REDUCED TRANSITION PROBABILITY FOR $^{124-130}$XENON TRANSITIONAL NUCLEI WITH CUBIC TERMS FROM CASIMIR INVARIANT OPERATORS AND IBM-1

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Abstract: *Interacting Boson Model may give appropriate approach to intensify the resolution for mystery lying within the structural aspects of Xenon nuclei dealing with triaxiality. Previous researches have provided suggestions regarding triaxiality in Xenon. This study indicates reduced transition probabilities B(E2) of Xenon isotopes with even neutrons from N=68 to 74. The parameter values have been determined by formation of cubic terms by Casimir invariant operators. Three – Three boson interaction are formed by addition of cubic terms breaking O(6) symmetry of IBM Hamiltonian. Transition rate R for some low-lying quadrupole collective state is studied in comparative way with experimental data and even-even $^{124-130}$Xenon isotope is found at O(6) symmetry. Calculation results also strengthen the available experimental data.*

Keywords: *IBM (Interacting boson model), Casimir invariant operators, Cubic terms, $^{124-130}$Xenon isotopes.*

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INTRODUCTION

Researchers have been indulged in solving the problems related to nuclear structure of various elements since recent past decades. For that, they have developed various approaches like harmonic vibrator, axial rotors and gamma soft deformed nuclei. During 1975, two researchers among them invented a new approach to nuclear collective motion i.e. Interactive Boson Model. These researchers were Iachello and Arima whose IBM-1 represents an even-even nucleus as a system of N bosons capable to catchup two levels. Two Bosons s and d having L=0 and 1 respectively and energies of these bosons represented as

$\in = \in_d - \in_s$

Where $\in$ represents gross energies of both bosons

L is angular momentum and total number of bosons N is sum of both s and d bosons.

Interacting Boson Model 1 (IBM-1) and Interacting Boson Model 2 (IBM-2) both use nucleons but in IBM-2 nucleons are used distinctly distinguishing between neutrons and protons. IBM expresses nuclear structure of an element in three limits i.e. U(5), SU(3) and SO(6) while differentiating wave function of neutrons and protons. Investigating wave function of both nucleons involve probing the B(E2) i.e. reducing transition probability. Here, we are calculating the value of B(E2) of $^{124-130}$Xenon transition nuclei with cubic terms from Casimir invariant operators and IBM-1.

Theoretical calculation

The general Hamiltonian of Interacting boson model-1 is given by

$\hat{H} = E0 + \hat{H}1 + \hat{H}2 + \hat{H}3 + \ldots\ldots\ldots$ (1)

The separation energy is denoted by E0 and it is constant. The one body term is given by :

$\hat{H}1 = \in s\,[s^{\dagger}\tilde{s}]^{(0)} + \in d\,\sqrt{5}[d^{\dagger}\tilde{d}]^{(0)} = \in s\, s^{\dagger}\cdot\tilde{s} + \in d\, d^{\dagger}\cdot\tilde{d} \equiv \in s\,\hat{n}s + \in d\, n\, d$ (2)

The third term which is two body part:

$$\widehat{H}_2 = \sum_{J=0,2,4} \frac{1}{2}(2J+1)^{\frac{1}{2}} C_J [[d^\dagger \times d^\dagger]^{(J)} [\tilde{d} \times \tilde{d}]^{(J)}]_0^{(0)}$$
$$+ \frac{1}{2}\nu_2 [[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} + [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]_0^{(0)}$$
$$+ \frac{1}{2}\nu_0 [[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]_0^{(0)}$$
$$+ u_2 [[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)}]_0^{(2)} + \frac{1}{2} u_0 [[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]_0^{(0)}$$

The IBM Hamiltonian of one body interaction contain two terms whereas two body interaction have seven terms. The total number of bosons is constant in this Hamiltonian and fourth term represents three body interaction.

$$\widehat{H}_3 = \frac{1}{2}\sum_{L=0,2,3,4,6} P_L \sqrt{2L+1} \{[[d^\dagger \times d^\dagger]^{(L_0)} \times d^\dagger]^{(L)} \times [[\tilde{d} \times \tilde{d}]^{(L_0)} \times \tilde{d}]^{(L)}\}^{(0)}$$
$$+ \frac{1}{2}\sum_{L=0,2,4} A_L \sqrt{2L+1} \left(\left\{ \left[[d^\dagger \times d^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times \left[[\tilde{d} \times \tilde{d}]^{(L_0)} \times \tilde{s} \right]^{(L)} \right\}^{(0)} \right.$$
$$\left. + \left\{ \left[s^\dagger \times [d^\dagger \times d^\dagger]^{(L_0)} \right]^{(L)} \times \left[\tilde{s} \times [\tilde{d} \times \tilde{d}]^{(L_0)} \right]^{(L)} \right\}^{(0)} \right)$$
$$+ a_2 \left(\left\{ \left[[d^\dagger \times d^\dagger]^{(L_0)} \times d^\dagger \right]^{(L)} \times \left[[\tilde{d} \times \tilde{d}]^{(L_0)} \times \tilde{s} \right]^{(L)} \right\}^{(0)} \right.$$
$$\left. + \left\{ \left[s^\dagger \times [d^\dagger \times d^\dagger]^{(L_0)} \right]^{(L)} \times \left[\tilde{d} \times [\tilde{d} \times \tilde{d}]^{(L_0)} \right]^{(L)} \right\}^{(0)} \right)$$
$$+ \frac{1}{2} a_0 \left(\left\{ \left[[d^\dagger \times d^\dagger]^{(L_0)} \times d^\dagger \right]^{(L)} \times [[\tilde{s} \times \tilde{s}]^{(L_0)} \times \tilde{s}]^{(L)} \right\}^{(0)} \right.$$
$$\left. + \left\{ \left[[s^\dagger \times s^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times \left[\tilde{d} \times [\tilde{d} \times \tilde{d}]^{(L_0)} \right]^{(L)} \right\}^{(0)} \right)$$
$$+ f_0 \left(\left\{ \left[[d^\dagger + d^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times \left[[\tilde{d} \times \tilde{d}]^{(L_0)} \times \tilde{s} \right]^{(L)} \right\}^{(0)} \right)$$
$$+ \sum_{L=0,2,4} B_L \sqrt{2L+1} \left(\left\{ \left[[s^\dagger \times s^\dagger]^{(L_0)} \times d^\dagger \right]^{(L)} \times [[\tilde{s} \times \tilde{s}]^{(L_0)} \times \tilde{d}]^{(L)} \right\}^{(0)} \right)$$
$$+ \frac{1}{2} f_0 \left(\left\{ \left[[s^\dagger \times s^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times [[\tilde{s} \times \tilde{s}]^{(L_0)} \times \tilde{s}]^{(L)} \right\}^{(0)} \right)$$
$$+ t_0 \left(\left\{ \left[[s^\dagger \times s^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times [[\tilde{s} \times \tilde{s}]^{(L_0)} \times \tilde{d}]^{(L)} \right\}^{(0)} \right.$$
$$\left. + \left\{ \left[d^\dagger \times [s^\dagger + s^\dagger]^{(L_0)} \right]^{(L)} \times \left[\tilde{s} \times [\tilde{s} \times \tilde{s}]^{(L_0)} \right]^{(L)} \right\}^{(0)} \right)$$
$$+ t_2 \left(\left\{ \left[[d^\dagger \times d^\dagger]^{(L_0)} \times s^\dagger \right]^{(L)} \times [[\tilde{s} \times \tilde{s}]^{(L_0)} \times \tilde{d}]^{(L)} \right\}^{(0)} \right.$$
$$\left. + \left\{ \left[d^\dagger \times [s^\dagger \times s^\dagger]^{(L_0)} \right]^{(L)} \times \left[\tilde{s} \times [\tilde{d} \times \tilde{d}]^{(L_0)} \right]^{(L)} \right\}^{(0)} \right)$$

The Hamiltonian for IBM-1can be expressed as a linear combination of the U(6) and its subgroup linear and quadratic Casimir operator.

H= $a_1\hat{C}_1[U(5)] + a'_1\hat{C}_2[U(5)] + a_2\hat{C}_2[SU(3)] + a_3\hat{C}_2[SO(6)] + a_4\hat{C}_2[SO(5)] + a_5\hat{C}_2[SO(3)] + a_6\hat{C}_1[U(6)] + a'_6\hat{C}_2[U(6)] + a_7\hat{C}_1[U(6)]\hat{C}_1[U(5)] + f_1[\hat{C}_1[U(5)]]^3 + f_2\hat{C}_2[SO(5)]\hat{C}_1[U(5)] + f'_2\hat{C}_2[SO(3)]\hat{C}_1[U(5)] + f_3\hat{C}_2[U(6)]\hat{C}_1[U(6)] + f_4\hat{C}_1[U(6)]\hat{C}_2[U(5)] + f_5\hat{C}_2[SO(5)]\hat{C}_1[U(6)] + f_6\hat{C}_2[SO(3)]\hat{C}_1[U(6)] + f_7[\hat{C}_1[U(6)]]^3$

The Casimir operators of U(6) and its subgroup[21] in the chains are given :

$\hat{C}_1[U(6)] = N$, $\hat{C}_1[U(5)] = n_d$[8], $\hat{C}_2[U(5)] = n_d(n_d + 4)$, $\hat{C}_2[U(6)] = N(N + 5)$,

$\hat{C}_2[SO(6)] = 2[N(N + 4) - 4P^\dagger.P]$

$\hat{C}_2[SO(5)] = 4[\frac{1}{10} L.L + U.U]$

$\hat{C}_2[SO(3)] = 2[L.L]$

To calculate the B(E2) value the reduced matrix elements of T^{E2} (E2 transition operator) have the form [7]

$$T^{E2} = \alpha_2[d^\dagger s + s^\dagger d]^{(2)} + \beta_2[d^\dagger d]^{(2)}$$

Where α2 and β2 are two parameters, α2 represents effective boson charge and β2 is related to α2. (s, d) and (s † d†) represents annihilation and creation operators. Transition operators are associated with IBM computed collective states. B(E2) values in IBM-1 are given for the limit U(5)-O(6) [22]

$$(E2, L + 2 \rightarrow L) \downarrow = \frac{1}{4}\alpha_2^2\,(L + 2)(2N - 1)$$

Where N is the Boson number, L is the Nuclear translating state and α_2^2 represents square of effective charge. Its value is used to compute the transition 8^+ to 6^+, 6^+ to 4^+,4^+ to 2^+ and 2^+ to 0^+ [11]. B(E2) in units of e2b2 is associated to B(E2) in units of W. u (Weisskopf single particle transition) [23].

$$(E2)e^2b^2 = 5.94 \times 10^{-6}\, A^{4/3} \times B(E2)w.\,u$$

e represents the charge of electron and 'barn' is the unit of area.

1barn = 10^{-28} m^2

RESULT AND DISCUSSION

$^{124-130}$Xe isotopes shows immense possibilities for investigation of low lying E2 strengths in the transitional regions regarding nuclear shape i.e. deformed to spherical. There is table below in which transition level, boson number and downward reduced transition possibilities B(E2) for the ground state band whose value lies from 8^+ to 6^+, 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+ of $^{124-130}$Xe isotopes. Using known experimental B(E2)↓ from $2_1^+ \rightarrow 0_1^+$transition B(E2)↓ value of $4_1^+ \rightarrow 2_1^+$, $6_1^+ \rightarrow 4_1^+$ and $8_1^+ \rightarrow 6_1^+$ transition of Xe isotopes are computed using IBM-1.

By using transition experiment B(E2)↓ value $2_1^+ \rightarrow 0_1^+$, the reduced transition possibilities of $^{124-130}$Xe isotopes of $4_1^+ \rightarrow 2_1^+$, $6_1^+ \rightarrow 4_1^+$, $8_1^+ \rightarrow 6_1^+$ are computed with cubic terms from Casimir invariant operators and IBM-1. The computed results are compared with experimental data.

Isotopes	**N**	**Transitional level**	**B(E2) values of e^2b^2**	
			Experimental	**IBM-1**
^{124}Xe	8	$2^+ \rightarrow 0^+$	0.193	0.192
		$4^+ \rightarrow 2^+$	0.258	0.260
		$6^+ \rightarrow 4^+$	0.321	0.280
		$8^+ \rightarrow 6^+$	0.253	0.272
^{126}Xe	7	$2^+ \rightarrow 0^+$	0.123	0.154
		$4^+ \rightarrow 2^+$	0.211	0.205
		$6^+ \rightarrow 4^+$	0.264	0.216
		$8^+ \rightarrow 6^+$	0.281	0.203
^{128}Xe	6	$2^+ \rightarrow 0^+$	0.097	0.094
		$4^+ \rightarrow 2^+$	0.163	0.175
		$6^+ \rightarrow 4^+$	0.195	0.184
		$8^+ \rightarrow 6^+$	0.195	0.183

^{130}Xe	5	$2^+ \rightarrow 0^+$ $4^+ \rightarrow 2^+$ $6^+ \rightarrow 4^+$ $8^+ \rightarrow 6^+$	0.073 0.117 0.131 0.117	0.087 0.112 0.138 0.113

CONCLUSION

Reduced transition possibilities B(E2)↓ is an observable criteria to examine change in nuclear shape. So, we calculated the B(E2) values for $^{124-130}$Xe isotopes with cubic term from Casimir invariant operators and IBM-1. The calculated value is consistent with experimental results. The value of energy ratio R(4/2) of $^{124-130}$Xe isotopes show O(6) symmetry.

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