

## EPQ MODEL CONSIDERING CP TECHNIQUE IN AN UNCERTAIN ENVIRONMENT



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### Abstract

The EPQ model can be used to maximize inventory management processes and minimize production impact on the environment when adopting cleaner production techniques. In this research an unreliable production process involving rework can be significantly impacted by the machine acclimatization period in an uncertain environment. The machine might need to be modified and maintained for the duration of the acclimatization period in order to function at its best. During the initial phases of production, this may lead to an increase in downtime and a decrease in production efficiency. The machine may not be working at its best during the machine acclimatization period, which can increase the likelihood of flaws and mistakes during the early stages of production. This acclimatization period decreases waste while enhancing machine performance, which leads to a decrease in damaged objects and transforming the process to be more eco-friendly. The aim of this research is to determine the optimal production quantity for minimizing overall costs. We transformed the model's imprecise cost parameters into Triangular Intuitionistic Fuzzy numbers as a way to address their uncertainty. The effectiveness of the model has been verified through numerical solutions.

**Keywords:** Economic Production Quantity, Triangular Intuitionistic Fuzzy numbers, Acclimatization period, Cleaner Production Technique.

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## 1. Introduction

Cleaner production process is an approach to manufacturing that strives to minimize the negative impacts of industrial processes on the environment, natural resources, and human health. Cleaner production has numerous advantages for businesses, such as lowering costs associated with resource usage, waste disposal, and regulatory compliance. Additionally, this approach supports sustainable development and protects the environment and public health by limiting the release of pollutants into the air, water, and soil. (Taft, 1918) [9] was the first to discuss the importance of considering the production rate of a machine when determining the Economic Production Quantity (EPQ). The Economic Production Quantity (EPQ) model is a significant tool for implementing cleaner production process as it assists businesses in optimizing their inventory management practices. It achieves this by reducing the inventory held in stock, which, in turn, reduces the consumption of raw materials and energy in production and the amount of waste generated. By using this model, businesses can achieve a balance between minimizing inventory costs and maintaining adequate stock levels to satisfy customer demand. (Hayek & Salameh, 2001) [4] conducted a study to investigate how expected standard impacts the finite capacity. (Chiu, 2003) [2] developed a model for lot-size production that accounts for items that can be scrapped or reworked. (Cárdenas-Barrón et al., 2015) [1] optimized a model for discrete lot-sizing that included factors such as delivery shipments, immediate rework, and random defective rates. The purpose of their study was to identify the most efficient values for the production quantity and the number of shipments in each cycle. (Liao, 2013) [6] proposed an optimal model for lot-sizing in a parallel process that involves defects, rework, minimal repair, preventive maintenance, and free-repair warranty. (Taleizadeh et al., 2016) [10] introduced a combined pricing and ordering policy for a manufacturing system that includes imperfect production and multiple shipments. The aim of their study was to determine the optimal policy that would maximize profits for the system. (Singh & Yadav, 2016) [8] proposed a new method to solve the fully intuitionistic fuzzy transportation problem. (Khan et al., 2017) [5] studied the impact of screening errors, learning in manufacturing, and random lead-time demands on a vendor-buyer EPQ model that includes defective items. The objective of their research was to examine the role of these factors in the model and how they affect the decision-making process. (Cunha et al., 2018) [3] introduced an EPQ model for a manufacturing process that involves defects, partial back ordering,

and a discount for defective items. The aim of their research was to determine the optimal production and ordering policy that would minimize costs and maximize profits for the system. (Nobil et al., 2019) [7] proposed an economic production quantity model that considers the warm-up period in a cleaner production environment. This research paper discusses a lot sizing problem in the context of a manufacturing process that has lost control in the acclimatization period for the production machine. During the acclimatization period, the machine operates at a lower rate to identify and repair any defects or failures before regular production begins. This results in an increase in the machine's useful life and a reduction in the probability of defective products. The model also takes into account the process of rework to prevent scrap production. An exact algorithm is used to determine the optimal production quantity, which minimizes the total cost of the system.

### Assumption

- (i) The rework process is capable of transforming defective items into those of perfect quality.
- (ii) After the acclimatization period and corrective maintenance, the machine's production rate increases.
- (iii) The rework process requires a certain amount of time for setup.
- (iv) Backorder management is allowed.
- (v) During the acclimatization period, normal manufacturing period, and rework period, the production rate of items that meet the criteria for perfect quality exceeds the demand rate.

### Notations

- $D_r$  – Customer demand.  
 $\varphi_1$  – The rate at which items are produced during the acclimatization period.  
 $\varphi_2$  – The rate of production during a steady-state manufacturing period.  
 $R_d$  – Repair rate for faulty items.  
 $\zeta_1$  – The defective product rate during the acclimatization period.  
 $\omega(\zeta_1)$  – Probability distribution of  $\zeta_1$   
 $\zeta_2$  – The defective product rate during steady-state production.  
 $\omega(\zeta_2)$  – Probability distribution of  $\zeta_2$   
 $\sigma_s$  – Before the acclimatization period, there is a set-up phase for producing items.  
 $\sigma_r$  – The time required to prepare for fixing or correcting faulty items.  
 $\sigma_m$  – After the acclimatization period, the period for corrective maintenance begins.  
 $\sigma_n$  – Acclimatization period  
 $\theta_p$  – The cost associated with starting or establishing a production process for creating goods.

$\theta_r$  – The cost incurred in preparing a production process to correct or fix defective items.  
 $\gamma_p$  – The cost of producing one unit of an item.  
 $\gamma_r$  – The cost incurred in correcting or fixing one unit of a defective item.  
 $\gamma_m$  – The cost associated with repairing or fixing a piece of equipment or machinery after it has malfunctioned or broken down.  
 $\alpha_1$  – The number of items manufactured during an acclimatization period for each production cycle.  
 $\alpha_2$  – The number of items manufactured during a regular production period for each production cycle.  
 $\Gamma$  – The total quantity of items produced within each production cycle.

$I_c$  – The overall cost of Inventory.  
 $A_c$  – Cost of acquisition per unit  
 $B_r$  – The price per unit of backordered goods over a period of twelve months.  
 $F_b$  – The proportion of items that are on backorder.

### Computational Models

The aggregate number of items that were fabricated during the acclimatization period before full-scale production is  $\alpha_1 = \varphi_1 \sigma_n$   
 The quantity of items manufactured during a typical production period is  $\alpha_2$   
 Each manufacturing cycle produces a combined total of both defective and non-defective items is given here

$$\Gamma = \alpha_1 + \alpha_2 = \varphi_1 \sigma_n + \alpha_2 \quad \dots(1)$$

Otherwise,  $\alpha_2 = \Gamma - \varphi_1 \sigma_n \quad \dots(2)$

To calculate the total number of defective items during each cycle, the following method is used:

$$S = \zeta_1 \alpha_1 + \zeta_2 \alpha_2 \quad \dots(3)$$

In Equation (3), if we replace the variable  $\alpha_2$  with the expression for  $\alpha_2$  that is given in Equation (2), we get a new equation.

$$S = \zeta_1 \varphi_1 \sigma_n + \zeta_2 (\Gamma - \varphi_1 \sigma_n) = \varphi_1 \sigma_n (\zeta_1 - \zeta_2) + \zeta_2 \Gamma \quad \dots(4)$$

The value of S is positive because  $\zeta_1 > \zeta_2$ .

Determine the highest amount of inventory, denoted by  $J_1, J_m, J_2, J_3$ , and  $J_r$  that can be held at the end of each of the following periods: acclimatization time, maintenance unit, average manufacturing cycle, setup time for rework, and rework cycle.

$$J_1 = ((1 - \zeta_1) \varphi_1 - D_r) \sigma_n = l_1 \sigma_n \quad \dots(5)$$

To simplify the expression, we can replace the term " $(1 - \zeta_1) \varphi_1 - D_r$ " with " $l_1$ ".

$$J_m = J_1 - D_r \sigma_n = l_1 \sigma_n - D_r \sigma_m \quad \dots(6)$$

$$J_2 = J_m + ((1 - \zeta_1) \varphi_1 - D_r) \sigma_p = l_1 \sigma_n - D_r \sigma_m + l_2 \frac{\alpha_2}{\varphi_2} = l_1 \sigma_n - D_r \sigma_m + l_2 \frac{\Gamma}{\varphi_2} - l_2 \frac{\varphi_1 \sigma_n}{\varphi_2} \quad \dots(7)$$

We can replace the term " $(1 - \zeta_2) \varphi_2 - D_r$ " with " $l_2$ ".

$$J_r = J_2 - D_r \sigma_r - D_r \sigma_m + l_2 \frac{\Gamma}{\varphi_2} - l_2 \frac{\varphi_1 \sigma_n}{\varphi_2} - D_r \sigma_r \quad \dots(8)$$

$$J_3 = J_r + (R_d - D_r) \sigma_r = J_r + \frac{l_3 S}{R_d} = l_1 \sigma_n - D_r \sigma_m + l_2 \frac{\Gamma}{\varphi_2} - l_2 \frac{\varphi_1 \sigma_n}{\varphi_2} - D_r \sigma_r + \frac{l_3 \varphi_1 \sigma_n (\zeta_1 - \zeta_2)}{R_d} + \frac{l_3 \zeta_2 \Gamma}{R_d} \quad \dots(9)$$

The cost associated with starting or establishing a production process for creating goods is  $\theta_p$  in each phase, the total  $\theta_p$  cost would be  $\theta_p = \frac{\theta_p}{v} = \frac{D_r \theta_p}{\Gamma}$ . As well as the cost incurred in preparing a production process to correct or fix defective items is  $\theta_r$  in each phase, the total  $\theta_r$  cost would be  $\theta_r = \frac{\theta_r}{v} = \frac{D_r \theta_r}{\Gamma}$ . The annual expenditure of production would be  $H \gamma_p \Gamma = D_r \Gamma$ . After the acclimatization period, the period for corrective maintenance  $\sigma_m$  for each phase. The total  $\theta_m$  cost would be  $H \sigma_m = \frac{\theta_m}{v} = \frac{D_r \theta_m}{\Gamma}$ . The cost of acquisition  $A_c$  for each phase. The total  $A_c$  would be  $D_r A_c$ . The backordering cost  $F_b B_r$  for each phase. The total  $F_b B_r$  cost would be  $D_r F_b B_r$ . The cost incurred in correcting or the number of defective items is S, and the time required to fix one unit of a defective item is  $\gamma_r$ . Therefore the total cost of restoring 'C' the following can be calculated

$$C = H \gamma_p S = \frac{D_r \gamma_r S}{\Gamma} = \frac{D_r \gamma_r \varphi_1 \sigma_n (\zeta_1 - \zeta_2)}{\Gamma} + D_r \gamma_r \zeta_1 \quad \dots(10)$$

The total sustaining charge (N) can be calculated as,

$$N = \frac{D_r \gamma}{\Gamma} \left\{ \frac{l_1 \times \sigma_n}{2} + \frac{(l_1 + J_m) \sigma_m}{2} + \frac{(J_m + J_2) \sigma_p}{2} + \frac{(J_2 + J_r) \sigma_1}{2} + \frac{(J_r + J_3) \sigma_R}{2} + \frac{J_3 \times \sigma_d}{2} \right\} \quad \dots(11)$$

The total sustaining charge can be expressed in an alternative way, according to the calculations presented in (Nobil et al.2018) [7].

N =

$$\frac{D_r \gamma \varphi_1 \sigma_n \sigma_m (1 - \zeta_1)}{\Gamma} + \frac{\gamma (R_d - D_r) \varphi_1^2 \sigma_n^2 (\zeta_1 - \zeta_2)^2 (2R_d - D_r)}{2R_d^2 \Gamma} + \frac{\gamma ((1 - \zeta_2) \varphi_2 - D_r) (1 - \zeta_1) \varphi_1^2 \sigma_n^2}{\varphi_2 \Gamma} - \frac{\gamma ((1 - \zeta_1) \varphi_1 - D_r) (1 + \zeta_1 - 2\zeta_2) \varphi_1^2 \sigma_n^2}{2\Gamma} - \frac{D_r \gamma \varphi_1 \sigma_n \sigma_r (\zeta_1 - \zeta_2)}{\Gamma} + \frac{\gamma ((1 - \zeta_2) \varphi_2 - D_r) (1 + \zeta_2)}{2\varphi_2} \Gamma + \frac{\gamma (R_d - D_r) \zeta_2^2}{2R_d} \Gamma +$$

$$\frac{y((1-\zeta_1)\varphi_1-D_r)(1-\zeta_2)\sigma_n}{2} + \frac{y((1-\zeta_1)\varphi_1-D_r)(1-\zeta_2)\varphi_2\sigma_n}{R_d} + y((1-\zeta_1)\varphi_1-D_r)(\zeta_1-\zeta_2)\varphi_1\sigma_n^2 + \frac{y((1-\zeta_2)\varphi_2-D_r)(\zeta_1-2\zeta_2)\varphi_1\sigma_n}{\varphi_2} + \frac{y(R_d-D_r)\zeta_2\varphi_1\sigma_n(\zeta_1-\zeta_2)(R_d-D_r)}{2R_d^2} - \frac{y((1-\zeta_2)\varphi_2-D_r)(1-\zeta_2)\varphi_1\sigma_n}{\varphi_2} - D_r y(\sigma_m + \zeta_2\sigma_r) \dots (12)$$

Thus, the total inventory cost that takes into account the total cost associated with starting or establishing a production process for creating goods for production and remanufacturing, the overall cost for the time interval for corrective

maintenance, the annual manufacturing cost, the total cost of cost of acquisition, restoring, backordering and the total sustaining charge can be determined.

$$I_c = \frac{\beta_1}{\Gamma} + \beta_2\Gamma + \beta_3 \dots (13)$$

Then,

$$\beta_1 = D_r(\theta_p + \gamma_m + \theta_r + \gamma_r\varphi_1\sigma_n(\zeta_1 - \zeta_2)) + D_r y\varphi_1\sigma_n\sigma_m(1 - \zeta_1) + \frac{y(R_d-D_r)\varphi_1^2\sigma_n^2(\zeta_1-\zeta_2)^2(2R_d-D_r)}{2R_d^2} + \frac{y((1-\zeta_2)\varphi_2-D_r)(1-\zeta_1)\varphi_1^2\sigma_n^2}{\varphi_2} - \frac{y((1-\zeta_1)\varphi_1-D_r)(1+\zeta_1-2\zeta_2)\varphi_1^2\sigma_n^2}{2} - D_r y\varphi_1\sigma_n\sigma_r(\zeta_1 - \zeta_2) \dots (14)$$

$$\beta_2 = \frac{y((1-\zeta_2)\varphi_2-D_r)(1+\zeta_2)}{2\varphi_2} + \frac{y(R_d-D_r)\zeta_2^2}{2R_d} \dots (15)$$

$$\beta_3 = D_r(\gamma_p + \gamma_r\varphi_2 - A_c - F_b B_r) + \frac{y((1-\zeta_1)\varphi_1-D_r)(1-\zeta_2)\sigma_n}{2} + \frac{y((1-\zeta_1)\varphi_1-D_r)(1-\zeta_2)\varphi_2\sigma_n}{R_d} + y((1-\zeta_1)\varphi_1 - D_r)(\zeta_1 - \zeta_2)\varphi_1\sigma_n^2 + \frac{y((1-\zeta_2)\varphi_2-D_r)(\zeta_1-2\zeta_2)\varphi_1\sigma_n}{\varphi_2} + \frac{y(R_d-D_r)\zeta_2\varphi_1\sigma_n(\zeta_1-\zeta_2)(R_d-D_r)}{2R_d^2} - \frac{y((1-\zeta_2)\varphi_2-D_r)(1-\zeta_2)\varphi_1\sigma_n}{\varphi_2} - D_r y(\sigma_m + \zeta_2\sigma_r) \dots (16)$$

This proposed problem has two limitations that need to be considered.

(i) The time required for setting up the production process of items would be  $\sigma_n \leq \frac{J_3}{D_r}$

If we substitute  $J_3$  using Equation (9), the resulting expression will be:

$$\Gamma_m = \left[ \frac{D_r(\sigma_s + \sigma_m + \sigma_r) - ((1-\zeta_1)\varphi_1 - D_r)\sigma_n + \frac{((1-\zeta_2)\varphi_2 - D_r)\varphi_1\sigma_n}{\varphi_2} - \frac{(R_d - D_r)\varphi_1\sigma_n(\zeta_1 - \zeta_2)}{R_d}}{\left(\frac{((1-\zeta_2)\varphi_2 - D_r)}{\varphi_2}\right) + \left(\frac{(R_d - D_r)\zeta_2}{R_d}\right)} \right] \dots (17)$$

(ii) Setup period for the process of reworking would be  $J_3 \geq 0$

From equation (8), we obtain the following result

$$\Gamma_1 = \left[ \frac{D_r(\varphi_2(\sigma_m + \sigma_r) - ((1-\zeta_1)\varphi_1 - D_r)\sigma_n + \frac{((1-\zeta_2)\varphi_2 - D_r)\varphi_1\sigma_n}{\varphi_2} - \frac{(R_d - D_r)\varphi_1\sigma_n(\zeta_1 - \zeta_2)}{R_d}}{\left(\frac{((1-\zeta_2)\varphi_2 - D_r)}{\varphi_2}\right) + \left(\frac{(R_d - D_r)\zeta_2}{R_d}\right)} \right] \dots (18)$$

The stated lot-sizing model is expressed as

$$\text{Min } I_c = \frac{\beta_1}{\Gamma} + \beta_2\Gamma + \beta_3 \dots (19)$$

Depending on:  $\Gamma \geq \Gamma_m, \Gamma \geq \Gamma_1, \Gamma > 0$

### Algorithm

Illustration I: If  $\beta_1 \geq 0$ , then this model is a convex nonlinear programming problem.

STEP 1: Determine the optimum level of  $\Gamma$

$$\text{using } \Gamma = \sqrt{\frac{\beta_1}{\beta_2}}$$

STEP 2: Using the formula  $\Gamma_{\min} = \max\{\Gamma_m, \Gamma_1\}$ , determine the minimum amount of production required.

STEP 3:  $\Gamma^*$  is assigned the value of  $\Gamma$  if  $\Gamma \geq \Gamma_{\min}$ ; otherwise,  $\Gamma^*$  is assigned the value of  $\Gamma_{\min}$ .

Illustration II: If  $\beta_1 < 0$ , then this model is a concave nonlinear programming problem.

STEP 1: If the expressions  $(1 - \zeta_1)\varphi_1 - D_r$ ,  $(1 - \zeta_2)\varphi_2 - D_r$ , and  $(R_d - D_r)$  are all greater than zero, then the problem has a viable solution

and should proceed to STEP 2. If any of these expressions are not positive, then skip to STEP 9.

STEP 2: To proceed to pace 3, compute the values of  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  using Equation (14–16).

STEP 3: Use Equation (17) to calculate  $\Gamma_m$  and Equation (18) to calculate  $\Gamma_1$ . Next, determine the value of  $\Gamma_{\min}$  by computing  $\max\{\Gamma_m, \Gamma_1\}$ . Once  $\Gamma_{\min}$  has been calculated, proceed to STEP 4.

STEP 4: Proceed to pace 5 if  $\beta_1$  is greater than or equal to zero. If  $\beta_1$  is less than zero, skip to (7)

STEP 5: Calculate the value of  $\Gamma$  by using the formula  $\Gamma = \sqrt{\frac{\beta_1}{\beta_2}}$

STEP 6: If  $\Gamma \geq \Gamma_{\min}$ , then  $\Gamma^* = \Gamma$ . Otherwise,  $\Gamma^* = \Gamma_{\min}$ . And, go to STEP 8.

STEP 7: Determine the optimal value of  $\Gamma$  by computing  $\Gamma^* = \Gamma_{\min} = \max\{\Gamma_m, \Gamma_1\}$ . After finding the optimal value of  $\Gamma$ , proceed to STEP 8.

STEP 8: After determining the optimal value of  $\Gamma^*$  using the formula  $\Gamma^* = \Gamma_{\min} = \max\{\Gamma_m, \Gamma_1\}$ , calculate the optimal value of  $I_c^*$  by applying Equation (21). Once the optimal value of  $I_c^*$  has been found, proceed to STEP 9.

STEP 9: end.

### Proposed Inventory Model That Incorporates Intuitionistic Fuzziness

Fuzzy theory is a mathematical framework for dealing with uncertainty and vagueness. Here are some basic definitions in fuzzy theory [8].

- $\hat{A}$  is an intuitionistic fuzzy set (IFS) defined over the universe of discourse  $X$  by the set of ordered triples,

$$\hat{A} = \{(\alpha, \sigma_{\hat{A}}(\alpha), \tau_{\hat{A}}(\alpha)) : \alpha \in X\} \quad \dots(20)$$

Where  $\sigma_{\hat{A}}(\alpha)$  and  $\tau_{\hat{A}}(\alpha)$  are functions from  $X$  to the interval  $[0, 1]$ . The functions  $\sigma_{\hat{A}}(\alpha)$  and  $\tau_{\hat{A}}(\alpha)$  satisfy the condition that  $0 \leq \sigma_{\hat{A}}(\alpha) + \tau_{\hat{A}}(\alpha) \leq 1$  for all  $\alpha$  in  $X$ . The terms  $\sigma_{\hat{A}}(\alpha)$  and

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$$\sigma_{\hat{A}}(\alpha) = \begin{cases} \frac{\alpha - c_1}{c_2 - c_1} & c_1 < \alpha \leq c_2 \\ \frac{c_3 - \alpha}{c_3 - c_2} & c_2 < \alpha \leq c_3 \\ 0 & \text{Otherwise} \end{cases}$$

$$\tau_{\hat{A}}(\alpha) = \begin{cases} \frac{c_2 - \alpha}{c_2 - c_1} & c_1 < \alpha \leq c_2 \\ \frac{\alpha - c_2}{c_3 - c_2} & c_2 < \alpha \leq c_3 \\ 1 & \text{Otherwise} \end{cases}$$

Where  $c_1' \leq c_1 < c_2 < c_3 \leq c_3'$ .

- The score function of the membership and non-membership functions  $\sigma_{\hat{A}}(\alpha)$  and  $\tau_{\hat{A}}(\alpha)$  is determined as

$$S(\sigma_{\hat{A}}) = \frac{c_1 + 2c_2 + c_3}{4} \text{ and } S(\tau_{\hat{A}}) = \frac{c_1' + 2c_2 + c_3'}{4}. \text{ Here is the accuracy ranking function of } \hat{A} = (c_1, c_2, c_3; c_1', c_2, c_3')$$

$c_3')$  is obtained by the formula  $\mathcal{R}(\hat{A}) = \frac{S(\sigma_{\hat{A}}) + S(\tau_{\hat{A}})}{2}$ .

### Numerical Solutions

Two experimental examples are provided, where the input data is sourced from (AH Nobil 2018) [7]. These experiments are carried out on a computer that features an Intel i5 processor running at 2.30 GHz, along with 4.00 GB of RAM. The programming language used for conducting these experiments is Python 3.10, specifically Anaconda3.

**Example 1:** By utilizing the aforementioned solution algorithm, we were able to successfully solve the given example. The values of  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  were determined to be 294896.89, 0.672, and 16240.2596 respectively. As  $\beta_1$  was found to be positive, this indicates that the first illustration has

$\tau_{\hat{A}}(\alpha)$  in the definition of  $\hat{A}$  are referred to as the degree of membership and degree of non-membership, respectively, for  $\alpha \in X$ . Additionally, the degree of hesitation  $\eta(\alpha)$ , for  $\alpha \in X$  is calculated using the following relation:

$$\eta(\alpha) = 1 - \sigma_{\hat{A}}(\alpha) - \tau_{\hat{A}}(\alpha) \quad \dots(21)$$

- For an intuitionistic fuzzy set (IFS)  $\hat{A} = \{(\alpha, \sigma_{\hat{A}}(\alpha), \tau_{\hat{A}}(\alpha)) : \alpha \in X\}$  defined over the universe of discourse  $X$ , two conditions must be satisfied.

(i) There exists a real number  $r$  such that  $\sigma_{\hat{A}}(r) = 1$  and  $\tau_{\hat{A}}(r) = 0$ .

(ii) The functions  $\sigma_{\hat{A}}(\alpha)$  and  $\tau_{\hat{A}}(\alpha)$  are piecewise continuous mappings from the set of real numbers to the interval  $[0, 1]$ , subject to the condition  $0 \leq \sigma_{\hat{A}}(\alpha) + \tau_{\hat{A}}(\alpha) \leq 1$  for all  $\alpha \in X$ .

- The triangular intuitionistic fuzzy number (TIFN)  $\hat{A} = (c_1, c_2, c_3; c_1', c_2, c_3')$  can be described by its membership and non-membership functions in the following way:

occurred. Consequently, we calculated the value of  $\Gamma$  using  $\Gamma = \sqrt{\frac{\beta_1}{\beta_2}}$ , which resulted in  $\Gamma = 662.27$ . By applying Equations (17) and (18), we were able to determine the values of  $\Gamma_m$  and  $\Gamma_1$ , which were found to be 19.85 and 13.73 respectively. Thus, the value of  $\Gamma_{\min}$  is equal to  $\Gamma_m$ , which is 19.85. As  $\Gamma$  is greater than  $\Gamma_{\min}$ , the optimal production run is  $\Gamma^* = 662.27$ , leading to an optimal total cost of \$17130.82.1. pdf

**Example 2:** The given example was solved using the aforementioned solution procedure. The values of  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  were calculated to be -0.7879, 0.5891, and 2241.589 respectively. As  $\beta_1$  was found to be negative, this indicates that illustration

2 has occurred. As a result, we determined the values of  $\Gamma_m$  and  $\Gamma_l$  using Equations (17) and (18) respectively, which were found to be 913.249 and 400.686. Therefore, the value of  $\Gamma_{\min}$  is equal to  $\Gamma_m$ , which is 913.249. Hence, the optimal production run is  $\Gamma^* = \Gamma_{\min} = 913.249$ , leading to an optimal total cost of \$2779.616. [2.pdf](#)

## 2. Conclusion

The concept of sustainable manufacturing has gained increasing importance in recent years, as it aims to minimize the negative impact of manufacturing processes on the environment and society while producing sustainable products. In this context, a model for sustainable manufacturing systems has been proposed, which takes into account the reworking of all defective items to prevent waste production. To ensure the efficient functioning of the manufacturing system and avoid generating waste. The manufacturing machinery undergoes an acclimatization period at the beginning of each production cycle, defects are detected and identified. The proposed model seeks to minimize the total cost incurred in the manufacturing system, including setup, maintenance, manufacturing, holding, acquisition cost, backordering cost and rework costs. An exact solution procedure were used to solve the problems, and two examples are used to demonstrate the effectiveness of the proposed methodology. The proposed model provides a comprehensive approach to achieve sustainable manufacturing by minimizing costs and preventing waste production. We might extend the model to include constraints on the warehouse, stochastic and scrapped goods.

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