Topological Indices of Mycielkian graph

Vishu Kumar M[1] Siva Kumar Pathuri[1] Silvia Leera Sequeira[2] and Veena K[3]

1. Department of Mathematics, School of Applied Sciences, REVA University, Bangalore-560 064, Karnataka, India.
2. Department of Mathematics, BMS College of Engineering, Bull Temple Road, Bangalore-19, Karnataka, India.
3. Department of Computer Science and Engineering, R L Jalappa Institute of Technology, Doddaballapur, Bangalore-561 203, Karnataka, India.

Abstract : There are many topological indices. Among the degree based topological indices, Randic index Zagreb indices, Banhatti indices, Atom bond connectivity index, Harmonic index, Geometric arithmetic index and Revan vertex degree index etc. For a given graph $G(V,E)$ of order *n* with vertex set $V=\{v_1,v_2,v_3,...,v_n\}$. The Mycielski graph of *G*, denoted by $M(G)$, is the graph with vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_iv_j^1 / v_iv_j\in E\}\cup\{v_i^1u / v_i^1\in V^1\}$. The vertex v_i^1 is called the copy of the vertex v_i and u is called the root of $M(G)$. In this paper we obtain Randic index, Zegreb index, Atom Bond Connectivity index, Harmonic index, Augmented Zagreb index, Sum Connectivity index, geometric arithmetic index and Revan index of Myceilskian graph.

MSC: 05C05, 05C07, 05C12, 05C35

Key words: Randic index, Zegreb index, Atom Bond Connectivity index, Harmonic index, Augmented Zagreb index, Sum Connectivity index, geometric arithmetic index and Revan index, Myceilskian graph.

1. Introduction

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex *v* is the number of vertices adjacent to *v*. $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree among the vertices of *G*. We refer [1] for undefined term and notation. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous such topological indices have been considered in theoretical chemistry and have some applications.

In chemical graph theory, study of characteristics of the chemical graph is important to understand the behaviour of the chemical combination,to understand the growth of the chemical combination and to make its comparison with other chemical combinations.

The first topological index in chemical graph theory is the Wiener index. In 1947, Wiener [2] introduced the topological index (Wiener index), while studying the boiling point of paraffin. This Wiener index has many applications in chemical graph theory [3, 4]. After Wiener index many topological indices has been investigated such as Randic index introduced by Randic, The first general Zagreb index was introduced by Li and Zhao [5]. Estrada et al. introduced the Atom-Bond Connectivity (ABC) index[6].

2. Preliminaries

Definition 2.1 For a given graph $G(V,E)$ of order *n* with vertex set $V=\{v_1,v_2,v_3,...,v_n\}$. The Mycielski graph of *G*, denoted by $M(G)$, is the graph with vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_iv_j^1 / v_iv_j\in E\}\cup\{v_i^1u / v_i^1\in V^1\}$. The vertex v_i^1 is called the copy of the vertex v_i and u is called the root of $M(G)$.

Definition 2.2 For a graph, the maximum degree denoted by $\Delta(G)$, is the vertex with greatest number of edges incident to it. The minimum degree denoted by $\delta(G)$, is the degree of the vertex with least number of edges incident to it.

Definition 2.3 The first and second Revan indices of a graph *G*, defined as $R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$ and $R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v)$.

Definition 2.4 The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ is defined as $M_1(G)=\sum_{v_i\in V(G)}(dv_i)^2$ and $M_2(G)=\sum_{uv\in E} d_G(u)d_G(v)$

Definition 2.5 Let $G(V,E)$ be a graph, $d_u(G)$ is the degree of vertex *u* of *G* and uv the edge connecting the vertices $u\,\&\,v$. Then atom bond connectivity index is defined as

$$ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_u(G)+d_v(G)-2}{d_u(G)d_v(G)}}$$

Definition 2.6 Let $G(V,E)$ be a graph, $d_u(G)$ is the degree of vertex *u* of *G* and uv the edge connecting the vertices $u\,\&\,v$. ThenHarmonic index defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_u(G)+d_v(G)}.$$

Definition 2.7 Let $G(V,E)$ be a graph, $d_u(G)$ is the degree of vertex *u* of *G* and uv the edge connecting the vertices $u\,\&\,v$. Then Augmented Zagreb index is defined as

$$AZI(G)=\sum_{uv\in E(G)}\left(\frac{d_u(G)d_v(G)}{d_u(G)+d_v(G)-2}\right)^3.$$

Definition 2.8 Let $G(V,E)$ be a graph, $d_u(G)$ is the degree of vertex *u* of *G* and uv the edge connecting the vertices $u\,\&\,v$. Then Sum connectivity index is defined as

$$SCI(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u(G)+d_v(G)}}.$$

Definition 2.9 Let $G(V,E)$ be a graph, $d_u(G)$ is the degree of vertex u of G and uv the edge connecting the vertices $u \& v$. Then Geometric arithmetic index is defined as $GA(G)=\sum_{uv\in E(G)} \frac{\sqrt{d_u(G)d_v(G)}}{\frac{1}{2}\left[d_u(G)+d_v(G)\right]}$.

3. Main Results

Theorem 3.1 Let $M(P_n)$ is the Mycielskian path graph then

1. $R_1(M(P_n)) = 7n^2 - 15n + 22$
2. $R_2(M(P_n)) = 3n^3 - 11n^2 + 26n - 20$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph by the algebraic method there are six types of edges on the degree of end vertices as follows

$E_{23} = \{uv \in E(G) / d_G(u) = 2 \& d_G(v) = 3\}$, $|E_{23}| = 2$

$E_{24} = \{uv \in E(G) / d_G(u) = 2 \& d_G(v) = 4\}$, $|E_{24}| = 4$

$E_{2n} = \{uv \in E(G) / d_G(u) = 2 \& d_G(v) = n\}$, $|E_{2n}| = 2$

$E_{3n} = \{uv \in E(G) / d_G(u) = 3 \& d_G(v) = n\}$, $|E_{3n}| = n-2$

$E_{43} = \{uv \in E(G) / d_G(u) = 4 \& d_G(v) = 3\}$, $|E_{43}| = 2n-6$

$E_{44} = \{uv \in E(G) / d_G(u) = 4 \& d_G(v) = 4\}$, $|E_{44}| = n-3$

Thus we have two types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G) = n+2$.

$RE_{n(n-1)} = \{uv \in E(G) / r_G(u) = n \& r_G(v) = n-1\}$, $|RE_{n(n-1)}| = 2$

$RE_{n(n-2)} = \{uv \in E(G) / r_G(u) = n \& r_G(v) = n-2\}$, $|RE_{n(n-2)}| = 4$

$RE_{2n} = \{uv \in E(G) / r_G(u) = 2 \& r_G(v) = n\}$, $|RE_{2n}| = 2$

$RE_{2(n-1)} = \{uv \in E(G) / r_G(u) = 2 \& r_G(v) = n-1\}$, $|RE_{2(n-1)}| = n-2$

$RE_{(n-2)(n-2)} = \{uv \in E(G) / r_G(u) = n-2 \& r_G(v) = n-2\}$, $|RE_{(n-2)(n-2)}| = n-3$

$RE_{(n-2)(n-1)} = \{uv \in E(G) / r_G(u) = n-2 \& r_G(v) = n-1\}$, $|RE_{(n-2)(n-1)}| = 2n-6$

1. To compute $R_1(M(P_n))$, we see that

$$R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$$
$$=\sum_{RE_{n(n-1)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{n(n-2)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{2n}}\left|r_G(u)+r_G(v)\right|+$$
$$\sum_{RE_{2(n-1)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{(n-2)(n-2)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{(n-2)(n-1)}}\left|r_G(u)+r_G(v)\right|$$
$$=2\big(n+(n-1)\big)+4(n+n-2)+2(2+n)+(n-2)\big(2+(n-1)\big)+$$
$$(n-3)(n-2+n-2)+(2n-6)(n-2+n-1)$$
$$=2(2n-1)+4(2n-2)+(4+2n)+(n-2)(n+1)+(n-3)(2n-4)+(2n-6)(2n-3)$$
$$=4n-2+8n-8+4+\left(2n+n^2-n-2\right)+\left(2n^2-10n+12\right)+\left(4n^2-18n+18\right)$$
$$R_1(G)=7n^2-15n+22$$

2. To compute $R_2(M(P_n))$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$
$$R_2(G)=\sum_{RE_{n(n-1)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{n(n-2)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{2n}}\left|r_G(u)r_G(v)\right|+$$
$$\sum_{RE_{2(n-1)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{(n-2)(n-2)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{(n-2)(n-1)}}\left|r_G(u)r_G(v)\right|$$
$$=2\big(n(n-1)\big)+4\big(n(n-2)\big)+(n-2)\big(2(n-1)\big)+(n-3)\left((n-2)^2\right)+$$
$$(2n-6)\big((n-2)(n-1)\big)+2(2n)$$
$$=2\left(n^2-n\right)+4\left(n^2-2n\right)+(n-2)(2n-2)+(n-3)\left(n^2-4n+4\right)+$$
$$(2n-6)\left(n^2-3n+2\right)$$
$$=2n^2-2n+4n^2-8n+\left(2n^2-6n+4\right)+\left(n^3-4n^2+4n-3n^2+12n-12\right)+$$
$$\left(2n^3-6n^2+4n-6n^2+18n-12\right)$$
$$R_2(G)=3n^3-11n^2+26n-20$$

∎

Theorem 3.2 Let $M(P_n)$ is the Mycielskian path graph then Zebgreb index

1. $M_1\left(M(P_n)\right)=n^2+25n-34$
2. $M_2\left(M(P_n)\right)=3n^2+38n-76$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\left\{v_i^1 / v_i\in V\right\}$ and edge set $E\cup\left\{v_iv_j^1 / v_iv_j\in E\right\}\cup\left\{v_i^1u / v_i^1\in V^1\right\}$. $\left|V\left(M(P_n)\right)\right|=2n+1$ and $E\left(M(P_n)\right)=4n-3$.

1. To compute $M_1(G)$, we see that

Sl.No	$d_v(G)$	Number of vertices
1.	2	4
2.	3	$n-2$
3.	4	$n-2$
4.	n	1

$$M_1(G)=\sum_v (d_v(G))^2$$

$$=4(2)^2+(n-2)(3)^2+(n-2)(4)^2+n^2$$

$$=4(4)+(9n-18)+(16n-32)+n^2$$

$$M_1(G)=n^2+25n-34$$

2. To compute $M_2(G)$, we see that

Sl.No.	$d_v(G)\&d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

$$M_2(G)=\sum_v d_u(G)d_v(G)$$

$$=2(2\times3)+4(2\times4)+2(2n)+(n-2)(3n)+(2n-6)(4\times3)+(n-3)(4\times4)$$

$$=2(6)+4(8)+2(2n)+(3n^2-6n)+(24n-72)+(16n-48)$$

$$=12+32+4n+3n^2-6n+24n-72+16n-48$$

$$M_2(G)=3n^2+38n-76$$

□

Theorem 3.3 Let $G=M(P_n)$ is the Mycielskian path graph then Harmonic index

$$H(G)=\frac{345n^3+2426n^2+3055n+846}{420n^2+2100n+2520}$$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_iv_j^1 / v_iv_j\in E\}\cup\{v_i^1u / v_i^1\in V^1\}$. $|V(M(P_n))|=2n+1$ and $|E(M(P_n))|=4n-3$.

Sl.No.	$d_v(G)\&d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_u(G)+d_v(G)}$$

$$=2\left[\frac{2}{2+3}\right]+4\left[\frac{2}{2+4}\right]+2\left[\frac{2}{2+n}\right]+(n-2)\left[\frac{2}{3+n}\right]+(2n-6)\left[\frac{2}{4+3}\right]+(n-3)\left[\frac{2}{4+4}\right]$$

$$=\frac{4}{5}+\frac{8}{6}+\frac{4}{2+n}+\frac{2(n-2)}{3+n}+\frac{2(2n-6)}{7}+\frac{2(n-3)}{8}$$

$$=\frac{4(336)+8(280)+2(2n-6)240+2(n-3)210}{1680}+\frac{4}{2+n}+\frac{2(n-2)}{3+n}$$

$$=\frac{1344-2240+960n-2880+420n-1260}{1680}+\frac{4}{2+n}+\frac{2(n-2)}{3+n}$$

$$=\frac{1380n-556}{1680}+\frac{4}{2+n}+\frac{(2n-4)}{3+n}$$

$$=\frac{(2+n)(3+n)(1380n-556)+4(1680)(3+n)+1680(2n-4)(2+n)}{1680(2+n)(3+n)}$$

$$=\frac{(n^2+5n+6)(1380n-556)+(20160+6720n)+(3360n^2-13440)}{1680n^2+8400n+10080}$$

$$=\frac{1380n^3+9704n^2+12220n+3384}{1680n^2+8400n+10080}$$

$$H(G)=\frac{345n^3+2426n^2+3055n+846}{420n^2+2100n+2520}$$

□

Theorem 3.4 Let $G=M(P_n)$ is the Mycielskian path graph then Augmented Zagreb index

$$AZI(G)=64+\frac{27n^4-54n^3}{n^3+3n^2+3n+1}+\frac{3456n-10368}{125}+\frac{512n-1536}{27}$$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1/v_i\in V\}$ and edge set $E\cup\{v_iv_j^1/v_iv_j\in E\}\cup\{v_i^1u/v_i^1\in V^1\}$. $\left|V(M(P_n))\right|=2n+1$ and $\left|E(M(P_n))\right|=4n-3$.

Sl.No.	$d_v(G) \& d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

To compute Augmented Zagreb index, we see that

$$AZI(G)=\sum_{uv\in E(G)}\left(\frac{d_u d_v}{d_u+d_v-2}\right)^3$$

$$=2\left(\frac{2\times3}{2+3-2}\right)^3+4\left(\frac{2\times4}{2+4-2}\right)^3+2\left(\frac{2n}{2+n-2}\right)^3+(n-2)\left(\frac{3n}{3+n-2}\right)^3$$

$$(2n-6)\left(\frac{4\times3}{4+3-2}\right)^3+(n-3)\left(\frac{4\times4}{4+4-2}\right)^3$$

$$=2\left(\frac{6}{3}\right)^3+4\left(\frac{8}{4}\right)^3+2\left(\frac{2n}{n}\right)^3+(n-2)\left(\frac{3n}{n+1}\right)^3+(2n-6)\left(\frac{12}{5}\right)^3+(n-3)\left(\frac{16}{6}\right)^3$$

$$=16+32+16+(n-2)\left(\frac{27n^3}{n^3+1+3n^2+3n}\right)+(2n-6)\left(\frac{1728}{125}\right)+(n-3)\frac{(512)}{27}$$

$$=64+\frac{27n^4-54n^3}{n^3+3n^2+3n+1}+\left(\frac{3456n-10368}{125}\right)+\left(\frac{512n-1536}{27}\right)$$

$$AZI(G)=64+\frac{27n^4-54n^3}{n^3+3n^2+3n+1}+\frac{3456n-10368}{125}+\frac{512n-1536}{27}$$

□

Theorem 3.5 Let $G=M(P_n)$ is the Mycielskian path graph then Randic index

$$R(G)=\frac{2}{\sqrt{6}}+\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2n}}+\frac{n-2}{\sqrt{3n}}+\frac{n-3}{\sqrt{3}}+\frac{n-3}{4}$$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_i v_j^1 / v_i v_j\in E\}\cup\{v_i^1 u / v_i^1\in V^1\}$. $|V(M(P_n))|=2n+1$ and $|E(M(P_n))|=4n-3$.

Sl.No.	$d_v(G) \& d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

To find Randic index, we see that

$$R(G)=\sum_{uv\in E}\frac{1}{\sqrt{d_u d_v}}$$
$$=2\left(\frac{1}{\sqrt{2\times 3}}\right)+4\left(\frac{1}{\sqrt{2\times 4}}\right)+2\left(\frac{1}{\sqrt{2n}}\right)+(n-2)\left(\frac{1}{\sqrt{3n}}\right)+(2n-6)\left(\frac{1}{\sqrt{4\times 3}}\right)+(n-3)\left(\frac{1}{\sqrt{4\times 4}}\right)$$

$$R(G)=2\left(\frac{1}{\sqrt{6}}\right)+4\left(\frac{1}{\sqrt{8}}\right)+2\left(\frac{1}{\sqrt{2n}}\right)+(n-2)\left(\frac{1}{\sqrt{3n}}\right)+(2n-6)\left(\frac{1}{\sqrt{12}}\right)+(n-3)\left(\frac{1}{\sqrt{16}}\right)$$
$$=\frac{2}{\sqrt{6}}+\frac{4}{2\sqrt{2}}+\frac{2}{\sqrt{2n}}+\frac{n-2}{\sqrt{3n}}+\frac{2n-6}{2\sqrt{3}}+\frac{n-3}{4}$$
$$R(G)=\frac{2}{\sqrt{6}}+\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2n}}+\frac{n-2}{\sqrt{3n}}+\frac{n-3}{\sqrt{3}}+\frac{n-3}{4}$$

□

Theorem 3.6 Let $G=M(P_n)$ is the Mycielskian path graph then Sum connectivity index

$$SCI(G)=\frac{2}{\sqrt{5}}+\frac{4}{\sqrt{6}}+\frac{2}{\sqrt{2+n}}+\frac{n-2}{\sqrt{3+n}}+\frac{2n-6}{\sqrt{7}}+\frac{n-3}{\sqrt{8}}$$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_i v_j^1 / v_i v_j\in E\}\cup\{v_i^1 u / v_i^1\in V^1\}$. $\left|V(M(P_n))\right|=2n+1$ and $\left|E(M(P_n))\right|=4n-3$.

Sl.No.	$d_v(G) \& d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

To find Sum connectivity index, we see that

$$SCI(G)=\sum_{uv\in E}\frac{1}{\sqrt{d_u+d_v}}$$

$$=2\left(\frac{1}{\sqrt{2+3}}\right)+4\left(\frac{1}{\sqrt{2+4}}\right)+2\left(\frac{1}{\sqrt{2+n}}\right)+(n-2)\left(\frac{1}{\sqrt{n+3}}\right)+(2n-6)\left(\frac{1}{\sqrt{4+3}}\right)$$
$$(n-3)\left(\frac{1}{\sqrt{4+4}}\right)$$

$$SCI(G)=\frac{2}{\sqrt{5}}+\frac{4}{\sqrt{6}}+\frac{2}{\sqrt{2+n}}+\frac{n-2}{\sqrt{3+n}}+\frac{2n-6}{\sqrt{7}}+\frac{n-3}{\sqrt{8}}$$

□

Theorem 3.7 Let $G=M(P_n)$ is the Mycielskian path graph then Atom Bond Connectivity index

$$ABC(G)=8\sqrt{\frac{1}{2}}+(n-2)\sqrt{\frac{n+1}{3n}}+(n-3)\sqrt{\frac{5}{3}}+(n-3)\sqrt{\frac{3}{8}}$$

Proof. Let G be the graph $M(P_n)$. In the Mycielskian graph the vertex set $V\cup V^1\cup\{u\}$, where $V^1=\{v_i^1 / v_i\in V\}$ and edge set $E\cup\{v_iv_j^1 / v_iv_j\in E\}\cup\{v_i^1u / v_i^1\in V^1\}$. $|V(M(P_n))|=2n+1$ and $|E(M(P_n))|=4n-3$.

Sl.No.	$d_v(G)\&d_u(G)$	Number of edges
1.	2 & 3	2
2.	2 & 4	4
3.	2 &n	2
4.	3 &n	$n-2$
5.	4 & 3	$2n-6$
6.	4 & 4	$n-3$

To find Atom Bond Connectivity index, we see that

$$ABC(G)=\sum_{uv\in E}\sqrt{\frac{d_u+d_v-2}{d_ud_v}}$$

$$=2\left(\sqrt{\frac{2+3-2}{2\times3}}\right)+4\left(\sqrt{\frac{2+4-2}{2\times4}}\right)+2\left(\sqrt{\frac{2+n-2}{2n}}\right)+(n-2)\left(\sqrt{\frac{3+n-2}{3n}}\right)+$$
$$(2n-6)\left(\sqrt{\frac{4+3-2}{4\times3}}\right)+(n-3)\left(\sqrt{\frac{4+4-2}{4\times4}}\right)$$

$$ABC(G)=2\left(\sqrt{\frac{3}{6}}\right)+4\left(\sqrt{\frac{4}{8}}\right)+2\left(\sqrt{\frac{n}{2n}}\right)+(n-2)\left(\sqrt{\frac{n+1}{3n}}\right)+(2n-6)\left(\sqrt{\frac{5}{12}}\right)+(n-3)\sqrt{\frac{6}{16}}$$

$$=2\sqrt{\frac{1}{2}}+4\sqrt{\frac{1}{2}}+2\sqrt{\frac{1}{2}}+(n-2)\sqrt{\frac{n+1}{3n}}+(n-3)\sqrt{\frac{5}{3}}+(n-3)\sqrt{\frac{3}{8}}$$

$$ABC(G)=8\sqrt{\frac{1}{2}}+(n-2)\sqrt{\frac{n+1}{3n}}+(n-3)\sqrt{\frac{5}{3}}+(n-3)\sqrt{\frac{3}{8}}$$

□

References

[1] V R Kulli, Collage Graph Theory, Vishwa International publications, Gulbarga India (2012).

[2] H. Wiener, Structural determination of paraffin boiling points, J.Am.Chem. Soc. 69, 17–20 (1947).

[3] A.A. Dobrynin, R. Entringer, I. Gutman, Wiener index of trees: theory and applications, Acta Appl. Math. 66, 211–249 (2001).

[4] I. Gutman, O.E. Polansky, Mathematical Concepts in Organic Chemistry, Springer-Verlag, New York (1986).

[5] X. Li and H. Zhao, Trees with the first three smallest and largest generalized topological, MATCH Commun. Math. Comput. Chem. 50, 57–62 (2004).

[6] Gutman, Degree–based topological indices, Croat. Chem. Acta 86 (2013) 351–361.

[7] I. Gutman, J. Monsalve, and J. Rada, “A relation between a vertex-degree-based topological index and its energy,” Linear Algebra and Its Applications, vol. 636, pp. 134–142, 2022.

[8] V. R. Kulli, “On K Banhatti indices of graphs,” Journal of Computer and Mathematical Sciences, vol. 7, pp. 213–218, 2016.

[9] V. R. Kulli, “Revan indices of oxide and honeycomb networks,” Int. J. Mathematics and its Applications, vol. 5, no. 4-E, pp. 663–667, 2017.

[10] H. Gonzglez-Diaz, S. Vilar, L. Santana, and E. Uriarte, “Medicinal chemistry and bioinformatics-current trends in drugs discovery with networks topological indices,” Current Topics in Medicinal Chemistry, vol. 7, no. 10, pp. 1015–1029, 2007.