CHARACTERISATION OF ORDERED SEMIRINGS BY ORDERED IDEALS AND ORDERED h- IDEALS

Muddsar Yaseen Malik[1*,] D. Sivakumar[2]

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Abstract

The main aim of this paper is to study ordered regular semirings and ordered h regular semirings by the characteristics of ordered ideals and ordered h ideals it has been shown that each h-regular ordered semiring is ordered h-regular semiring but the converse is not followed. It has also been shown that each regular ordered semiring is ordered h-regular semiring but the converse is not followed. Main and important results relating to operator closure and h-regular ordered semirings are given.

Keywords:- regular ordered semirings, h-regular ordered semirings , ordered h-regular semirings and ordered h-ideals etc.

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[1*]Ph.D Research Scholar,Department of Mathematics,Annamalai University, Annamalai Nagar,Chidambaram 608002,Tamilnadu,India. (E-mail:muddsarmalik807@gmail.com) ORCID: 0000-0001-9421-8470
2Professor, Department of Mathematics,Annamalai University, Annamalai Nagar,Chidambaram-608002,Tamilnadu,India. (E-mail:sivakumarmaths1965@gmail.com) ORCID: 0000-0002-1396-7849

***Correspondence Author:-** Muddsar Yaseen Malik
[*]Ph.D Research Scholar,Department of Mathematics,Annamalai University, Annamalai Nagar,Chidambaram 608002,Tamilnadu,India. (E-mail:muddsarmalik807@gmail.com) ORCID: 0000-0001-9421-8470

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Introduction

In 1935, Von Neumann proposed the concept of regularity in rings and demonstrated that if the semigroup $(S, \cdot)$ is regular, then the ring $(S, +, \cdot)$ is likewise regular. In 1951,[2] Bourne demonstrated that a semiring $(S, +, \cdot)$ is also regularif $y \in S \exists a, b \in S$, such that $y + yay = yby$. Ideals are essential in the structure theory of semirings [3].In 2011, the term "ordered semiring" was defined by [5] as a semiring with a partially ordered set $(S, \leq)$ such that $\leq$ is consistent partial order relation with the operations $+$ and $\cdot$ of S.The idea of several left and right ordered ideals is defined in this essay. We have offered ordered ideals in a variety of ordered semirings including regular ordered semiring , h-regular ordered semiring, and ordered h-regular semiring. h-ideal theory was applied by Jianming Zhan and Xueling M A in [5].We expanded the idea of h-ideals in ordered semirings and their types and extended concept of h-ideals to demonstrate related results in different cases.

Preliminaries:

Note: When we say S is semiring than it is understood we are talking about $(S, +, \cdot)$.

Definition:[3,7,15] A semiring $(S, +, \cdot)$ is structure having two bilateral operations $+$ and $\cdot$ such that additive reduct $(S, +)$ and multiplicative reduct $(S, \cdot)$ are semigroups and the distributive laws hold:
that is $u(v + z) = uv + uz$ and $(u + v)z = uz + vz \ \forall u, v, z \in S$.

Examples: $(N, +, \cdot)$ and $(W, +, \cdot)$ are both semirings.

Definition: [3,15] Suppose $I \neq \varphi$ and I be subset of $(S, +, \cdot)$ than I is left ideal or right ideal if following properties are satisfied.
(1) $a + b \in I \ \forall a, b \in I$.
(2) $SI \subseteq I$ or $IS \subseteq I$.
(3) If I is left and right ideal than I is an ideal of semiring S or two sided ideal of S.

Ordered Semiring

Note: Whenever we say S is an ordered semiring that means we are talking about $(S, +, \cdot, \leq)$.

Definition:[7,15] An ordered semiring is algebraic system $(S, +, \cdot, \leq)$ such that $(S, +, \cdot)$ is a semiring , $(S, \leq)$ is a partially ordered set and the relation $\leq$ is compatible to the operations $+$ and $\cdot$ on S.
Note:- $\leq$ is just a symbol which represents partial order relation on set.
Example: Let us take the semiring $(N, +, \cdot)$, where N is the set of $+ve$ integers. Defining the relation R on N by $p R q \Leftrightarrow q \leq p \ \forall p, q \in N$.
Then (N, R) is a poset (partially ordered set) so $(N, +, \cdot, R)$ is an ordered semiring.

Ordered h-ideals in ordered semiring

Definition : [15] Let H , M be the non-void subsets of an ordered semiring S then $(H]$ is defined as $(H] = \{g \in S : g \leq h\}$ for some $h \in H$ and $HM = \{hm : h \in H, m \in M\}$.

Definition: [7] Let $H \neq \varphi, H \subseteq S$ where S is ordered semiring, then the h-closure of H is denoted by $\overline{H}$ and is defined as $\overline{H} = \{g \in S, \exists x_1, x_2 \in H : g + x_1 + h \leq x_2 + h, h \in H\}$.

Definition : [7,15] Suppose S is ordered semiring, $H \neq \varphi$ and $H \subseteq S$ satisfying the given conditions:
(1) H is left ideal or right ideal of S.
(2) If $s \leq t$ for some $t \in H$, then $s \in H$.
Then H is a left ordered ideal or right ordered ideal. If H is both left ordered ideal and right ordered ideal of S than H is ordered ideal of S.

Example: Assume the an ordered semiring $(N, +, \cdot)$, where N is the set of counting numbers. Defining the relation ρ on N by $s \, \rho \, h \iff s \geq h \, \forall \, s, h \in N$.
Therefore $(N, +, \cdot, \rho)$ is a poset (partially ordered set). So $(N, +, \cdot, \rho)$ is an ordered semiring and $(2N]$ is ordered ideal in $(N, +, \cdot, \rho)$.

Definition: [7,4] Suppose H is a non-empty subset of semiring S then H is a left ordered h-ideal of S if the below conditions are satisfied:
(1) H is a left ordered ideal of S.
(2) If $e + a + t = b + t$ for some $a, b \in H$, $t \in H$ then $e \in H$.
In the same manner, right ordered h-ideal is defined if H is both a left ordered h-ideal and a right ordered h-ideal of S then H is referred as ordered h-ideal of S.
Definition:[21] Suppose S is an ordered semiring if $\forall \, x \in S \, \exists$ some $t \in S$ such that $x \leq xtx$ than S is said to be regular ordered semiring.

Definition: [7, 21] Let S be an ordered semiring then $a \in S$ is referred to be an ordered h-regular if $a \in \overline{(aSa]}$. If every member of S is ordered h-regular,then S is an ordered h-regular semiring.

Example: Assume the an ordered semiring $(N, +, \cdot)$, where N is the set of counting numbers. Defining the relation ρ on N by $s \, \rho \, h \iff s \geq h \, \forall \, s, h \in N$.
Therefore $(N, +, \cdot, \rho)$ is a poset (partially ordered set). So $(N, +, \cdot, \rho)$is an ordered semiring. Also having the property
$s + shs + t \leq shs + t \quad \forall \, s, h \in N, t \in N$ therefore $(N, +, \cdot, \rho)$ is an ordered h-regular semiring. After the next definition example has been shown which is ordered h-regular semiring but not h-regular ordered semiring.

Definition :[7] Suppose S is an ordered semiring if for every $a \in S \; \exists, e, h, c \in S$ such that $a + aea + c \leq aha + c$.Then S is h-regular ordered semiring.

Example: Given below example shows that D is ordered h-regular but not h –$r$egular ordered.
Let $D = \{d_1, d_2, d_3\}$. Define binary operation $+$ and $\cdot$ on D as follow:

$+$	d_1	d_2	d_3
d_1	d_1	d_1	d_1
d_2	d_1	d_2	d_3
d_3	d_1	d_3	d_3

$\cdot$	d_1	d_2	d_3
d_1	d_2	d_2	d_2
d_2	d_2	d_2	d_2
d_3	d_2	d_2	d_2

The relation is as $\leq \, = \{(d_1, d_1), (d_2, d_2), (d_3, d_3), (d_1, d_2), (d_1, d_3), (d_2, d_3)\}$
Than $(D, +, \cdot, \leq)$ is an ordered semiring.
Also $\forall \, a \in D$ following properties hold.
(1) $a + d_1 + c \leq d_2 + c, c \in D$ .Where a and c are arbitrarily choosen.
(2) $d_1, d_2 \in (aDa]$ i.e. $d_1 \leq ad_ia$ and $d_2 \leq ad_ia$, for some $ad_ia \in aDa$.Therefore D is an ordered h-regular semiring.Where d_i is some elements of D.
On the other side $d_3 + d_3ad_3 + d_2 \leq d_3cd_3 + d_2$ doesn't have solution so D is not an h-regular ordered semiring.

Theorem : Assume S is ordered semiring and let H and M are non-void subsets of S then

(1) $(\overline{H}] \subseteq \overline{(H]}$
(2) If $H \subseteq M$ then $\overline{H} \subseteq \overline{M}$
(3) $\overline{(H]}\, M \subseteq \overline{(HM]}$ and $H\overline{(M]} \subseteq \overline{(HM]}$

Proof: (1) Let $x \in (\overline{H}]$ .Then $\exists\ h \in \overline{H}$ such that $x \leq h$. Since $h \in \overline{H}$,then there exist $a, b \in H$ such that $h + a + k \leq b + k, k \in H$.
It follows that $x + a + k \leq h + a + k \leq b + k$.
Since $H \subseteq (H]$, $a, b \in (H], k \in (H]$, $x \in \overline{(H]}$ *i.e* $(\overline{H}] \subseteq \overline{(H]}$

(2) Assume $H \subseteq M$ and let $x \in \overline{H}$ than there exist $a, b \in H$ such that $x + a + k \leq b + k, k \in H$
By the supposition, we get $a, b, k \in M \Rightarrow x \in \overline{M}$, so $\overline{H} \subseteq \overline{M}$
(3) Let $x \in \overline{(H]}$ and $w \in M$.
So, there exist $u, v \in (H]$ such that $x + u + s \leq v + s, s \in (H]$.
So $xw + uw + sw \leq vw + sw$. Since $u, v, s \in (H]$,$u \leq a$ and $v \leq c$ and $s \leq d$ for some $a, c, d \in H$.
So $uw \leq aw \in HM$ and $vw \leq cw \in HM$ and $sw \leq dw \in HM$.
$\Rightarrow xw \in \overline{(HM]}$. So $\overline{(H]}\, M \subseteq \overline{(HM]}$.
In the same way we get $H\overline{(M]} \subseteq \overline{(HM]}$.

Theorem: Suppose H and M are non-empty subsets of ordered semiring S
with $H + H \subseteq H$ and $M + M \subseteq M$.
(1) $H \subseteq (H] \subseteq \overline{H} \subseteq \overline{(H]}$
(2) $H + M \subseteq \overline{H} + \overline{M} \subseteq \overline{H + M}$
(3) $\overline{(H]} + \overline{(M]} \subseteq \overline{\overline{(H]} + \overline{(M]}}$
(4) $\overline{(H]} + \overline{(M]} \subseteq \overline{(H + M]}$

Proof: **(1)** Clearly $H \subseteq (H]$
Let $g \in (H]$ so by definition of operator ( ] $\exists\ p \in H$ such that $g \leq p$
This implies $g + p + p \leq p + p + p$ implies $g \in \overline{H} \Rightarrow (H] \subseteq \overline{H}$.
Since $H \subseteq (H] \Rightarrow \overline{H} \subseteq \overline{(H]}$.
(2) From part (1) we have $H \subseteq \overline{H}$ and $M \subseteq \overline{M}$ implies $H + M \subseteq \overline{H} + \overline{M}$
Now we show that $\overline{H} + \overline{M} \subseteq \overline{H + M}$

Suppose $g \in \overline{H} + \overline{M}$ so there exist $h \in \overline{H}$ and $m \in \overline{M}$ such that $g = h + m$.
Since $h \in \overline{H}$ and $m \in \overline{M}$ so by definition of h-closure $\exists\ p, q \in H$ and $r, s \in M$ such that $h + p + t \leq q + t$ where $t \in H$ and $m + r + u \leq s + u$ where $u \in M$.

Which implies $g + p + t + r + u = h + m + p + t + r + u$
Implies $g + (p + r) + (t + u) \leq q + t + s + u$
Implies that $= q + s + t + u$
Which means $g + (p + r) + (t + u) \leq (q + s) + (t + u)$
As $(t + u) \in H + M$ then by definition of h-closure, we get $g \in \overline{H + M}$
Implies $\overline{H} + \overline{M} \subseteq \overline{H + M}$.

(3) Suppose $g \in \overline{(H]} + \overline{(M]}$, then there exist $p_1 \in \overline{(H]}, q_1 \in \overline{(M]}$,
such that $g = p_1 + q_1$
Now $g + (p_1 + q_1) + h = (p_1 + q_1) + (p_1 + q_1) + h$
Implies $g + (p_1 + q_1) + h = (p_1 + p_1) + (q_1 + q_1) + h$
Since $p_1 + p_1 \in \overline{(H]}$ and $q_1 + q_1 \in \overline{(M]}$, then by definition of h-closure,
we get $g \in \overline{\overline{(H]} + \overline{(M]}} \Rightarrow \overline{(H]} + \overline{(M]} \subseteq \overline{\overline{(H]} + \overline{(M]}}$.

(4) Suppose $g \in \overline{(H]} + \overline{(M]}$, so there exist $p_1 \in \overline{(H]}$, $q_1 \in \overline{(H]}$, such that $g = p_1 + q_1$.
Since $p_1 \in \overline{(H]}$ and $q_1 \in \overline{(H]}$
So by using definition of h-closure, $p, q \in (H]$ and $r, s \in (M]$ such that
$p_1 + p + t \leq q + t, t \in (H]$
and $q_1 + r + u \leq s + u$, $u \in (M]$.

Now $g + p + r + t + u = p_1 + q_1 + p + r + t + u.$
. $g + p + r + t + u \leq q + s + t + u.$
Since $(p + r), (q + s) \in (H + M]$,
$g + (p + r) + \overline{(t + u)} \leq \overline{q + s} + \overline{(t + u)}.$
This implies $g \in \overline{(H + M]} \Rightarrow \overline{(H]} + \overline{(M]} \subseteq \overline{(H + M]}.$

Theorem: Let S be an ordered semiring, then:
(1) Intersection of any family of left ordered h-ideals of S is a left ordered h-ideal.
(2) Intersection of any family of right ordered h-ideals of S is a right ordered h-ideal.
(3) Intersection of any family of ordered h-ideals of S is an ordered h-ideal.

Proof: (1) Let A_n be left ordered h-ideal of S forall $n \in I$, as $\bigcap_{n \in I} A_n \neq \phi$
Since A_n is a left ordered h-ideal, we get A_n is a left ordered ideal for all $n \in I$.
Then $\bigcap_{n \in I} A_n$ is left ordered ideal.
Consider there exist $g \in S$ and $r_1, r_2 \in \bigcap_{n \in I} A_n$, $h \in \bigcap_{n \in I} A_n$
such that $g + r_1 + h = r_2 + h.$
Since $\bigcap_{n \in I} A_n \subseteq A_n$ for all $n \in I$ we get, $r_1, r_2, h \in A_n.$
Since A_n is a left ordered h-ideal and $r_1, r_2 \in A_n, g + r_1 + h = r_2 + h, h \in A_n \ \forall n \in I$
So by using definition of left ordered h-ideal, we get $g \in A_n \ \forall n \in I$
implies $g \in \bigcap_{n \in I} A_n$
Therefore $r_1, r_2 \in \bigcap_{n \in I} A_n, g + r_1 + h = r_2 + h, h \in \bigcap_{n \in I} A_n$
implies $g \in \bigcap_{n \in I} An.$
By definition of left ordered h-ideal, we get $\bigcap_{n \in I} A_n$ is a left ordered h-ideal of S.
(2) Let A_n be right ordered h-ideal of S for all $n \in I$, as $\bigcap_{n \in I} A_n \neq \phi$
Since A_n is a right ordered h-ideal, we get A_n is a right ordered ideal for all $n \in I$.
$\Rightarrow \bigcap_{n \in I} A_n$ is right ordered ideal.
Consider there exist $g \in S$ and $r_1, r_2 \in \bigcap_{n \in I} A_n$, $h \in \bigcap_{n \in I} A_n$
such that $g + r_1 + h = r_2 + h.$
Since $\bigcap_{n \in I} A_n \subseteq A_n$ for all $n \in I$ we have $r_1, r_2, h \in A_n.$
As A_n is a right ordered h-ideal and $r_1, r_2 \in A_n, g + r_1 + h = r_2 + h, h \in A_n \ \forall n \in I$
So by using definition of right ordered h-ideal , we get $g \in A_n \ \forall n \in I$
implies $g \in \bigcap_{n \in I} A_n$
Therefore $r_1, r_2 \in \bigcap_{n \in I} A_n, g + r_1 + h = r_2 + h, h \in \bigcap_{n \in I} A_n$
implies $g \in \bigcap_{n \in I} A_n.$
By definition of right ordered h-ideal, we get $\bigcap_{n \in I} A_n$ is a right ordered h-ideal of S.

(**3**) From (1) and (2), we get $\bigcap_{n \in I} A_n$ is a left and right ordered h-ideal of S.Therefore, $\bigcap_{n \in I} A_n$ is an ordered h-ideal of S. Hence proved.

Theorem:-Suppose S be an ordered semiring and H be left ideal or right ideal or ideal, then following statements are equivalent: -
(1) H is left ordered h-ideal or right ordered h-ideal or ordered h-ideal of S.
(2) Assume $x \in S, x + r_1 + h \leq r_2 + h$ for some $r_1, r_2 \in H, h \in H$ then $x \in H.$
(3) $\overline{H} = H$
Proof: (1) implies **(2)** Assume H is a left ordered h-ideal.
Let $x \in S$ such that $x + r_1 + h \leq r_2 + h$ for some $r_1, r_2 \in H, h \in H$ so by definition of left ordered h-ideal, we obtain $x \in H$.
(2) $\Rightarrow$) **(3)**
Assume (2) is true. Let $x \in \overline{H}$ then there exist $r_1, r_2 \in H$ such that

$$x + r_1 + h \leq r_2 + h, h \in H.$$

By (2), we obtain $x \in H$. So $\overline{H} \subseteq H$.Since $H \subseteq \overline{H}$, therefore $\overline{H} = H$.
(3) implies **(1)** Assume that $\overline{H} = H$. Let $x \in S$ be such that $x + r_1 + h \leq r_2 + h$ for Some $r_1, r_2 \in H, h \in H$ then $x \in \overline{H}$. Since $\overline{H} = H$, so $x \in \overline{H} = H$ .Then $x \in H$.

Since $x + r_1 + h \leq r_2 + h$ for some $r_1, r_2 \in H, h \in H$ then $x \in H$, so by definition of left ordered h-ideal or right ordered h-ideal or ordered h-ideal, we obtain H is left ordered h-ideal or right ordered h-ideal or ordered h-ideal of S.

Theorem: Let S be an ordered semiring and H be a nonempty subset of S . Then following conditions hold.
(1) Assume H a left ideal, then $\overline{(H]}$ will be smallest left ordered h-ideal which contains H.
(2) Assume H a right ideal, then $\overline{(H]}$ will be smallest right ordered h-ideal which contains H.
(3) Assume H is an ideal, then $\overline{(H]}$ will be smallest ordered h-ideal which contains H.

Proof: Let H be a left ideal .We know $\overline{(H]}$ is closed under addition.
Let $x \in \overline{(H]}$ and $l \in H$, then by h-closure,there exist $r, w \in (H]$
such that $x + r + h \leq w + h,\ h \in (H]$
Hence $lx + lr + lh \leq lw + lh.$
So by using definition of ( ], we have $lh \in (H]$.
Since $(lr), (lw) \in (H], lx + (lr) + (lh) \leq (lw) + (lh), (lh) \in (H]$:
So by h-clousre, we obtain $lx \in \overline{(H]}$. Hence, $\overline{(H]}$ is a left ordered h-ideal.
It is known that $\overline{(H]}$ is a left ordered h-ideal which contains H.
Let J is a left ordered h-ideal which contains H. We will have $(H] \subseteq (J] = J$
then $\overline{(H]} \subseteq \overline{J} = J$
Hence, $\overline{(H]}$ will be smallest left ordered h-ideal which contains H.
(2) It is same as we did in part (1).
(3) with the help of part (1) and part (2), we have shown that $\overline{(H]}$ will be smallest left and right ordered h-ideal which contains H.
Hence $\overline{(H]}$ will be smallest ordered h-ideal which contains H.

Theorem: Let S be an ordered semiring. If S is an ordered h-regular than
$H \cap M = \overline{(HM]}$ $\forall$ right ordered h-ideals H, left ordered h-ideals M of S.
Proof: Suppose S is an ordered h-regular semiring and H is right ordered h-ideal , M is left ordered h-ideal of S.Then, we have $HM \subseteq H$ and $HM \subseteq M$.Thus $(HM] \subseteq (H] = H$ and $(HM] \subseteq (M] = M$.
$\Longrightarrow \overline{(HM]} \subseteq \overline{H} = H$ and $\overline{(HM]} \subseteq \overline{M} = M$.Thus $H \cap M \supseteq \overline{(HM]}$ ...(A)
Let $a \in H \cap M$. As S is an ordered h-regular $\exists\ b, c \in (aSa]$.
such that $a + b + o \leq c + o,\ o \in (aSa]$.
Since $b, c, o \in (aSa]$ then by definition of ( ], $\exists\ j, k, l \in S$ such that $b \leq aja$
$c \leq aka$,$o \leq ala$.
Since H is a right ordered h-ideal, M is a left ordered h-ideal, we have $aja, aka, ala \in HM$
Since $b \leq aja \in HM,\ c \leq aka \in HM, o \leq ala \in HM$, so by using definition of ( ].
we have $b, c, o \in (HM]$ so $a \in \overline{(HM]} \Longrightarrow H \cap M \subseteq \overline{(HM]}$ …(B)
From (A) and (B) $H \cap M = \overline{(HM]}$

Definition: Let S is an ordered semiring. Let $a \in S$ if $a \in \overline{(Sa^2]}$ then a is said to be left ordered h-regular and if $a \in \overline{(a^2S]}$ than a is said to be right ordered h-regular.If every element of S is left ordered h-regular (right ordered h-regular) than S is called left ordered h-regular (right ordered h-regular).

Theorem: Let S be left ordered h-regular semiring then
(1) for all left ordered h-ideal H of S , $\overline{(H^2]} = H$
(2) $J \cap H = \overline{(JH]}$ for all left ordered h-ideal H and for all ordered h-ideal J of S.
Proof: (1) Let H be left ordered h-ideal of S.Then, we obtain $\overline{(H^2]} \subseteq \overline{(H]} = H$
Assume $r \in H$. Since S is a left ordered h-regular, so $r \in \overline{(Sr^2]}$
We have $\overline{(Sr^2]} \subseteq \overline{(SH^2]} \subseteq \overline{(H^2]} \Longrightarrow r \in \overline{(H^2]}$ .Hence $H \subseteq \overline{(H^2]} \Longrightarrow \overline{(H^2]} = H$.
(2) Assume that H is left ordered h-ideal and J is ordered h-ideal of S.
So we get $\overline{(JH]} \subseteq \overline{(J]} = J$ and $\overline{(JH]} \subseteq \overline{(H]} = H$ .Therefore, $\overline{(JH]} \subseteq J \cap H$. … $*$
Suppose $a \in J \cap H$ since S is left ordered h-regular.
Which implies $a \in \overline{(Sa^2]} \subseteq \overline{(SJH]} \subseteq \overline{(JH]}$ implies $a \in \overline{(JH]}$.
Hence, $J \cap H \subseteq \overline{(JH]}$. … $**$

Combining $**$ and $*$ we get
Thus $J \cap H = \overline{(JH]}$.

Conclusion

The concepts of the ord. h- ideals in semirings and their key characteristics were
discussed. The ordered h-ideal features have been used to describe the classes of semirings like ordered h-regular. The concepts of the ordered h-ideals can be applied to non-associative structures like those in (13 , 15 , 20). Additionally, in semiring theory, ordered h-ideals can be extended for fuzzifcation.

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