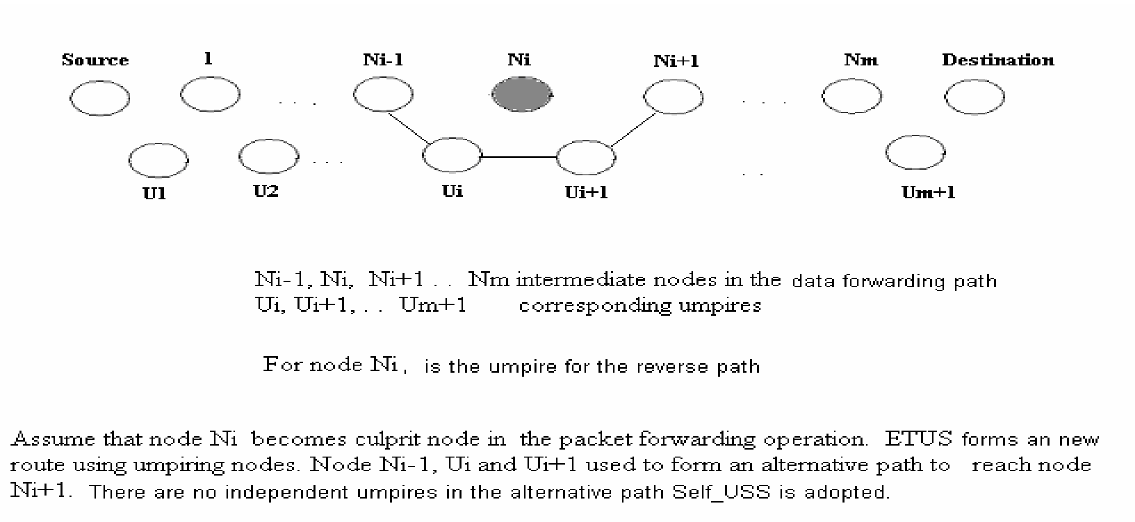

Fig. 2: Self_USS.

Observation 3.3 Every triple connected dominator coloring set is a TUS and every triple connected dominating set is a dominating set. During data forwarding phase if node N_i misbehaves i.e. it drops packets, it is observed by nodes N_{i-1} , U_{i-1}, and U_{i+1} and they send M-Flag messages, and convict the guilty node; further they unicast messages among themselves to establish an alternative path via N_{i-1} , U_i, U_{i+1} and N_{i+1}. In this segment Self_USS will operational as shown in the Fig. 2. On the other hand if node N_i goes out of communication link but the umpiring nodes do not receive the hello messages from N_i and simply switch over to alternative path, thus booking of innocent node is avoided. Thirdly umpire U_i may go out of communication link with U_{i-1}, N_{i-1}, N_i, and U_{i+1} .In such a case N_{i-1} monitors N_i and N_i monitors N_{i+1} under Self_USS.

The ad-hoc networking environment have been proposed, but little performance information on each protocol and node tailed performance comparison between the protocols has previously been available.

4. Triple connected dominator coloring set

Definition 4.1

A triple connected dominator coloring of a graph G is a proper three coloring of G in which every vertex dominates every vertex of at least one color class. The minimum number of colors required for a triple connected dominator coloring set of G is called the triple connected dominator chromatic number of G and is denoted by $\chi_{tcc}(G)$.

Example 4.1 For the 3-regular graph in Fig. 3, the triple connected dominator chromatic number of $\chi_{tcc}(G) = 3$.

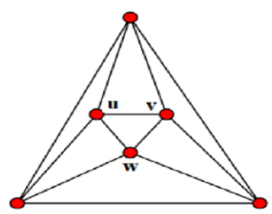

Fig. 3. Graph with $\chi_{tcc}(G) = 3$

Observation 4.1. A triple connected dominator coloring set (tcc- set) does not exists for all graphs and if exists, then $\chi_{tcc}(G) \geq 3$.

Definition 4.2
A subset S of V is said to be a complementary connected dominating set (ccd-set) if S is a dominating set and $\langle V-S \rangle$ is connected. The chromatic number $\gamma_{ccd}(G)$ is the minimum number of colours required to colour all the vertices such that no two adjacent vertices receive the same color.

5. Regular and 3- Regular Graphs using Triple connected dominator coloring set

Let G be a connected 2 -regular graph. Then clearly G is a cycle. Also, for even cycles $\chi_{tcc}(G)=3,$ for odd cycles, $\chi_{tcc}(G)=3.$ So for cycles $\gamma_{ccd}(G)=\chi_{tcc}(G),$ then $\gamma_{ccd}(G)=3.$ But we have $\gamma_{ccd}(C_n)=n-2,$ for $n \geq 3.$ If $\gamma_{ccd}(C_n)=2, n=4,$ and so $G \equiv C_4$ and $\gamma_{ccd}(C_n)=3,$ then G C5. Thus, G is C_4 or C_5. Hence, we can conclude that C_4 and C_5 are the only connected 2-regular graphs for which $\gamma_{ccd}(G)=\chi_{tcc}(G).$ Next, we characterize $\gamma_{ccd}(G)=\chi_{tcc}(G)=3$-regular graphs.

Theorem 5.1 If $\gamma_{ccd}(G)=\chi_{tcc}(G)=3$ with 3 - regular graph on 6 vertices is not a regular graph.

Proof Let G be a 3 -regular graph on 6 vertices with $\gamma_{ccd}(G)=\chi_{tcc}(G)=3.$ Then the perimeter of G is 3 or 4. If it is 3, then $G \cong C_3$. Otherwise, $G \cong K_{3,3}$. But $\gamma_{ccd}(C_3)=\chi_{tcc}(K_{3,3})=2,$ which is a contradiction.

Theorem 5.2 Let G be a 3 -regular graph on 8 vertices. Then $\gamma_{ccd}(G)=\chi_{tcc}(G)=3$ iff G is G_1 or G_2.

Proof Let G be a 3 -regular graph on 8 vertices with $\gamma_{ccd}(G)=\chi_{tcc}(G)=3.$ Then G has 12 edges. Let $v_1, v_2, ..., v_8$ be the vertices of G and let $S=\{v_6, v_7, v_8\}$ be a $\gamma_{ccd}(G)$-set. Take $V_s = V-S$. Since S is a $\gamma_{ccd}(G)$-set, $\langle V_s \rangle$ is connected. Also $\Delta(V_s)=2.$ and hence $\langle V_s \rangle$ is C_5.

Case (i) If $\langle V_s \rangle$ is isomorphic to C_5, then $E(V,S)\ S=5.$ This implies that $\langle S \rangle$ has 2 edges and 3 vertices. Hence, $\langle S \rangle = P_3 = v_6 v_7 v_8$. Without loss of generality, v_7 can be assumed to be close to v_1. If $N(v_6)=\{v_4, v_5, v_7\},$ then $N(v_8)=\{v_2, v_3, v_7\},$ and the resulting graph $\gamma_{ccd}(G)=2$ with $S=\{v_5, v_6\},$ a contradiction. When the neighbors of v_6 are $\{v_3, v_5, v_7\}$ (or $\{v_2, v_5, v_7\},$ we get a graph G_2 (or G_1).

Case (ii) $\langle V_s \rangle P_5$. In this case $E(V,S)\ S=7.$ Hence, $\langle S \rangle$ has only one edge. That is, $\langle S \rangle = P_2$. Without loss of generality, we can assume that v_8 is an isolated vertex in $\langle S \rangle$. It suffices to deal with the case where $N(v_6)$ and $N(v_8)$ have a common neighbor or the case where $N(v_7)$ and $N(v_8)$ have a common neighbor. We obtain $\gamma_{ccd}(G)=2,$ which is impossible. Let the neighbors of v_8 be $\{v_1, v_2, v_5\}$. When $N(v_7)=\{v_1, v_3, v_6\},$ the given graph $S=\{v_2, v_6\}$ with $\gamma_{ccd}(G)=2,$ is infeasible. The graph G_1 is obtained when the neighbors of v_7 are $\{v_1, v_4, v_6\}$. On the other hand, let $\{v_1, v_3, v_5\}$ be neighbors of v_8. If $N(v_7)=\{v_1, v_2, v_6\},$ then $N(v_6)=\{v_4, v_5, v_7\}.$ Here $\{v_1, v_4\}$s the ccd-set, a contradiction. If $N(v_7)=\{v_1, v_4, v_6\},$ then the resulting graph is G_2.

Conversely, for $i=1, 2$, $rad(G_i)=diam(G_i)=2$, any two nonadjacent vertices have a common neighbor, so that $\gamma_{ccd}(G)\neq 2$. And it is easy to see that $\gamma_{ccd}(G_1)=\gamma_{ccd}(G_2)=3$. Additionally, $\chi_{tcc}(G_1)=\chi_{tcc}(G_2)=3$.

Exact value for some special graphs:

1) The Franklin graph is a 3-regular graph with 12 vertices and 18 edges given in Fig. 4.

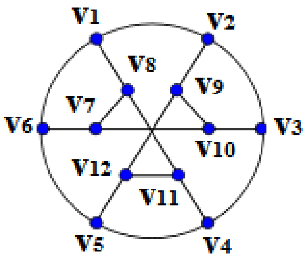

Fig. 4

For any Franklin graph G, triple connected dominator chromatic number $\chi_{tcc}(G)=3$, and also $\gamma_{tc}(G)=4$. Here $S=\{v_1, v_4, v_6, v_7\}$ is a minimum dominating set.

2) The Bull graph is a simple connected graph with 5 vertices, 5 edges, as shown in Fig. 5.

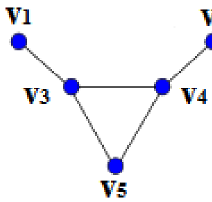

Fig. 5

For any Bull graph G, triple connected dominator chromatic number $\chi_{tcc}(G)=3$, and also $\gamma\ (G)=2$. Here $S=\{v_3, v_4,\}$ is a minimum dominating set.

3) The Goldner–Harary graph is a simple connected graph with 11 vertices, 27 edges, as shown in Fig. 6.

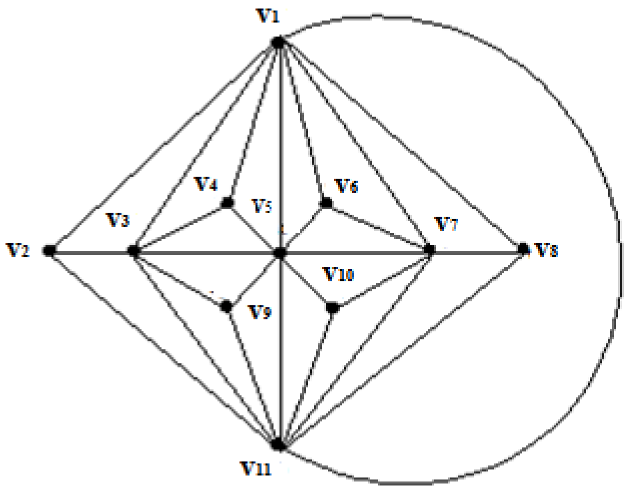

Fig. 6

For any Goldner- Harary graph G, triple connected chromatic number $\chi_{tcc}(G) = 3$, and also $\gamma\ (G) = 2$. Here $S = \{v_1, v_{11}\}$ is a minimum dominating set.

4) The Bidiakis cube is a 3-regular graph with 12 vertices and 18 edges given in Fig. 7.

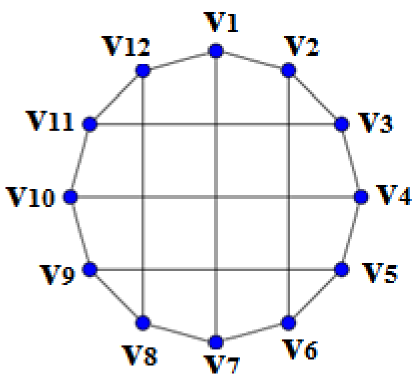

Fig. 7

For any Bidiakis cube graph G, triple connected dominator chromatic number $\chi_{tcc}(G) = 3$, and also $\gamma_{tc}(G) = 4$, Here $S = \{v_1, v_4, v_7, v_{10}\}$ is a minimum dominating set.

Theorem 5.3 For any connected graph G with $p \geq 3$, we have $\left\lceil \frac{p}{\Delta + 1} \right\rceil \leq \gamma_{tc}(G) \leq p - 1$ and the bounds are sharp.

6. Relation With Other Graph Theoretical Parameters

Theorem 6.1 For any connected graph G with $p \geq 3$ vertices, the bound is sharp if and only if $\gamma_{tc}(G) + k(G) \leq 2p - 2$ and $G \cong K_3$.

Proof. Let G be a connected graph with $p \geq 3$ vertices. $k(G) \leq p - 1$ and by Theorem 5.3, $\gamma_{tc}(G) \leq p - 1$. Hence $\gamma_{tc}(G) + k(G) \leq 2p - 2$. Suppose G is isomorphic to K_3. Then clearly $\gamma_{tc}(G) + k(G) \leq 2p - 2$. Conversely, $\gamma_{tc}(G) + k(G) \leq 2p - 2$. This is only possible if

$\gamma_{tc}(G) = p-1$ and $k(G) = p-1$. But $k(G) = p-1$, and so $G \cong K_p$ for which $\gamma_{tc}(G) = 3 = p-1$ so that $p = 3$. Hence $G \cong K_3$.

Theorem 76.2 For a connected G with $p \geq 3$ vertices, the connection is sharp iff $\gamma_{tc}(G) + \chi_{tcc}(G) \leq 2p-1$ and $G \cong K_3$.

Proof. Let G be a connected graph with $p \geq 3$ vertices. $\chi_{tcc}(G) \leq p$ and by Theorem 5.3, $\gamma_{tc}(G) \leq p-1$. Hence $\gamma_{tc}(G) + \chi_{tcc}(G) \leq 2p-1$. Suppose G is isomorphic to K_3. Then clearly $\gamma_{tc}(G) + \chi_{tcc}(G) = 2p-1$. Conversely, $\gamma_{tc}(G) + \chi_{tcc}(G) = 2p-1$. This is only possible if $\gamma_{tc}(G) = p-1$ and $\chi_{tcc}(G) = p$. Since $\chi_{tcc}(G) = p$, G is isomorphic to K_p for which $\gamma_{tc}(G) = 3 = p-1$ so that $p = 3$. Hence $G \cong K_3$.

7. Conclusion

The triple connected dominator color set problem on circular graphs with only bidirectional connections (CGB). The graphs can be used to model wireless ad hoc networks where nodes have different transmission ranges. The main approach in our TUS is to create a maximally independent set and then merge them. Through mathematical analysis, we have shown that using extreme graphs with the minimum number of vertices to interconnect the maximally independent set can help reduce the size of the triple connected dominator chromatic number. Graph color and dominance play an important role in many real-world applications and is still the subject of exciting research.

References

1. R. Jothiraj and A. Kathirvel, "A Mathematical Based Performance Comparison of TUS and Strong Triple Connected Domination Sets for MANET", International Journal on Recent Researches In Science, Engineering & Technology, vol.4,Issue 2, Feb. 2016.
2. A.Kathirvel, and Dr. R. Srinivasan, "ETUS: An Enhanced Triple Umpiring System for Security and Performance Improvement of Mobile Ad Hoc Networks", International Journal of Network Management, John Wiley & Sons, Vol. 21, No. 5, September/October 2011, pp. 341 - 359.
3. A.Kathirvel, and Dr. R. Srinivasan, "ETUS: An Enhanced Triple Umpiring System for Security and Robustness of Mobile Ad Hoc Networks", International Journal of Communication Networks and Distributed Systems, Inderscience, Vol. 7, Nos.1 / 2, 2011, pp. 153 – 187.
4. Paulraj Joseph J., Angel Jebitha M.K., Chithra Devi P. and Sudhana G,Triple connected graphs, Indian Journal of Mathematics and Mathematical Sciences,Vol. 8, No.1, (2012), pp. 61-75.
5. Jothiraj R, Jayakumar S and Venugopal P ,"Triple Connected Dominator Coloring to Cyclic Open Shop Scheduling of Unit Circular- Arc Graph", International Journal of Research and Analytical Reviews,Vol. 6,Issue 1, (2019),pp. 579-583.
6. Jothiraj R, Jayakumar S and Venugopal P," Cyclic Open Shop Scheduling in a Paired Triple Connected Dominator Coloring of Circular- Arc Graph" Journal of Applied Science and Computations,Vol.7, Issue 1,(2018), pp. 675-680.
7. Bondy, 1. A., Bounds for the chromatic number of a graph, Journal of Combinatorial Theory, 7 (1969), pp. 96-8.