Changing and Unchanging d^d – distance for Edge Removal in Some Special Graphs

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Abstract: Let G = (V, E) be a simple graph. Let *u* and *v* be two vertices of a connected graph *G*. Then the *d-length* of a *u-v* path defined as $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$, where $d(u, v)$ is the shortest distance between the vertices *u* and *v*. In this paper d^d – distance of edge removal in some special graphs are determined.

Keywords: distance , degree**,** d^d-distance, fan graph, star graph.

DOI: 10.48047/ecb/2023.12.si6.703

Introduction

Let G(V,E) be a simple, connected graph where V(G) is its vertex set and E(G) is its edge set. The degree of any vertex v in G is the number of edges incident with v and is denoted by deg v. The minimum degree of a graph is denoted by δ(G) and the maximum degree of a graph G is denoted by Δ(G) . A vertex of degree 0 is called an isolated vertex and a vertex of degree 1 is called a pendent vertex. The standard or usual distance d(u, v) between u and v is the length of the shortest u − v path in G . In this paper, d^d – distance of some corona relaed graphs are determined. In this paper, change or otherwise of the values of d^d – distance when a edge is removed from a graph is studied.

2. Edge Removal

Definition 2.1: Let a graph G(V,E) be a connected graph with d^d distance in graphs.

$$E^0_{d^d}(G) = \{e \in E(G) : d^d (G) = d^d (G - e)\}$$

$$E^+_{d^d}(G) = \{e \in E(G) : d^d (G) < d^d (G - e)\}$$

$$E^-_{d^d}(G) = \{ e \in E(G) : d^d G) > d^d G - e)\}$$

Example 2.2: Let G = $K_{1,3}$. Let v be non adjacent vertics and e be any edge of G, G – e = $K_{1,2} \cup K_1$ by defintion d^d (G) = 2+ 1 + 1 + 1 = 5 and d^d (G - e) = 2 + 1 + 1 + 1 = 5. Hence d^d (G) = d^d (G - e).

Example 2.3 : Let G = P_5. Let V(G) = $\{v_1, v_2, v_3, \ldots, v_5\}$ and v be v_1 and v_5 . d^d (G) = 7. Let e = v_3v_4 and G – e = $P_3 \cup P_2$, d^d (G - e) = d^d (P_5) + d^d (P_7) = 5+ 4 = 9. Hence d^d (G) < d^d (G - e).

Example 2.4: Consider the graphs G = C_3 and G – e = P_3 . Let v be neighbhourhood vertex . By definition , d^d (G) = 1 + 2 + 2 + 4 = 9 and d^d (G - e) = 1 + 1 + 2 + 2 = 6. **H**ence d^d (G) > d^d (G - e).

The above examples 2.1,2.2 and 2.3 show that removal of a edge from a graph G may increase/ decrease/ does not affect d^d (G).

Result 2.5: When G – e = (G-uv) $\cup K_1$ has no d^d – distance in graphs.

Result 2.6: If G = P_n , n ≥ 4. Let e be any edge of G and u,v be end vertices . Then d^d – distance does not exist.

Proof: u and v be end vertices of G and e be any edge. The graph G – e = $P_{n-1/2} \cup P_{n-1/2} = P_{n/2} \cup P_{n/2}$. Hence d^d – distance does not exists.

Theorem 2.7: Let G = K_n be a complete graph of order n, n ≥ 3 and e = $v_{n-1}v_n$. Then e ∈ $E^{+}_{d_d}$(G).

Proof: Let V(K_n) = {$u_1, u_2, ..., u_n$}. Let e = $v_{n-1}v_n$. d^d of K_n is 1 + (n - 1) + (n - 1) + $(n - 1)^2 = n^2$. In K_n – e, d^d of K_n – e is 2 + (n - 2) + (n - 2) + $(n - 2)^2 = n^2 - 2n + 2$. Hence d^d of K_n > d^d of K_n - e. Therefore e ∈ $E^{+}_{d_d}$(G).

Theorem 2.8: Let G = f_n. Let V(G) = {u, u_i : 1 ≤ i ≤ n }and u = v_1 and v = v_n. Let e = u_iu_{i+1}, 2 ≤ i ≤ n - 1. Then e ∈ $E^{0}_{d^d}$(G).

Proof: $d^d(G) = d^d(u,v) = 2 + 2 + 2 + 4 = 10$. G – e = $f_i \cup f_{n-i}$. d^d (G - e) = 2 + 2 + 2 + 4 = 10 = $d^d(G)$ Therefore e ∈ $E^{0}_{d^d}$(G).

Illustration : Let G = f_6 given in the following figure 1

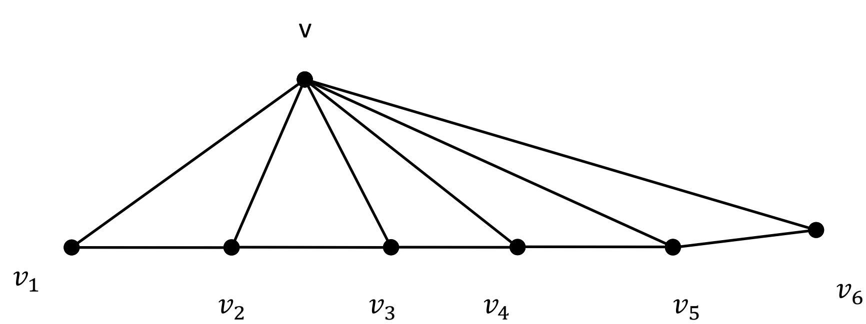

Figure 1

d^d(v_1, v_6) = d(v_1, v_6) + deg v_1 + deg v_6 + degv_1 degv_6 = 2 + 2 + 2 + 4 = 10.

Let G – v_2 v_3 = H. H is given in the following figure 2

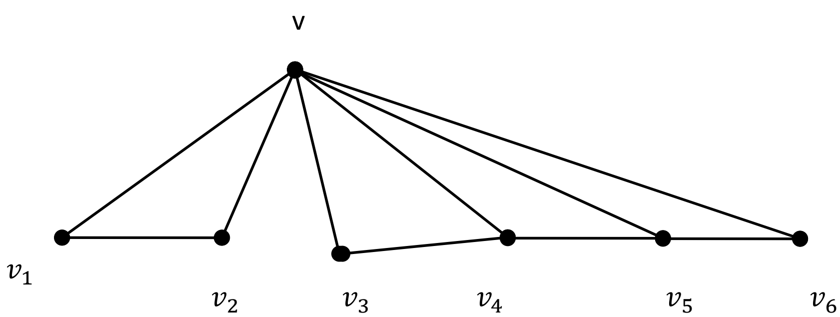

Figure 2

d^d(G – v_2v_3 = H) = d(v_1, v_6) + deg v_1 + deg v_6 + degv_1 degv_6 = 2 + 2 + 2 + 4 = 10.

Therefore d^d (G – v_2v_3 = H) = 10 = d^d (G).

Theorem 2.9: Let G = J_n , n ≥ 1 be a jewel graph. Let E(G) = {ux, uy,vx, vy, xy, uv_i, vv_i,: 1 ≤ i ≤ n}. Let e = xy. Then e ∈ $E^{-}_{d^d}$(G).

Proof: Let V(G) = {u, x, v, y, v_i : 1 ≤ i ≤ n} and E(G) = {ux, uy,vx, vy, xy, uv_i, uv_n, vv_i, vv_n,v_iv_{i+1} : 1 ≤ i ≤ n - 1}. Let u = x and v = y. $d^d(G = (u,v))$ = 1 + 3 + 3 + 9 = 16. Let e = xy. $d^d(G$ - e) = 2 + 2 + 2 + 4 = 10. Hence $d^d(G)$ > $d^d(G - e)$. Hence e ∈ $E^{-}_{d^d}$(G).

Illustration : Let G = J_2 and G – xy are given in the figure 3 and 4

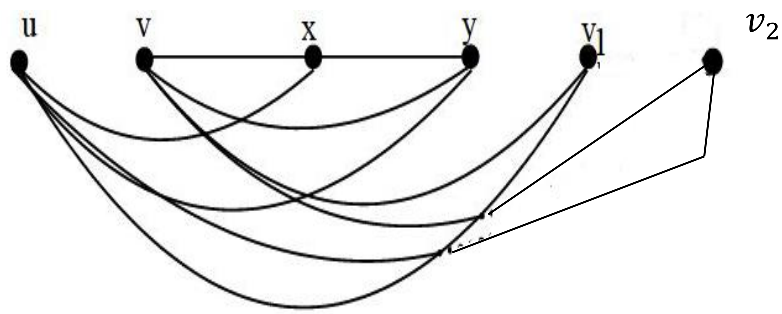

Figure 3

$d^d(G)$ = 1 + 3 + 3 + 9 = 16.

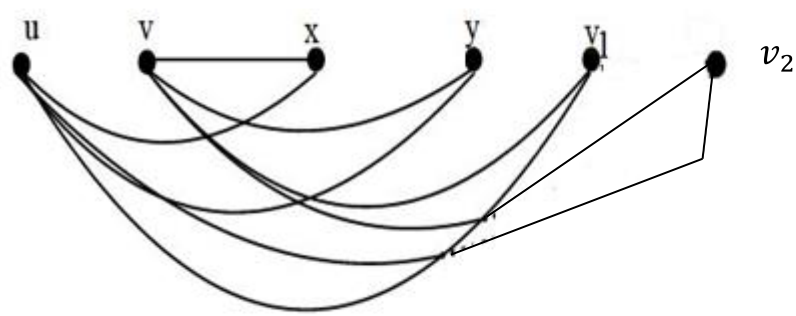

Figure 4

$d^d(G$ - e) = 2 + 2 + 2 + 4 = 10. Therefore $\gamma_{du}(G)$ > $\gamma_{du}(G - xy)$.

Conclusion:

Many researchers are concentrating various distance concepts in graphs. We introduced d^d – disance in graps. In this paper we disccuss about d^d-distance in join two graphs.

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