LOW TEMPERATURE MODIFICATION TO FLUCTUATIONS OF CONSERVED CHARGES OF SYSTEMS UNDER STRONG INTERACTION

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Abstract: The study of exotic matter produced in high energy experiments is extremely important yet difficult due to involved symmetries and phase transitions. Lattice-QCD provides a theoretical framework for the same. QCD-inspired phenomenological models hold parallel importance. Here we try to build a suitable algorithm to investigate fluctuations of different conserved charges, which are supposedly viable signatures for in-medium transitions. To account for the same, we use medium-dependent pion masses, which are the lowest lying hadrons and thus expected to contribute most significantly to the thermodynamics of the system. We obtain encouraging results to indicate successful implementation and further examinations in this direction.

Keywords: *Conserved charges, fluctuations, strongly interacting systems, effective model.*

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1. Introduction:

Strongly interacting matter and exploration of its different properties are at the zenith of interest for researchers all over the World. Different relativistic heavy-ion collision experiments are targeted to study and investigate various such properties at finite temperatures and densities. On the theoretical front, the lattice QCD framework gives a first-hand benchmark estimate. However, the complex fermion determinant problem at finite chemical potential cripples its simplicity, though techniques have been developed to address this issue. Lattice QCD have achieved their continuum results with T_c as 155 MeV and 150 MeV from HotQCD and Wuppertal-Budapest (WuB) collaborations respectively [1,2]. It is interesting and worthwhile to simulate this study of strongly interacting matter using the realm of a QCD-inspired phenomenological model like the Nambu-Jona-Lasinio or NJL model which effectively incorporates crucial feature of chiral symmetry breaking and its restoration. However, the necessity of properly addressing the gluonic degrees of freedom led to amendment of the Polyakov or P-loop to the NJL model, thus giving rise to the Polyakov-Nambu-Jona—Lasinio or PNJL model. Thus, the PNJL model beautifully incorporates the chiral and deconfinement features within a single framework. Different thermodynamical studies within the mean-field framework have been found to have close similarity to lattice and experimental results. Recent lattice continuum results called for the need to reframe or reparametrize the model to bring about a quantitative agreement with recent lattice data [1,2]. A significant mismatch in the low temperature sector urged us to introduce different techniques to deal with the same. Realizing the importance of the hadronic degrees of freedom in the low temperature sector, we built a hybrid model [3] in conjunction with the Hadron Resonance Gas or HRG model where coupling between the two models PNJL and HRG [4] took place via a switching function. Though the method culminated fairly well, we were in search for a more natural framework. This led to develop another method, rather another new modified version of PNJL model, where instead of using any external element like the switching function, we bring about a more natural implementation by additively including the lowest lying hadronic contributions, that is of pions into the pressure. Here the role of switching function is played by the medium-modified pion masses. This investigation re-establishes the importance of pion as the sole heavy-weight contributor in this regime and also highlighting the minimal role of other add-ons. This also makes it essential to see how including only pion as hadron contribution, the physics extracted from PNJL model approaches the lattice results. Exploring the behavior of pressure, we study the nature of fluctuations of conserved charges which help to extract the corresponding physics here.

In the next section we describe the model framework followed by the theoretical estimation of pion mass depending on the medium properties and its entanglement with the thermodynamic potential of PNJL model. The results will be shown and discussed afterwards to finally conclude.

2. Model formalism

The PNJL model [5], as discussed, has been built by adding the P-loop to the NJL model [6, 7, 8]. While the NJL part takes care of the chiral contribution, the P-loop takes care of the confinement-deconfinement physics. There have been previous explorations using PNJL 2 and 2+1 flavors [7, 9-18]. Here, we use 2+1 flavor version of the model, incorporating upto six-quark type interactions. The thermodynamic potential is given by [5],

$$
\begin{aligned}
&\Omega(\varphi,\bar{\varphi},\sigma_f,T,\mu) \\
&= 2g_S\sum_{f=u,d,s}\sigma_f{}^2-\frac{g_D}{2}\sigma_u\sigma_d\sigma_s-6\sum_f\int_0^\Lambda\frac{d^3p}{(2\pi)^3}E_f\Theta(\Lambda-|\vec{p}|) \\
&-2T\sum_f\int_0^\infty\frac{d^3p}{(2\pi)^3}\,ln\left[1+3\left(\varphi+\bar{\varphi}\,e^{-(E_f-\mu_f)/T}\right)e^{-(E_f-\mu_f)/T}+e^{-3(E_f-\mu_f)/T}\right] \\
&-\ -2T\sum_f\int_0^\infty\frac{d^3p}{(2\pi)^3}\,ln\left[1+3\left(\bar{\varphi}+\varphi e^{-(E_f+\mu_f)/T}\right)e^{-(E_f+\mu_f)/T}+e^{-3(E_f+\mu_f)/T}\right]+\mathcal{U}'(\varphi,\bar{\varphi},T)
\end{aligned}
\tag{1}
$$

where the first five terms are from NJL model with slight modification due to Polyakov loop. The term with g_S is the four-quark interaction. The six-quark interaction term with coefficient g_D explicitly breaks the axial U(1) symmetry. Here, $E_f=\sqrt{p^2+M^2}$. The dynamic mass generation is given by,

$$M_f = m_f - 2g_S\sigma_f + \frac{g_D}{2}\sigma_{f+1}\sigma_{f+2} \qquad (2)$$

where, σ_f refers to the quark condensate.
While the third term in right hand side of Eq. (1) gives zero-point energy, the fourth and fifth terms give the finite temperature and chemical potential contributions of quarks and anti-quarks respectively. These terms arise from fermionic determinant in NJL model which is duly modified by fields corresponding to traces of Polyakov loop and conjugate given by,

$$\varphi = \frac{Tr_c L}{N_c} \text{ and } \bar{\varphi} = \frac{Tr_c L^\dagger}{N_c}$$

L is the Polyakov loop given by,

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{1/T} d\tau\, A_4(\vec{x},\tau)\right]$$

A_4 is temporal component of background gluonic field.
$\mathcal{U}'$ contains the effective potential for φ and $\bar{\varphi}$ [11, 19-22]. The form of $\mathcal{U}'$ we use is,

$$\frac{\mathcal{U}'(\varphi,\bar{\varphi},T)}{T^4} = \frac{\mathcal{U}(\varphi,\bar{\varphi},T)}{T^4} - \kappa \ln[J(\varphi,\bar{\varphi})] \qquad (3)$$

Here, $\mathcal{U}(\varphi,\bar{\varphi},T)$ is the Landau-Ginzburg type potential chosen in commensuration with global Z(3) symmetry [7] of Polyakov loop given by,

$$\frac{\mathcal{U}(\varphi,\bar{\varphi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\varphi}\varphi - \frac{b_3}{6}(\varphi^3+\bar{\varphi}^3) + \frac{b_4}{4}(\bar{\varphi}\varphi)^2 \qquad (4)$$

Due to reasons discussed in [5], the coefficient $b_2(T)$ is chosen to have temperature dependence of the form,

$$b_2(T) = a_0 + a_1 \exp\left(-a_2\frac{T}{T_0}\right)\frac{T_0}{T} \qquad (5)$$

Here, b_3 and b_4 are constants. J,

$$J[\varphi,\bar{\varphi}] = (1 - 6\bar{\varphi}\varphi + 4(\bar{\varphi}^3+\varphi^3) - 3(\bar{\varphi}\varphi)^2) \qquad (6)$$

is the Jacobian of transformation from Polyakov loop to its traces. κ is a dimensionless quantity, phenomenologically determined. Tables I and II below summarize the various parameters used in the present framework.

Table I : Parameters for the NJL model

m_u (MeV)	m_s (MeV)	Λ (MeV)	$g_s\Lambda^2$	$g_D\Lambda^5$
5.5	134.758	631.357	3.664	74.636

Table II : Parameters for the Polyakov loop

T_0(MeV)	a_0	a_1	a_2	b_3	b_4	κ
175	6.75	-9.0	0.25	0.805	7.555	0.1

3. Calculation of Pion mass

We aim to see the role of the most important hadronic contribution in this scenario, viz. that coming from pions and observe their effects on the thermodynamics as well as fluctuation coefficients in finite temperature and zero chemical potential regimes. Rather than using a switching function [3], here we incorporate a more natural mechanism to take care of the pions. We here use the mean-field formalism. Beyond mean-field NJL calculations can be found in [23-25].
At the outset, we obtain the pion mass solving the pole condition,

$$1 - 2G_M\,\Pi_M\left(\omega = m_M, \vec{k} = 0\right) = 0 \qquad (7)$$

G_M here is effective vertex for the given flavor combination. $\Pi_M(k^2)$ is one-loop polarization function for the corresponding mesonic channel. Using random phase approximation, we obtain,

$$\Pi_M(k^2) = \int \frac{d^4p}{(2\pi)^4} Tr[\Gamma_M S(p+\frac{k}{2})\Gamma_M S(p-\frac{k}{2})] \quad (8)$$

Here, S(p) is the quark propagator.
We find the mesonic contribution to the thermodynamic potential as [19],

$$\delta\Omega_M = -\nu_M T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-\frac{E_p}{T}}\right) \quad (9)$$

Here, ν_M corresponds to statistical weight factor of corresponding species.

$$E_p = \sqrt{\vec{p}^2 + m_{pole}{}^2(T)}$$

Here, m_{pole} is the mass of the meson, in this case corresponding to pionic mass, which is obtained by solving Eq. (7).

4. Results

Firstly, we obtain the mean fields by minimizing the thermodynamic potential with respect to σ, φ and $\bar{\varphi}$. Putting values of these fields into Eqs. (7) and (8), we obtain the variation of pion mass with temperature. The pressure is then obtained by substituting these mean field values in Eq. (1) and pole masses in Eq. (7). The thermodynamic potential for pion is further obtained from Eq. (9). Hence, we obtain Fig. 1 which gives us the variation of scaled pressure with temperature. We have also given the continuum extrapolated lattice QCD data alongside, from both HotQCD and WuB collaborations, to show a distinct comparison with our results.
In the high temperature regime, both the old and new versions of PNJL model show good agreement with Lattice outcomes. This is expected as the hadronic contribution should dilute away as the temperature increases and partonic degrees of freedom prevail in the system. In the low temperature sector however, the hadronic, here pionic degrees of freedom are dominant and hence this modified version of the model framework provides much better results.

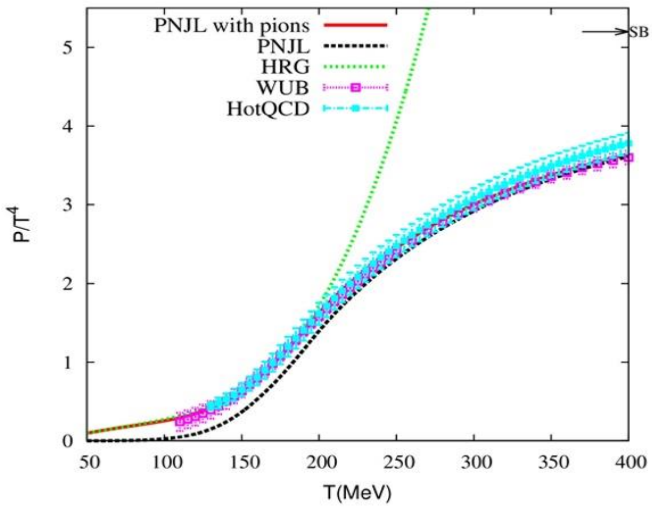

Fig 1: Pressure as a function of temperature

It might seem surprising though, since only pion has been considered for this study. But the answer lies in the result itself throwing light on the importance of pionic contribution in the low temperature sector. Pion itself suffices to give required explanation to the low temperature results. This also motivates to incorporate other degrees of freedom in this domain especially when strangeness comes into play and to study the finite chemical potential sector, that we plan to do elsewhere.
This work is a significant refurbishment to our previous results [3], where we explored various behavioral patterns. With special emphasis on the pionic contribution, we here investigate the behavior of various fluctuations of conserved charges like baryon number, electric charge and strangeness. The net number of different quark flavors are conserved in strong interactions.

However, these quark states being not physically observable, it becomes essential to relate flavor conservations to conservations of experimentally observed charges of hadrons like the baryon number B, electric charge Q and strangeness S. Study of fluctuations is very important for characterization of any system since they are expected to show specific signatures at phase transition or crossover thus acting as important signals. Fluctuation-dissipation theorem relates these fluctuations to the susceptibilities. $c_n^X(T)$ (where X=B, Q, S), the diagonal Taylor coefficients of nth order, can be expressed in terms of the fluctuations $\chi_n^X(T)$, of corresponding order as,

$$c_n^X(T) = \frac{1}{n!}\frac{\partial^n(P/T^4)}{\partial(\mu_X/T)^n} = \frac{T^{n-4}}{n!}\chi_n^X(T)$$

The expansion is carried out around $\mu_B=\mu_Q=\mu_S=0$, μ being the corresponding chemical potential. We first study the baryon number susceptibility shown in Fig.2.

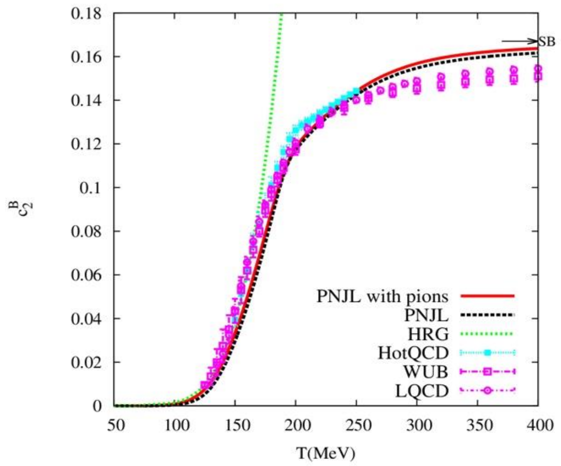

Fig 2 : Baryon number fluctuation as a function of temperature

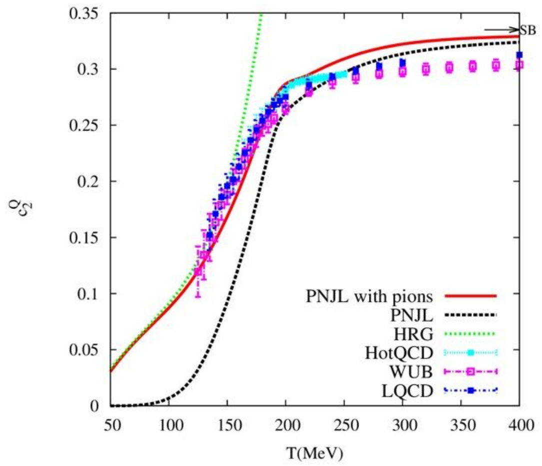

Fig 3 : Charge fluctuation as a function of temperature

Here, we see the variation of the baryon number susceptibility as a function of temperature. Model results are shown alongside that of lattice. The results for baryon number susceptibility obtained using both PNJL and this revised one match with that of lattice. However, it is clear that at low temperatures where the pionic contribution is expected to play the major role, this modified framework including pions distinctly shows a better match than PNJL, hence pointing the importance of including pions in our study as the most important contributor in the low temperature sector.

Fig.3 shows the behavior of electric charge susceptibility with temperature. This plot shows the stark difference between the old version of PNJL model and this modified model. The constituent quarks in the PNJL model correctly accounts for the baryon fluctuations, however in case of charge fluctuations, they cease to show a close match with lattice. This shows the importance of including pionic degrees of freedom below the cross-over temperature T_c. Due to the same reason, the HRG model appropriately throwing in contributions from light hadrons, closely matches lattice results for T<150 MeV.

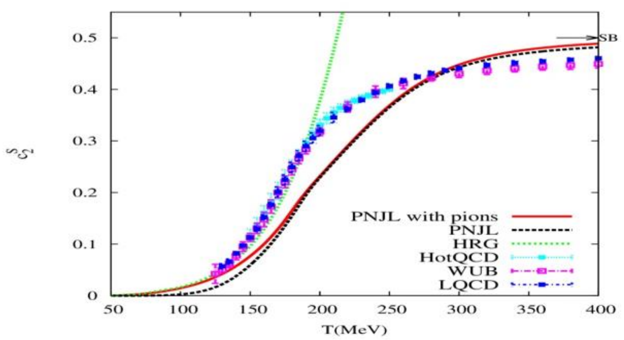

Fig 4: Strangeness fluctuation as a function of temperature

In Fig. 4 , we see the temperature variation of the strangeness susceptibility. Here, we see that the HRG results match with lattice upto a temperature ∼ 190 MeV, which is surprising compared to the other results, where HRG results match upto much lower temperature ∼ 160 MeV. A possible reason might be that HRG model is constructed from experimentally observed hadrons, while lattice framework might have significant contributions from additional strange hadron species [5]. The incorporation of lowest lying strange hadron like kaon, might further improve the result that we plan to explore elsewhere.

5. Conclusion

It is of utmost importance to carve a proper niche to comprehend the behavior of strongly interacting systems. Lattice QCD though being a robust ab-initio technique, the background of QCD-inspired phenomenological models often becomes easier to explore giving a smoother canvas for analysis of various observables. PNJL model, being one such model, attempted to explain various such quantities. However, availability of recent lattice continuum results made it essential to reframe the model and suitably address the low temperature regime. It is noteworthy that pion is the most important contributor in this regime and thus needed to be incorporated suitably to properly adhere to the scenario. Here, in this manuscript, we address this issue and study the fluctuation coefficients, which is of crucial importance to understand the physics involved. The incorporation of pion as degrees of freedom into the system effectively addresses the mismatch. The medium dependence of pion masses minimizes adjustment of any external parameter and acts as natural influencer for hadronic to partonic transitions. The fluctuations of conserved charges now show encouraging results, the crossover temperature being around 155MeV. This motivates enough for further robust investigations including many more hadronic degrees of freedom specially to account for strangeness and in the finite chemical potential domain, which is planned to execute as upcoming project.

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