PRIME MEAN CORDIAL LABELING OF GRAPHS

Jeena. R.V[1] **Dr. S. Asha**[2]

[1] Research Scholar (Full time), Register Number: 21113112092015,
Research Department of Mathematics, Nesamony Memorial Christian College, Marthandam,
Kanniyakumari District, Tamil Nadu, India.
Affiliated to Manonmaniam Sundarnar University, Tirunelveli- 627012, Abishekapatti, Tamil Nadu, India.
Corresponding Email ID: vrjeena@gmail.com

[2]Assistant Professor, Research Department of Mathematics,
Nesamony Memorial Christian College, Marthandam, Kanniyakumari District,
Tamil Nadu, India, Affiliated to Manonmaniam Sundarnar University, Tirunelveli- 627012, Abishekapatti,
Tamil Nadu, India.

Corresponding Email ID: ashanmcc@gmail.com

Abstract
Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to be a prime mean cordial labeling if there exists an injective function $f: V(G) \to \{0,1,2,\dots,q\}$ such that the induced edge labeling $f^*: E(G) \to \{0,1\}$ defined by $f^*(uv) = \begin{cases} 1 & if \left\lceil \frac{f(u)+f(v)}{2} \right\rceil is\ odd \\ 0 & otherwise \end{cases}$ satisfying the condition that for every $v \in V(G)$ with $deg(v) \geq 1, S_v = \sum\{f^*(e = uv)/uv \in E(G)\}$ is 1 or prime and $|e_f(i) - e_f(j)| \leq 1, i, j \in \{0,1\}$ where $e_f(x)$ denotes the number of edges labeled with x. . A graph with prime mean cordial labeling is called prime mean cordial graph. In this paper prime mean cordiality of some graphs are discussed.

Key words: Prime, Mean, Cordial, Mean Cordial, Prime Mean Cordial, Brush graph, Fusing.

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Introduction

Graphs we consider here are simple, finite, connected and undirected. The vertex set and edge set of a graph G are $V(G)$ and $E(G)$ respectively. The concept of cordial labeling was introduced by Cahit in the year 1987. The concept of mean labeling was introduced by S. Somasundaram and R. Ponraj. The concept of prime mean labeling was introduced by K. Palani. Motivated from the above works, we introduced a new type of labeling called prime mean cordial labeling.

Definition :2.1.

Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to be a prime mean cordial labeling if there exists an injective function $f: V(G) \to \{0,1,2,\dots,q\}$ such that the induced edge labeling $f^*: E(G) \to \{0,1\}$ defined by $f^*(uv) = \begin{cases} 1 & if \left\lceil \frac{f(u)+f(v)}{2} \right\rceil is\ odd \\ 0 & otherwise \end{cases}$ satisfying the condition that for every $v \in V(G)$ with $deg(v) \geq 1, S_v = \sum\{f^*(e = uv)/uv \in E(G)\}$ is 1 or prime and $|e_f(i) - e_f(j)| \leq 1, i, j \in \{0,1\}$ where $e_f(x)$ denotes the number of edges labeled with x. A graph with prime mean cordial labeling is called prime mean cordial graph.

Example: 1.2 A graph that admits a prime mean cordial labeling is given below:

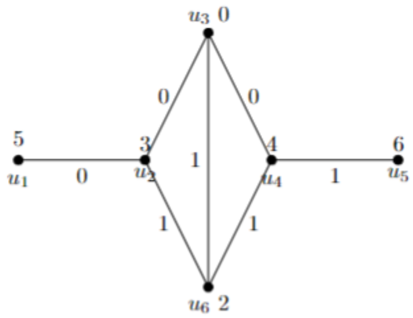


Figure 1

Here $e_f(0) = 3, e_f(1) = 4$ and $S_v = 1$ or prime.

Theorem: 1.3 *The Brush graph* $\boldsymbol{B}_n$ $(\boldsymbol{n} \geq \boldsymbol{3})$ *is prime mean cordial graph*

Proof:

Let $\boldsymbol{V}(\boldsymbol{B}_n) = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_n, \boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ and

$\boldsymbol{E}(\boldsymbol{B}_n) = \{\boldsymbol{u}_i\, \boldsymbol{u}_{i+1}: \boldsymbol{1} \leq \boldsymbol{i} \leq \boldsymbol{n} - \boldsymbol{1}\} \cup \{\boldsymbol{u}_i \boldsymbol{v}_i: \boldsymbol{1} \leq \boldsymbol{i} \leq \boldsymbol{n}\}$

Define $V(B_n) \rightarrow \{0,1,2, \dots q\}$ as follows:

Case (i): $n \equiv 0 \ (mod\ 3)$

Let $n = 3t$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t - 1$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t$

Then $e_f(0) = 3t - 1, e_f(1) = 3t$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case (ii): $n \equiv 1 \ (mod\ 3)$

Let $n = 3t + 1$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t + 1$

Then $e_f(0) = 3t, e_f(1) = 3t + 1$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case(iii): $n \equiv 2 \ (mod\ 3)$

Let $n = 3t + 2$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t + 1$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t+2$

Then $e_f(0) = 3t + 1, e_f(1) = 3t + 2$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

For $1 \leq i \leq n - 1, f^*(e_i) = 0$

For $1 \leq i \leq n, f^*(e_i') = 1$

$S_{u_1} = f^*(e_1) + f^*(e'_1) = 1$

For $2 \leq i \leq n-1, S_{u_i} = f^*(e_{i-1}) + f^*(e_i) + f^*(e'_i) = 1$

$S_{u_n} = f^*(e_{n-1}) + f^*(e'_n) = 1$

For every $v \in V(B_n), n \geq 3, S_v$ is equal to 1 or prime.

Therefore f is a prime mean cordial labeling.

Hence $B_n, n \geq 3$ is prime mean cordial graph

Example :1.4 Prime mean cordial labeling of the Brush graph $\boldsymbol{B}_6$ is given below:

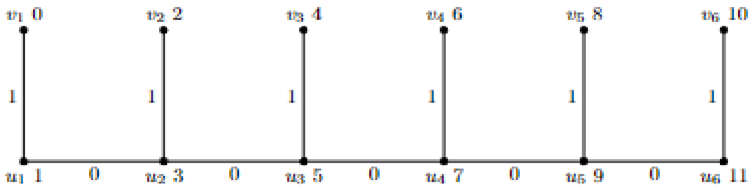


Figure 2

Theorem: 1.5 *The graph obtained by identifying (fusing) any two vertices in a brush graph B_n is a prime mean cordial graph.*

Proof:

Let $\boldsymbol{V}(\boldsymbol{B}_n) = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots \boldsymbol{u}_n, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots ., \boldsymbol{v}_n\}$

$E(B_n) = \{u_i u_{i+1}: 1 \leq i \leq n-1\} \cup \{u_i, v_i: 1 \leq i \leq n\}$

Let G_k be the graph obtained by fusing any two vertices in B_n. Here $|V(G_k)| = 2n - 1$.

Define $V(B_n) \rightarrow \{0,1,2, \ldots q\}$ as follows:

Case (i): $n \equiv 0 \ (mod\ 3)$

Let $n = 3t$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t-2$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t\text{-}1$

Then $f^*(u_n v_n) = 0$

Here $e_f(0) = 3t - 1, e_f(1) = 3t - 1$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case (ii): $n \equiv 1 \ (mod\ 3)$

Let $n = 3t + 1$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t-1$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t$

$f^*(u_n v_n) = 0$

Here $e_f(0) = 3t, e_f(1) = 3t$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case (iii): $n \equiv 2 \ (mod\ 3)$

Let $n = 3t + 2$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t$

$f^*(u_i v_i) = 1, 1 \leq i \leq 3t+1$

$f^*(u_n v_n) = 0$

Here $e_f(0) = 3t + 1, e_f(1) = 3t + 1$

Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

For $1 \leq i \leq n-2, f^*(e_i) = 0$
For $1 \leq i \leq n-1, f^*(e_i') = 1$
Then $f^*(e_n') = 0$
$Now\ S_{u_1} = f^*(e_1) + f^*(e_1') = 1$
For $2 \leq i \leq n-2, S_{u_i} = f^*(e_{i-1}) + f^*(e_i) + f^*(e_i') = 1$

$$S_{u_{n-1}} = f^*(e_{n-2}) + f^*(e_{n-1}') + f^*(e_n') = 1$$

Therefore for every $v \in V(B_n)$, S_v is equal to 1 or prime.
Therefore f is a prime mean cordial labeling.
Hence B_n is prime mean cordial graph.

Example: 1.6

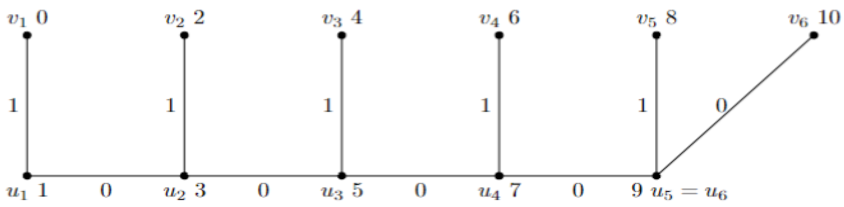


Figure 3. Fusion of $\boldsymbol{u_5}$ and $\boldsymbol{u_6}$ in $\boldsymbol{B_6}$

Theorem: 1.7 *A graph $G = (V, E)$ attaching $K_{1,2}$ to each pendant vertex of comb graph forms prime mean cordialgraph*

Proof:

Let $V\ ((Pn \odot)K_1 \odot K_{1,2}) = \{u_i, v_i, x_i : 1 \leq i \leq n\}$ and
$E\ ((Pn \odot K_1) \odot K_{1,2}) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i, v_i y_i, v_i x_i : 1 \leq i \leq n\}$
Define $V((Pn \odot K_1) \odot K_{1,2}) \to \{0,1,2, \dots q\}$ as follows:

Case (i): $n \equiv 0\ (mod\ 3)$

Let $n = 3t$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t-1$
$f^*(u_i v_i) = 1, 1 \leq i \leq 3t$
$f^*(v_i x_i) = 0, 1 \leq i \leq 3t$
$f^*(v_i y_i) = 1,\ 1 \leq i \leq 3t$
Here $e_f(0) = 6t - 1, e_f(1) = 6t$
Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case (ii): $n \equiv 1\ (mod\ 3)$

Let $n = 3t + 1$

Define $f^*(u_i u_{i+1}) = 1 \leq i \leq 3t$
$f^*(u_i v_i) = 1 \leq i \leq 3t + 1$
$f^*(v_i x_i) = 0,\ 1 \leq i \leq 3t+1$
$f^*(v_i y_i) = 1, 1 \leq i \leq 3t+1$
Here $e_f(0) = 6t + 1, e_f(1) = 6t + 2$
Therefore $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

Case (iii): $n \equiv 2 \ (mod\ 3)$

Let $n = 3t + 2$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t + 1$ $f^*(u_i v_i) = 1, 1 \leq i \leq 3t+2$

$f^*(v_i x_i) = 0, 1 \leq i \leq 3t$ $f^*(v_i y_i) = 1, 1 \leq i \leq 3t$

Here $e_f(0) = 6t + 1, e_f(1) = 6t + 2$

Therefore $e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1\}$

For $1 \leq i \leq n-1, f^*(e_i) = 0$

For $1 \leq i \leq n, f^*(e_i') = 1$

For $1 \leq i \leq n, f^*(e) = 0$

$S_{u_1} = f^*(e_1) + f^*(e_1') = 1$

For $2 \leq i \leq n-1, S_{u_i} = f^*(e_{i-1}) + f^*(e_i) + f^*(e_i') = 1$

$S_{u_n} = f^*(e_{n-1}) + f^*(e_n') = 1$

For $1 \leq i \leq n$, $S_{v_i} = f^*(e_1') + f^*(e_i'') + f^*(e_i''') = 2$

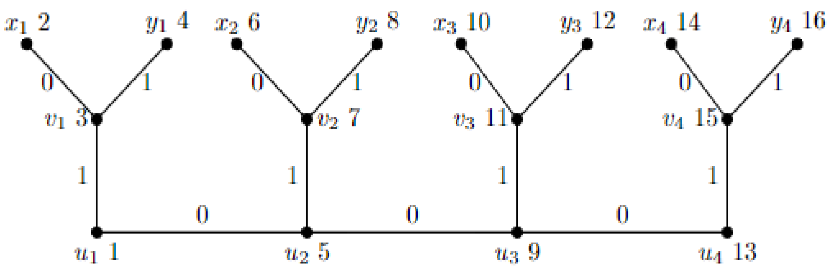
Therefore for every $v \in V\left((Pn \odot K_1) \odot K_{1,2}\right), S_v$ is equal to 1 or prime.

Therefore f is a prime mean cordial labeling.

Hence $(Pn \odot K_1) \odot K_{1,2})$ is a prime mean cordial graph.

Example: 1.8

Prime mean cordial labelling of $(P_4 \odot K_1) \odot K_{1,2}$



Theorem: 1.9. *The graph $TL_n \odot K_1$ is prime mean cordial.*

Proof:

Let TL_n be the triangular ladder.

Let $V(TL_n \odot K_1) = \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$

$E(TL_n \odot K_1) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i, u_i x_i, v_i y_i : 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} : 1 \leq i \leq n-1\}$

Define $V(TL_n \odot K_1) \rightarrow \{0,1,2, \ldots, q\}$ as follows:

Case (i): $n \equiv 0 \ (mod\ 3)$

Let $n = 3t$

Define $f^*(u_i u_{i+1}) = 0, 1 \leq i \leq 3t - 1$

$f^*(v_i v_{i+1}) = 1, 1 \leq i \leq 3t + 1$

$f^*(u_i x_i) = 1, 1 \leq i \leq 3t$

$f^*(u_iy_i) = 1, 1 \le i \le 3t$

$f^*(u_iv_i) = 0, 1 \le i \le 3t$

$f^*(u_iv_{i+1}) = 0, 1 \le i \le 3t - 1$

Then $e_f(0) = 9t - 2$ and $e_f(1) = 9t - 1$

Therefore $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0,1\}$

Case (ii): n $\equiv 1\ (mod\ 3)$

Let $n = 3t + 1$

Define $f^*(u_iu_{i+1}) = 0, 1 \le i \le 3t$

$f^*(v_iv_{i+1}) = 1, 1 \le i \le 3t$

$f^*(u_ix_i) = 1, 1 \le i \le 3t + 1$

$f^*(u_iy_i) = 1, 1 \le i \le 3t+1$

$f^*(u_iv_i) = 0, 1 \le i \le 3t$

$f^*(u_iv_{i+1}) = 0, 1 \le i \le 3t$

Then $e_f(0) = 9t + 1$ and $e_f(1) = 9t + 2$

Therefore $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0,1\}$

Case (iii): $n \equiv 2\ (mod\ 3)$

Let $n = 3t + 2$

Define $f^*(u_iu_{i+1}) = 0, 1 \le i \le 3t + 1$

$f^*(v_iv_{i+1}) = 1, 1 \le i \le 3t + 1$

$f^*(u_ix_i) = 1, 1 \le i \le 3t + 2$

$f^*(v_iy_i) = 1, 1 \le i \le 3t+2$

$f^*(u_iv_i) = 0, 1 \le i \le 3t + 2$

$f^*(u_iv_{i+1}) = 0, 1 \le i \le 3t + 1$

Then $e_f(0) = 9t + 4$ and $e_f(1) = 9t + 5$

Therefore $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0,1\}$

For $1 \le i \le n - 1, f^*(e_i) = 0$

For $n \le i \le 2n - 2, f^*(e_i) = 1$

For $1 \le i \le n, f^*(e_i') = 0$

For $1 \le i \le n, f^*(e_i'') = 0$

For $1 \le i \le 2n - 2, f^*(e_i''') = 0$

Then $S_{u_1} = f^*(e_1) + f^*(e_1') + f^*(e_1''') + f^*(e_2''') = 1$

For $2 \le i \le n - 1, S_{u_i} = f^*(e_{i-1}) + f^*(e_i) + f^*(e_i') + f^*(e_{2i-1}''') + f^*(e_{2i}''') = 1$

Then $S_{u_n} = f^*(e_{n-1}) + f^*(e_n') + f^*(e_{2n-1}''') = 1$

Now $S_{v_1} = f^*(e_n) + f^*(e_1'') + f^*(e_1''') = 1$

For $1 \le i \le n - 2, S_{v_{i+1}} = f^*(e_{3i}) + f^*(e_{3i+1}) + f^*(e_{i+2}''') + f^*(e_{i+1}'') + f^*(e_{i+1}''') = 1$

Then $S_{v_n} = f^*(e_{2n-2}) + f^*(e_n'') + f^*(e_{2n-2}''') + f^*(e_{2n-1}''') = 1$

Therefore for every $v \in V(TL_n \odot K_1), S_v$ is equal to 1 or prime.

Therefore f is a prime mean cordial labeling.
Hence $TL_n \odot K_1$ is a prime mean cordial graph.
Example:2.0

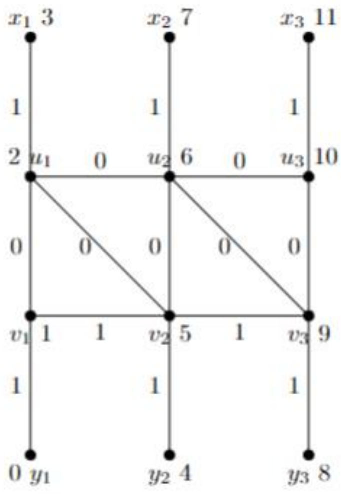


Figure 5. $\boldsymbol{TL_3 \odot K_1}$

Conclusion

In this paper we introduced the concept of prime mean cordial labeling and studied the prime mean cordial labeling behavior of few graphs.

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