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REPLENISHMENT POLICY FOR DETERIORATING ITEMS WITH TIME-DEPENDENT DEMAND BACKLOGGED PARTIALLY UNDER TRADE CREDIT PERIOD AND PRESERVATION TECHNOLOGY FOR QUEUED CUSTOMERS : COMPUTATIONAL APPROACH

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ABSTRACT:

This paper aims to frame an inventory model with Time-dependent demand, instantaneous deterioration rate with preservation technology, partially backlogged shortages, trade credit and queued customers. In this proposed model, the instantaneous deteriorating items for queued customers is developed under the assumption of partial backlogging and backlogging rate varies as the waiting time for the next replenishment. Demand follows time-dependent demand pattern. When the shortage occurs, some customers are willing to wait for back order and others would turn to buy from other sellers and in some situations customers or vendors are assumed to receive the demand in bulk of inventory are put in queue at a service facility. This model focused on three things. The first one is to reduce the deterioration rate by preservation technology, the second one is to trade-off the cost of providing a level of service capacity and the customers waiting for service and the third is using an appropriate trade credit period to minimize the total cost. The classical optimization technique is used to solve the problem. This study provides a reference for a manufacturer and a retailer on making inventory decisions under preservation technology and credit strategies. The main objective of this model is to minimize the total cost function with respect to optimal replenishment policy. Finally numerical examples are provided to illustrate the problem and sensitivity analysis has been carried out to depict the significance of the total cost and the cycle period.

Keywords: Inventory, Partial backlogging, Instantaneous deterioration, Time-dependent demand, Preservation technology and Queued customers.

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Introduction:

Inventory control is essential for companies to reduce their costs, maintaining stock, improving products quality, providing better services and managing customer demands, companies are facing greater challenges when they are working with deteriorating products: Deteriorating means decay, demangle, spoilage and / or obsolescence of products from its original condition. Alternatively companies can acquire preservation technology to reduce the rate to deterioration. Due to deterioration, inventory system faces the problem of shortages and loss of good will or loss of profit. So the deterioration must also be considered in inventory control.

Every company wants to attract their customers. Therefore, companies can offer credit policy to lighten the financial burden of the customers, and this, can attract more customers.

In some inventory systems such as fashionable commodities the length of waiting time for the next replenishment is the main factor in determining whether backlogging will be accepted or not, the longer the waiting time is the smaller the backlogging rate would be and vice versa.

Harris (1915) described the first economic order quantity (EOQ) model for constant and known demand. Then some researchers considered the variable demand such as time dependant and stock dependant. Ghare (1963) proposed a model for exponentially decaying inventory. Later, Covert and Philip (1973) prolonged the model replacing a constant deterioration rate by a two-parameter Weibull distribution deterioration rate .Shah and Jaiswal (1977) proposed an order-level inventory model for items with a constant rate o deterioration. Aggarwal (1978), Dave and Patel (1981),Giri and Chaudiri (1998),Chang and Dye (1999) have used constant deterioration rate in inventory models. Inventory models with time dependent demand rate were discussed by Sivazlian and Stanfel (1976). Dave and Patel (1981) proposed a (T, si)-policy inventory model for deteriorating items with time proportional demand. Chung and Ting (1993) proposed a heuristic for replenishment of deteriorating items with a linear trend in demand. They considered time dependant demand rate.

Backlogging occurs due to Shortages. Some researchers assumed partial backlogging while some other researchers considered Complete backlogging. In reality, it is seen that during the shortage period either all customers wait until the arrival of next order (completely backlogged) or all customers leave the system (completely lost). Some customers are able to wait for the next order to satisfy their demands during the stock out period, while others do not wish to wait and they have to fill their demands from other sources (partial back order case). Customers who experience stock-out will be less likely to buy again from the suppliers, they may turn to another store to purchase the goods. The sales for the product may decline due to the introduction of more competitive product or the change in consumers' preferences. The longer the waiting time, the lower the backlogging rate is. This leads to a larger fraction of lost sales and a less profit. As a result, taking the factor of partial backlogging into account is necessary. Abad (1996) studied an inventory model for optimal pricing and lot-sizing under conditions of perishability and partial backordering. Chang and Dye (1999) developed an EOQ model for deteriorating items with time varying demand and partial backlogging. Abad (2001) proposed an optimal price and order-size for a reseller under partial backlogging. Goyal and Giri (2001) proposed a model for recent trends in modeling of deteriorating inventory.

Trade credit policy is another interesting tool to stimulate customers demand. Trade credit is one of the facility where customers must pay the cost of the product after a predetermined time. Trade credit policy can help the customers to purchase products as the payment is not paid instantly, and it also boots the business of a company. The customers must pay the cost with some interest, if they do not pay within the predetermined time. First Goyal (1985) derived the retailers optimal EOQ when the supplier provides a permissible delay in payment (ie an upstream trade credit). Agarwal and Jaggi (1995) generalized the EOQ model from non-deteriorating items to deteriorating items. Jamal, Sarker and Wang (1997) extended the EOQ model to allow for shortages. Teng (2002) amended the model by differentiating the unit price and unit cost to establish an easier solution. Later, Haung's (2003) introduced a bi-level trade-credit policy. He considered a delay in payment from a supplier to a retailer and from the retailer to the customer. Huang's (2003) economic order quantity model has been revisited by Teng and Goyal (2007) emphasizing that the retailer's delay period must be shorter than the suppliers's one. Sarkar (2012) developed an EOQ model with delay-in-payment and time-varying deterioration rate

The deteriorating rate affects the customers demand, and finally ,the total cost. Therefore, a reduction in the deterioration rate will save a considerable cost significantly (Ouyang et al, 2006; Hsu et al, 2010). An investment is preservation technology , e.g. refrigeration equipment, can reduce the deterioration rate. Further research on inventory model with preservation technology has been conducted such as Dye and Hsieh (2012) , Lee and Dye (2012), Dye (2013), He and Huang (2013), Zhang et a.l (2014) and Yang et al. (2015). Umakanta Mishra, Jacobo Tijerina (2018) developed Retailer's joint ordering, pricing and preservation Technology investment policies for a deteriorating item under permissible delay in payments. Recently, Li et al. (2019) developed a replenishment, pricing and investment decision model for non – instantaneous deteriorating items with preservation technology investment and price dependent demand while Abu, Hui Ming and Yosef (2019) considered advertisement policy and Bardhan et al. (2019) considered stock- dependent demand.

There is a number of situations in which a customers or vendors of some sort are assumed to receive the demand in bulk of inventory are subject to put in queue at a service of facility. The goal of queuing is essentially to trade-off the cost of providing a level of service capacity and the customers waiting for service.

Thus in the present work we focused on developing an inventory model with time dependant demand rate, trade credit, preservation technology, partial backlogged and Queued Customers. The backlogging rate is inversely proportional to the length of the waiting time for next replenishment. The total cost function which consists of setup cost, holding cost backordering cost, lost sale cost (opportunity cost), deteriorating cost, waiting cost and preservation cost is constructed and subjected to the optimization which in turn gives us the system of non-linear equations. Further, a computing algorithm is proposed to find the solution of system by using the N-R method. We compute the optimal inventory period and total optimal average cost and some numerical examples are given to illustrate the theory. Sensitivity analysis are performed to see how the values of the key parameters affect the Total cost function.

Notations description:

K – Ordering cost of inventory, $per order

I(t) – The inventory level at time t.

D(t) – Demand rate function

θ – Deterioration rate, a fraction of the on hand inventory

P – Purchase cost, $per unit

h – Holding cost excluding interest charges, \$ per unit / year

s – Shortage cost,, \$ per unit / year

P_1 – Selling price, \$ per unit / year

α – Constant demand with respect to time (follows exponential distribution)

π – Opportunity cost due to lost sales, \$ / year

I_e – Interest charges which can be earned, \$ per unit

I_r – Interest charges which invested in inventory, \$ / year, $I_r \geq I_e$

q – Maximum inventory level

M – Permissible delay period to settle the accounts with the supplier and 0<M<1, M in years

T – The length of replenishment cycle

T_1 – Time at which shortages starts, $0 \leq T_1 \leq T$ (for instantaneous deteriorating items)

B – Demand during the shortage period

δ – The backlogging parameter (a positive constant) $0 \leq \delta \leq 1$

λ – The average arrival rate

μ – The average service rate

L_s – The number of customers waiting for inventory

C_W - Waiting cost per customer per unit time

ξ - Cost of preservation technology investment per unit time.

Assumptions:

1. The inventory system involves only one item.
2. The replenishment rate is infinite.
3. There is no replacement or repair of deteriorated units.
4. The demand rate function D(t) is deterministic and is a known function of time and it is given by,

$$D(t) = \begin{cases} Ae^{-\alpha t}, 0 \leq t \leq T_1 \\ B, T_1 \leq t \leq T \end{cases}$$

Where $A < 0$ and $0 < \alpha < 1$ is the constant governing decreasing rate of the demand

5. Shortages are allowed and the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative.

The backlogging parameter δ is a positive constant.

6. The rate of deterioration with the investment of preservation technology is $m(\xi) = e^{-v\xi}$, which satisfies the condition : $\frac{\partial m(\xi)}{\partial \xi} < 0$, $\frac{\partial^2 m(\xi)}{\partial \xi^2} > 0$. v is sensitive parameter of investment to the deterioration rate.
7. λ and μ are assumed to be constant and are related by $L_s = \left(\frac{\lambda}{\mu - \lambda}\right)$

Model Formulation:

Based on the assumptions mentioned earlier, this section presents the following inventory model formulation. In the beginning, an enterprise purchase Q units of goods. As in figure 1 and 2, the reduction of the inventory is due to the combined effect of the demand as well as the deterioration in the interval $[0, T_1)$ and the preservation technology $m(\xi) = e^{-v\xi}$ and the demand backlogged in the interval $[T_1, T)$. The Inventory system with respect to time and the preservation technology $m(\xi) = e^{-v\xi}$, can be written as follows.

$$\frac{dI(t)}{dt} = \begin{cases} -Ae^{-\alpha t} - \theta m(\xi) I(t), 0 \leq t \leq T_1 \\ -\frac{B}{1+\delta(T-t)}, T_1 \leq t < T \end{cases} \quad \text{……………..(1)}$$

With boundary condition $I(T_1)=0$.

The solution of Eq. (1) is

$$I(t)=\begin{cases} I_1(t) if\ 0\le t\le T_1 \\ I_2(t) if T_1\le t<T \end{cases}$$

Consider $0\le t<T_1$,

Solving the differential equation $\frac{dI_1(t)}{dt}=-Ae^{-\alpha t}-\theta m(\xi)I_1(t),$

We get $I_1(t)e^{\theta mt}=\int -Ae^{-\alpha t}.e^{\theta mt}dt+c_1$, where c_1 is the constant of integration.

Using the boundary condition $I_1(T_1)=0$, We can find c_1 and $I_1(t)$ is given as,

$$I_1(t)=\left(\frac{-Ae^{t(-\alpha+\theta m)}}{-\alpha+\theta m}+\frac{Ae^{y(-\alpha+\theta m)}}{-\alpha+\theta m}\right)e^{-\theta mt}$$

Now consider, $T_1\le t<T$

$$\frac{dI_2(t)}{dt}=\frac{-B}{1+\delta(T-t)}$$

Solving the differential equation we get,

$$I_2(t)=-B\log[1+\delta(T-t)]\left(\frac{-1}{\delta}\right)+c_2$$

Where c_2 is the constant of integration, using the given boundary condition $I_2(T_1)=0$, we given $I_2(t)$ as follows

$$I_2(t)=\frac{\log(1+\delta(T-t)B-\log(1+\delta T-\delta T_1)B}{\delta}$$

Therefore,

$$I_1(t)=\left(\frac{-Ae^{t(-\alpha+\theta m)}}{-\alpha+\theta m}+\frac{Ae^{y(-\alpha+\theta m)}}{-\alpha+\theta m}\right)e^{-\theta mt}, where 0\le t<T_1$$

$$I_2(t)=\frac{\log(1+\delta(T-t)B-\log(1+\delta T-\delta T_1)B}{\delta}, where T_1\le t\le T$$

……………………..(2)

From $I_1(t)$ we can find the holding cost in the interval $[0,T_1)$ denoted by HC.

$$HC=h\int_0^{T_1} I_1(t)dt$$

$$=h\left\{\int_0^{T_1}\left(\frac{-Ae^{t(-\alpha+\theta m)}}{-\alpha+\theta m}+\frac{Ae^{y(-\alpha+\theta m)}}{-\alpha+\theta m}\right)e^{-\theta mt}dt\right\}$$

$$= \frac{hA(-\theta m e^{y\alpha} + e^{y\theta m}\alpha - \alpha + \theta m)e^{-y\alpha}}{(-\alpha + \theta m)\alpha\theta m} \quad \text{……………………..(3)}$$

Deterioration cost in the interval $[0,T_1)$ denoted by DC is given as,

$$DC = P\theta\int_0^{T_1} I_1(t)dt$$

$$= P\theta\left\{\int_0^{T_1}\left[\frac{-Ae^{t(-\alpha+\theta m)}}{-\alpha+\theta m} + \frac{Ae^{y(-\alpha+\theta m)}}{-\alpha+\theta m}\right]e^{-\theta m t}dt\right\}$$

$$= PA\left[\frac{\left(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m\right)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha m}\right] \quad \text{……………………..(4)}$$

During the stock out period we have to consider two costs. First we have to derive the shortage cost for the backlogged items and then we have to obtain the opportunity cost due to lost sales. The shortage cost over the period $[T_1,T)$ denoted by SC is given by,

$$SC = s\int_{T_1}^{T} I_2(t)dt$$

$$= s\int_{T_1}^{T}\left(-\frac{B}{\delta}\right)\{\log[1+\delta(T-T_1)] - \log[1+\delta(T-t)]\}dt$$

On simplification, we get

$$SC = -\frac{sB}{\delta}\left\{(T-T_1)\log[1+\delta(T-T_1)] - (T-T_1)\log[1+\delta(T-T_1)] + (T-T_1) - \frac{1}{\delta}\log[1+\delta(T-T_1)]\right\}$$

$$= -\frac{sB}{\delta^2}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\}.$$

The cost cannot be negative. So the shortage cost is given by,

$$SC = \frac{sB}{\delta^2}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\}. \quad \text{………………….(5)}$$

Now, the opportunity cost due to lost sales during the replenishment cycle demand by OC is given as,

$$OC = \pi B\int_{T_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]dt$$

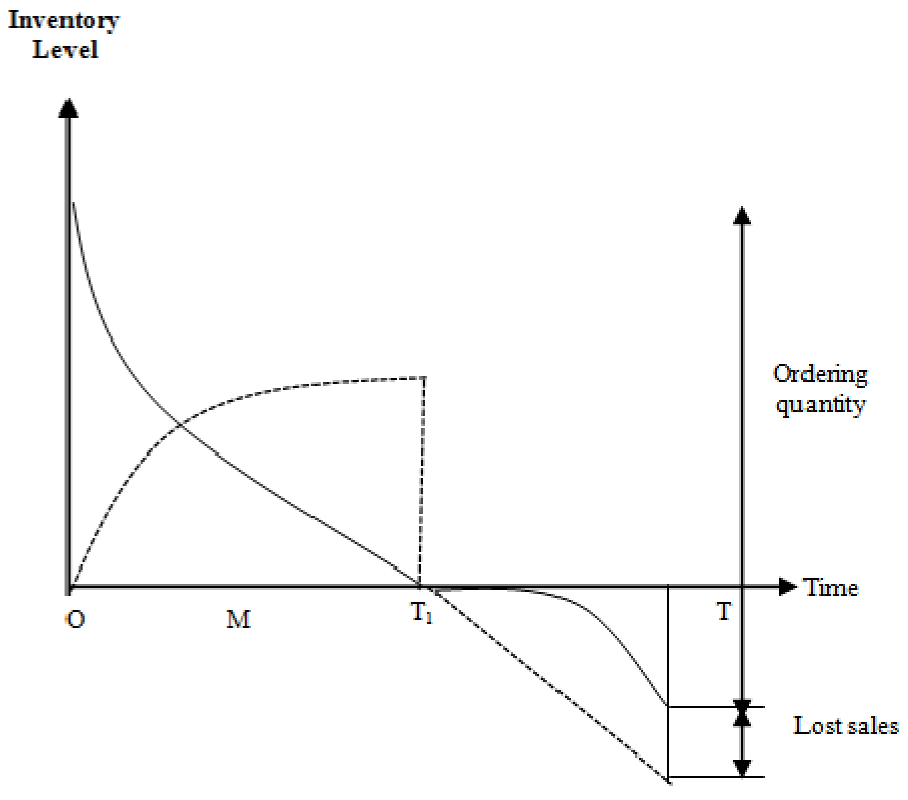
$$= \pi B\left\{ t - \frac{\log[1+\delta(T-t)]}{-\delta} \right\}_{T_1}^{T}$$

$$= \pi B\left\{ \delta(T-T_1) - \frac{1}{\delta}\log[1+\delta(T-T_1)] \right\}$$

$$OC = \frac{\pi B}{\delta}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\} \quad \text{.....................(6)}$$

Now, we have to consider M which is the permissible delay in settling the accounts offered by the supplier, there are two possibilities, case(1) $M \le T_1$ or case (2) $M > T_1$, We shall discuss first case(1) and then case(2),

Case (1): $M \le T_1$



Inventory Level for Case 1: M≤T1

Figure 1

Since in this case the length of the period with positive inventory of the items is larger than the credit period , the buyer can earn the interest with an annual rate I_e in [0,T1).

The interest earned denoted by IE_1 is

$$IE_1 = PI_e \int_0^{T_1} (T_1 - t)(Ae^{-\alpha t})dt$$

Now, $IE_1 = \dfrac{P_1 I_e A(T_1\alpha - 1 + e^{-T_1\alpha})}{\alpha^2}$ ……………….(7)

After the fixed credit period, the buyer has to pay the interest on the product still in stock with an annual rate I_r. Hence the interest payable denoted by IP is given by

$$IP = PI_r \int_M^{T_1} I_1(t)dt$$

$$= PI_r \left\{ \int_M^{T_1} \left[\frac{-Ae^{t(-\alpha+\theta m)}}{-\alpha+\theta m} + \frac{Ae^{y(-\alpha+\theta m)}}{-\alpha+\theta m} \right] e^{-\theta mt} dt \right\}$$

By simplifying we have,

$$IP = \left[\frac{-1}{(-\alpha+\theta m)\alpha\theta m}\right]\left[PI_r A\left(e^{T_1\alpha+\theta mM}\theta m - e^{T_1\theta m + M\alpha}\alpha + e^{M\alpha+\theta mM}\alpha - e^{M\alpha+\theta mM}\theta m\right)e^{-M\alpha - T_1\alpha - \theta mM}\right] \quad \text{……(8)}$$

The total waiting cost for the customers in the system,

$$WC = C_W L_S = C_w\left(\frac{\lambda}{\mu-\lambda}\right) \quad \text{...……………..(9)}$$

Preservation cost $(PRC) = \xi T$ ……………..(10)

The Total average cost in this case is

$$TC_1 = \frac{1}{T}\left[K + HC + DC + SC + OC + WC + PRC + IP - IE_1\right]$$

$$= \frac{1}{T}\left\{\begin{array}{l} K + \dfrac{hA(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha\theta m} \\ + PA\left[\dfrac{\left(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m\right)e^{-T_1\alpha}}{\left(-\alpha+\theta m\right)\alpha m}\right] + \dfrac{sB}{\delta^2}\left\{\delta\left(T - T_1\right) - \log\left[1 + \delta\left(T - T_1\right)\right]\right\} \\ + \dfrac{\pi B}{\delta}\left\{\delta\left(T-T_1\right) - \log\left[1+\delta\left(T-T_1\right)\right]\right\} + C_w\left(\dfrac{\lambda}{\mu-\lambda}\right) + \xi T \\ + \left[\dfrac{-1}{(-\alpha+\theta m)\alpha\theta m}\right]\left[PI_r A\left(e^{T_1\alpha+\theta mM}\theta m - e^{T_1\theta m+M\alpha}\alpha + e^{M\alpha+\theta mM}\alpha - e^{M\alpha+\theta mM}\theta m\right)e^{-M\alpha-T_1\alpha-\theta mM}\right] \\ - \dfrac{P_1 I_e A(T_1\alpha - 1 + e^{-T_1\alpha})}{\alpha^2} \end{array}\right\}$$

$$=\frac{1}{T}\left\{\begin{array}{l} K+(h+P)A\left[\dfrac{\left(-\theta m e^{T_1\alpha}+e^{T_1\theta m}\alpha-\alpha+\theta m\right)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha m}\right] \\ +\dfrac{sB}{\delta^2}\left\{\delta(T-T_1)-\log\left[1+\delta(T-T_1)\right]\right\} \\ +\dfrac{\pi B}{\delta}\left\{\delta(T-T_1)-\log\left[1+\delta(T-T_1)\right]\right\}+C_w\left(\dfrac{\lambda}{\mu-\lambda}\right)+\xi T \\ +\left[\dfrac{-1}{(-\alpha+\theta m)\alpha\theta m}\right]\left[PI_r A\left(e^{T_1\alpha+\theta mM}\theta m-e^{T_1\theta m+M\alpha}\alpha+e^{M\alpha+\theta mM}\alpha-e^{M\alpha+\theta mM}\theta m\right)e^{-M\alpha-T_1\alpha-\theta mM}\right] \\ -\dfrac{P_1 I_e A(T_1\alpha-1+e^{-T_1\alpha})}{\alpha^2} \end{array}\right\}$$

…………………………..(11)

The necessary conditions for the total annual cost $\partial TC_1(T,T_1,\xi)$ is convex with respect to $T,T_1 and \xi$ are $\frac{\partial TC_1(T,T_1,\xi)}{\partial T}=0, \frac{\partial TC_1(T,T_1,\xi)}{\partial T_1}=0 and \frac{\partial TC_1(T,T_1,\xi)}{\partial \xi}=0$.

Provided they satisfy the sufficient conditions

$$\left.\frac{\partial^2 TC_1(T,T_1,\xi)}{\partial T^2}\right|_{(T^*,T_1^*,\xi^*)}>0, \left.\frac{\partial^2 TC_1(T,T_1,\xi)}{\partial T_1^2}\right|_{(T^*,T_1^*,\xi^*)}>0 and \left.\frac{\partial^2 TC_1(T,T_1,\xi)}{\partial \xi^2}\right|_{(T^*,T_1^*,\xi^*)}>0$$

and $\begin{bmatrix} \frac{\partial^2}{\partial \xi^2}TC_1 & \frac{\partial^2}{\partial \xi\partial T_1}TC_1 & \frac{\partial^2}{\partial \xi\partial T}TC_1 \\ \frac{\partial^2}{\partial T_1\partial \xi}TC_1 & \frac{\partial^2}{\partial T_1^2}TC_1 & \frac{\partial^2}{\partial T_1\partial T}TC_1 \\ \frac{\partial^2}{\partial T\partial \xi}TC_1 & \frac{\partial^2}{\partial T\partial T_1}TC_1 & \frac{\partial^2}{\partial T^2}TC_1 \end{bmatrix}$ >0

We develop the following algorithm to find the optimal values of $T,T_1 and \xi$ (say $T^*,T_1^* and \xi^*$) that minimize $TC_1(T,T_1,\xi)$

Algorithm 1: Case(i)

The problem mentioned above is solved by using the following algorithm:

Step 1: Start

Step 2: Put $\frac{\partial TC_1(T,T_1,\xi)}{\partial T}, \frac{\partial TC_1(T,T_1,\xi)}{\partial T_1} and \frac{\partial TC_1(T,T_1,\xi)}{\partial \xi}$

Step 3: Solve the simultaneous equation $\frac{\partial TC_1(T,T_1,\xi)}{\partial T}=0, \frac{\partial TC_1(T,T_1,\xi)}{\partial T_1}=0 and \frac{\partial TC_1(T,T_1,\xi)}{\partial \xi}=0$ by fixing M and initializing the values of $K,P,h,s,P_1,\pi,\theta,I_e,I_r,M,\delta,\alpha,C_W,\lambda,\mu,v,A,B$

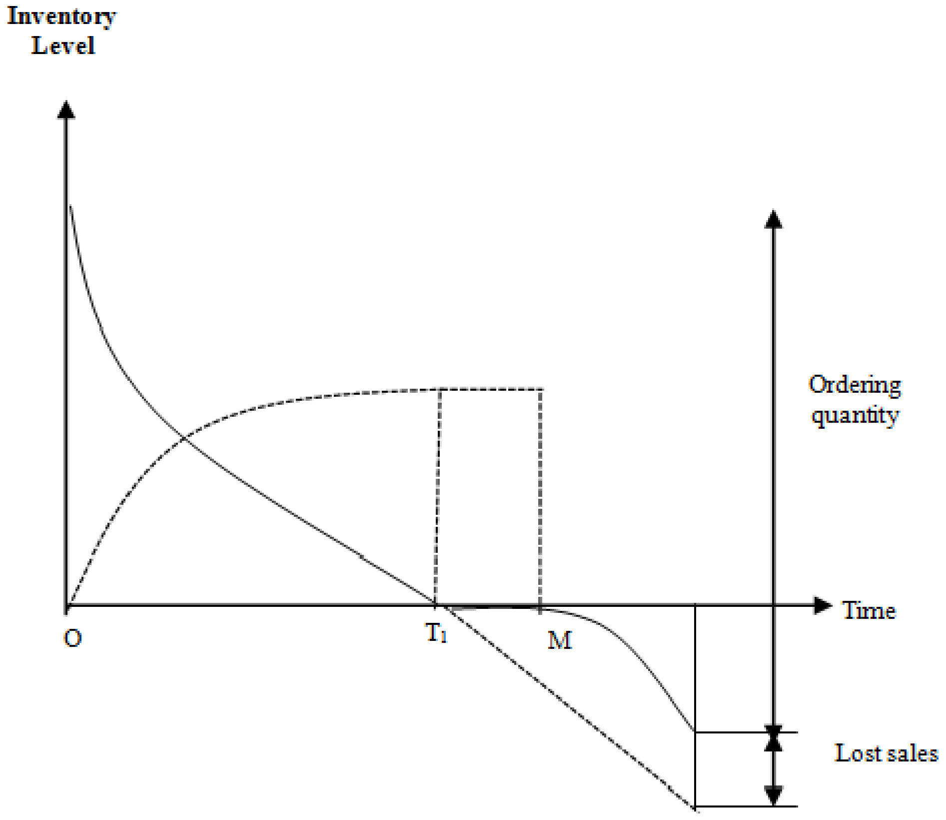
Step 4: Choose one set of solution from Step 3

Step 5: If the values in equation (10) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step3.

Step 6: Evaluate $TC_1(T^*, T_1^*, \xi^*)$

Step 7: Stop

Case (2): $T_1 < M$



Inventory Level for Case 2: M>T1

Figure 2

Since $T_1 < M$ the buyer earns the interest during the period [0,M) and pays no interest earned in this case denoted by IE_2 is given by,

$$IE_2 = PI_e \left\{ \int_0^{T_1} (T_1 - t)\left(Ae^{-\alpha t_1}\right)dt + (M - T_1)\int_0^{T_1} Ae^{-\alpha t}\, dt \right\}$$

$$= PI_e A\left[\frac{1}{2} Ae^{-\alpha T_1} y^2 - \frac{(M - T_1)A\left(-1 + e^{-\alpha T_1}\right)}{\alpha}\right]$$

The total average cost in this case is

$$TC_2 = K + HC + DC + SC + OC + WC + PRC - IE_2$$

$$= \frac{1}{T}\left\{\begin{array}{l} K + \dfrac{hA(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha\theta m} \\ + PA\left[\dfrac{\left(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m\right)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha m}\right] + \dfrac{sB}{\delta^2}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\} \\ + \dfrac{\pi B}{\delta}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\} + C_w\left(\dfrac{\lambda}{\mu-\lambda}\right) + \xi T \\ - PI_e A\left[\dfrac{1}{2}Ae^{-\alpha T_1}y^2 - \dfrac{(M-T_1)A\left(-1+e^{-\alpha T_1}\right)}{\alpha}\right] \end{array}\right\}$$

$$= \frac{1}{T}\left\{\begin{array}{l} K + (h+P)A\left[\dfrac{\left(-\theta m e^{T_1\alpha} + e^{T_1\theta m}\alpha - \alpha + \theta m\right)e^{-T_1\alpha}}{(-\alpha+\theta m)\alpha m}\right] \\ + \dfrac{sB}{\delta^2}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\} \\ + \dfrac{\pi B}{\delta}\{\delta(T-T_1) - \log[1+\delta(T-T_1)]\} + C_w\left(\dfrac{\lambda}{\mu-\lambda}\right) + \xi T \\ - PI_e A\left[\dfrac{1}{2}Ae^{-\alpha T_1}y^2 - \dfrac{(M-T_1)A\left(-1+e^{-\alpha T_1}\right)}{\alpha}\right] \end{array}\right\} \quad (12)$$

The necessary conditions for the total annual cost $\partial TC_2(T,T_1,\xi)$ is convex with respect to $T,T_1 and \xi$ are $\frac{\partial TC_2(T,T_1,\xi)}{\partial T} = 0, \frac{\partial TC_2(T,T_1,\xi)}{\partial T_1} = 0 and \frac{\partial TC_2(T,T_1,\xi)}{\partial \xi} = 0$.

Provided they satisfy the sufficient conditions

$$\left.\frac{\partial^2 TC_2(T,T_1,\xi)}{\partial T^2}\right|_{(T^*,T_1^*,\xi^*)} > 0, \left.\frac{\partial^2 TC_2(T,T_1,\xi)}{\partial T_1^2}\right|_{(T^*,T_1^*,\xi^*)} > 0 and \left.\frac{\partial^2 TC_2(T,T_1,\xi)}{\partial \xi^2}\right|_{(T^*,T_1^*,\xi^*)} > 0$$

and $\begin{bmatrix} \frac{\partial^2}{\partial \xi^2}TC_2 & \frac{\partial^2}{\partial \xi \partial T_1}TC_2 & \frac{\partial^2}{\partial \xi \partial T}TC_2 \\ \frac{\partial^2}{\partial T_1 \partial \xi}TC_2 & \frac{\partial^2}{\partial T_1^2}TC_2 & \frac{\partial^2}{\partial T_1 \partial T}TC_2 \\ \frac{\partial^2}{\partial T \partial \xi}TC_2 & \frac{\partial^2}{\partial T \partial T_1}TC_2 & \frac{\partial^2}{\partial T^2}TC_2 \end{bmatrix}$ >0

We develop the following algorithm to find the optimal values of $T,T_1 and \xi$ (say $T^*,T_1^* and \xi^*$) that minimize $TC_2(T,T_1,\xi)$

Algorithm 2: Case(ii)

The problem mentioned above is solved by using the following algorithm:

Step 1: Start

Step 2: Put $\frac{\partial TC_2(T,T_1,\xi)}{\partial T}, \frac{\partial TC_2(T,T_1,\xi)}{\partial T_1} and \frac{\partial TC_2(T,T_1,\xi)}{\partial \xi}$

Step 3: Solve the simultaneous equation
$\frac{\partial TC_2(T,T_1,\xi)}{\partial T} = 0, \frac{\partial TC_2(T,T_1,\xi)}{\partial T_1} = 0 and \frac{\partial TC_2(T,T_1,\xi)}{\partial \xi} = 0$ by fixing M and initializing the values of $K, P, h, s, P_1, \pi, \theta, I_e, I_r, M, \delta, \alpha, C_W, \lambda, \mu, v, A, B$

Step 4: Choose one set of solution from Step 3

Step 5: If the values in equation (11) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step3.

Step 6: Evaluate $TC_2(T^*, T_1^*, \xi^*)$

Step 7: Stop

Numerical Examples:

Example 1: Consider the inventory system with the following data
$K = 250, P = 15, h = 4, s = 18, P_1 = 20, \pi = 12, \theta = 0.05, I_e = 0.11, I_r = 0.12, M = 0.0113, \delta = 0.4, \alpha = 0.04,$
$C_W = 3, \lambda = 10, \mu = 8, v = 4, A = 50, B = 20,$
in appropriate units. In this case $M \le T_1$. Using the algorithm we obtain the optimal solution as $T = 1.30088, T_1 = 0.8322, \xi = 5.10336$. Hence the minimum total cost per unit time is $TC_1 = 5105.258$.

Example 2: Again the data are the same as in Example1, except that M=0.8 in appropriate units. Here we find that $M > T_1$.Therefore using the algorithm we find the optimal values as $T = 1.1006, T_1 = 0.69961, \xi = 4.59252$. Hence the minimum total cost per unit time is $TC_2 = 3724.495$.

In order to solve the examples given above , Maple (version 15) software is used.

Effect of Change in various parameters of the inventory

Changing parameter	Change in parameter	T	T_1	ξ	TC
θ	0.50	1.3240	0.6955	4.5165	3298.455
	0.54	1.1253	0.6930	4.5463	3968.173
	0.58	1.1231	0.6889	4.6311	4573.933
	0.60	1.1220	0.6869	4.9773	4751.870
δ	0.396	1.2778	0.6971	4.5391	3946.347
	0.398	1.2774	0.6972	4.5396	4189.630
	0.402	1.2775	0.6972	4.5405	4676.974
	0.404	1.2774	0.6973	4.5406	4761.840

C_w	1	1.1403	0.7032	4.5952	4491.225
	2	1.3411	0.7002	4.5673	4226.380
	3	1.1278	0.6971	4.5391	3946.347
	4	1.1215	0.6941	0.5707	3876.174
v	3.6	1.1156	0.5389	5.2822	2292.801
	3.8	1.1069	0.5279	4.7678	2812.586
	4.2	1.0916	0.5082	4.5873	3628.649
	4.4	1.0890	0.5024	4.5802	3699.028
M	7.6	1.1036	0.6950	4.5383	3572.585
	7.8	1.1022	0.6973	4.5654	3409.795
	8.1	1.1099	0.7007	4.6058	3208.554
	8.3	1.1096	0.7014	4.6258	3036.456

Figure 3: Effect of θ on Total Cost

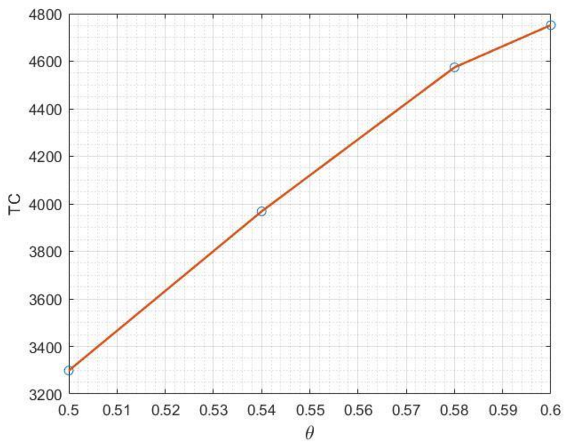


Figure 4: Effect of δ on Total Cost

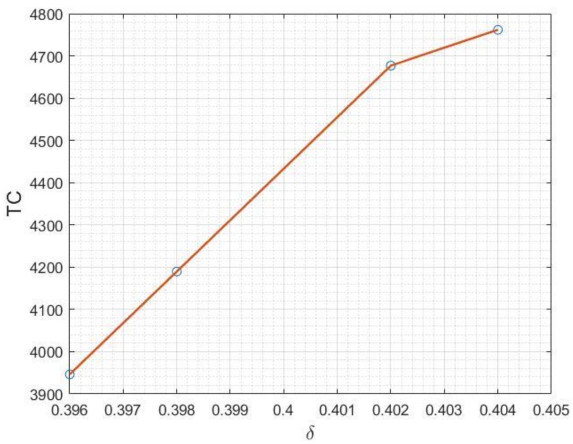


Figure 5: Effect of C_w on Total Cost

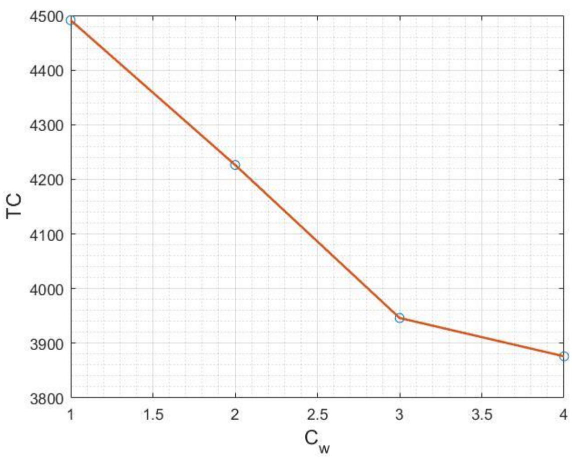


Figure 6: Effect of v on Total Cost

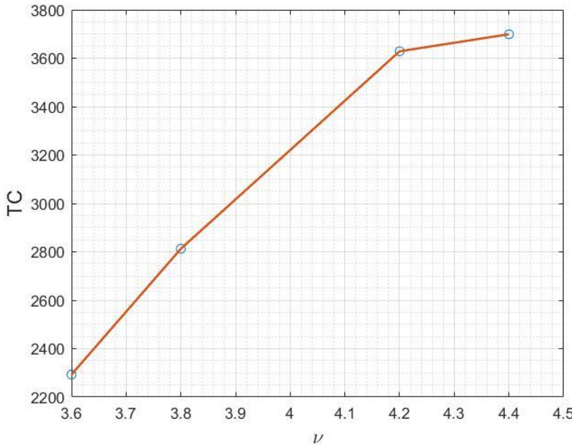
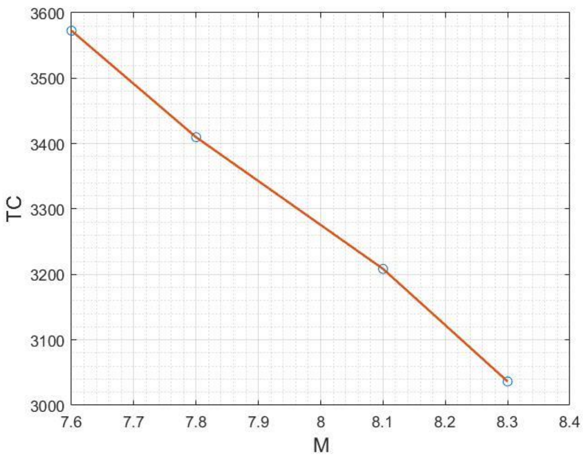


Figure 7: Effect of M on Total Cost



Managerial Implications:

We now study of changes in the values of the system parameters on the optimal replenishment policy of Example 2. We change one parameter at a time, keeping the other parameters unchanged. The results are summarized in Table 1. Based on the numerical results, we obtain the following managerial implications

1. When the parameter M increases, the total cost decreases. This shows that from managerial point of view, when the customers trade credit period offered by the retailer is increasing, the retailer should order less quantity more frequently. Hence, the retailer gets the longer permissible delay period from the supplier.
2. From Table, we see that the increases in rate of deterioration θ leads to an increase of total cost. Hence when the deterioration rate of products is more, the retailer should order less, frequently and invest most in preservation cost to reduce the deterioration.
3. The backlogging rate decreases with increase in the backlogging parameter δ . Hence, when the backlogging rate decreases, the total inventory cost increases and the order quantity decreases. To achieve minimum total inventory cost, the retailer should increases the backlogging rate by ordering more quantity.
4. The parameter v shows the significant effect on preservation cost. Preservation cost is decreased when v is increased. However, it has no notable effect on others.
5. From Table, we note that, when waiting cost increases, the optimal average cost decreases.

Conclusion:

In this paper we described a Replenishment policy for deteriorating items with time-dependant demand rate, trade credit and preservation technology for queued customers. Shortages are allowed and partially backlogged. In market there are certain items where during the seasonal period, the demand increases with time and when the season is off, the demand sharply decreases. Thus the proposed model can also be used in inventory control of seasonal items. This work helps the retailer to obtain optimal replenishment cycle time by incorporating the following facts: (1) the retailer has a chance to reduce the deterioration rate by investing in preservation technology and reduce the total average annual inventory cost by increasing backlogging rate (2) a supplier often offers a permissible delay in payments which will attract more customers (3) We can increase the customers by putting them in queue at service facility when they receive the demand in bulk of inventory, Since the goal of Queuing is essentially to trade-off the cost of providing a level of service capacity and the customers waiting for service. The classical optimization technique is used to derive the optimal average total cost. Behavior of different parameters have been discussed through the numerical example and sensitivity analysis. From sensitivity analysis carried out the rate of change in the parameters θ (deterioration rate), δ (backlogging parameter), C_W (waiting cost), v (sensitive parameter), M (permissible delay period) drastically decrease the total inventory cost and decreased order quantity to avail frequently the trade credit period which well suits for present business scenario. This paper can be further extended with the stock dependant demand rate and also fuzzy or stochastic uncertainty in inventory parameters may be considered.

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