DAKSHAYANI INDICES OF LINE GRAPH OF JAHANGIR GRAPH

B.Uma Devi[1*], M.Micheal Ezhilarasi[2], A.M. Anto[3]

Abstract

In theoretical chemistry, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices. The derived graph of a simple graph G, denoted by G^+, is the of graph having the same vertex set of G and two vertices are adjacent if and only if their distance in G is two. In this paper, we compute generalized Dakshayani indices, first and second neighbourhood Dakshayani indices, first and second hyper neighbourhood Dakshayani indices, the minus and square neighbourhood Dakshayani indices of the derived graph of Subdivision of some graphs.

ArticleHistory: **Received: 10.01.2022** **Revised: 14.03.2022** **Accepted: 05.04.2022**

Key words: Dakshayani Indices, derived graph, line graph.

AMS Subject classification: 05C90, 05C35, 05C12.

[1] * Associate Professor, Department Of Mathematics,S.T Hindu College, Nagercoil, India.
E-Mail:Umasub1986@Gmail.Com
[2]Register Number 21113162092013,Research Scholar, Department Of Mathematics,
S.T Hindu College, Nagercoil, India. E-Mail:Ezhil.Lihze@Gmail.Com
[3]Assistant Professor, Department Of Mathematics,St. Albert's College (Autonomous), Ernakulam, Kochi, Kerala, India. E-Mail:Antoalexam@Gmail.Com
Affiliated To Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India

***Corresponding Author:** - B.Uma Devi
* Associate Professor, Department Of Mathematics,S.T Hindu College, Nagercoil, India.
E-Mail:Umasub1986@Gmail.Com

DOI: 10.53555/ecb/2022.11.4.031

1. INTRODUCTION

Graph Theory is a branch of mathematics which used almost all fields of mathematics. Chemical graph theory deals with the discussion of chemical compounds using simple graphs. A graph G consists of vertices and edges such that chemical compound atoms can be taken by vertices and edges between the atoms are bonds. The degree $d_G(v)$ is the number of edges incident with the vertex v and is known as degree of a vertex v. The topological indices which are the numerical quantities that are used to determine the properties of chemical compounds. The chemical graph theory has its application in the development of chemical science and medical science. The mathematical Chemistry that has so many offers with topological indices used for QSAR / QSPR study. For discussion of topological indices, we see [2,4,5,6,7]. In this purposed work. We use some degree-based topological indices such as generalized Dakshayani index, the first and second neighbourhood Dakshayani indices, the first and second hyper neighbourhood Dakshayani indices, the minus and square neighbourhood Dakshayani indices on the derived graph of subdivision graph of a some graphs.

We consider here the graphs with $V(G)$ and $E(G)$ are the vertices and edges of G respectively. Denote $d_G(v)$ for the degree of vertex v. The Complement $\overline{G}$ of G is the graph with vertex as $V(G)$ and two vertices in $\overline{G}$ are adjacent if and only if they are not adjacent in G. Also the set of all vertices adjacent to v is called open neighbourhood of v and is denoted by $N_G(v)$. The closed neighbourhood of v is denoted by $N_G[v] = N_G(v) \cup \{v\}$. We consider the notation $D_G(v) = d_G(v) + \sum_{u \in N_G(v)} d_G(v)$ is the degree sum of closed neighbourhood of vertices of v. For any other undefined notations are terminology, we refer the readers to [8].

In [6], V. R. kulli, proposed the generalized Dakshayani index, which the defined

as $DK^{\alpha}(G) = \sum_{v \in V(G)} d_{\overline{G}}(v)\, d_G(v)^{\alpha}$ -------- (1.1)

where α is any real number

By the motivation of first and second Zagreb indices introduced by Gutman and Trinajstic [2], V.R. Kulli in [4], defined the new degree based topological indices as follows.

The first neighbourhood Dakshayani index is defined

as $ND_1(G) = \sum_{uv \in E(G)} [D_G(u) + D_G(v)]$ -------- (1.2)

The Second neighbourhood Dakshayani index is defined as

$ND_2(G) = \sum_{uv \in E(G)} D_G(u) D_G(v)$ -------- (1.3)

The first hyper neighbourhood Dakshayani index is defined as

$HND_1(G) = \sum_{uv \in E(G)} [D_G(u) + D_G(v)]^{2}$ -------- (1.4)

The second hyper neighbourhood Dakshayani index is defined as

$HND_2(G) = \sum_{uv \in E(G)} [D_G(u) D_G(v)]^{2}$ -------- (1.5)

The minus neighbourhood Dakshayani index is defined as

$MND(G) = \sum_{uv \in E(G)} [D_G(u) - D_G(v)]$ -------- (1.6)

The square neighbourhood Dakshayani index is defined as

$QND(G) = \sum_{uv \in E(G)} [D_G(u) - D_G(v)]^{2}$ -------- (1.7)

The line graph $L(G)$ of G is the graph whose vertices are in one-to-one correspondence with the edges of G and two vertices of $L(G)$ are adjacent if and only if the corresponding edges in G share a common vertex

2.Jahangir graphs

For $m, n \geq 3$, a Jahangir graph $J_{n,m}$ is the graph obtained by taking a cycle C_{nm} with one additional vertex which is adjacent to on vertices of C_{nm} at distance n to each other on C_{nm}.

Clearly $J_{n,m}$ is of order $nm + 1$ and size $m(n + 1)$.

The Jahangir graph $J_{3,4}$ and its line graph is shown in Figure 1

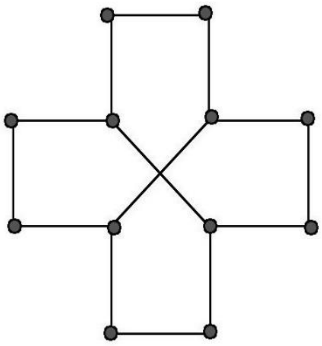


Figure 1 The graph $J_{3,4}$

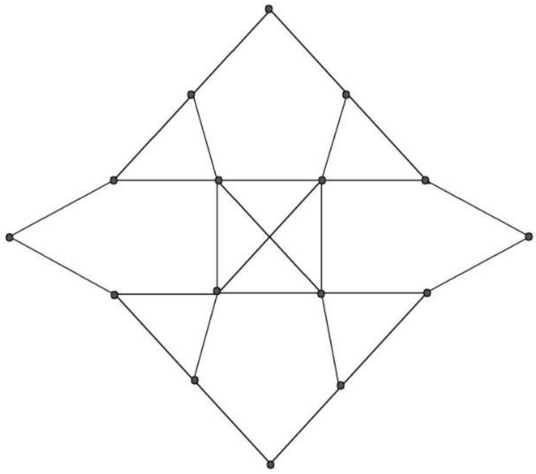


Figure 2 The line graph $J_{3,4}$

Table 1 The edge partition of the line graph of $J_{n,m}$ for $n, m \geq 3$.

$(d_{L(J_{n,m})}(u), d_{L(J_{n,m})}(v))$, where $uv \in E(L(J_{n,m}))$	(2,2)	(2,3)	(3,3)	$(3, m+1)$	$(m+1, m+1)$
Number of edges	$m(n-3)$	$2m$	m	$2m$	$\frac{m(n-1)}{2}$

Table 2 The edge partition of the line graph $J_{n,m}$ for $n = 3$ and $m \geq 3$.

$(d_{L(J_{n,m})}(u), d_{L(J_{n,m})}(v))$, where$uv \in E(L(J_{n,m}))$	$(8, m+9)$	$(m+9, m+9)$	$(m+9, m^2+m+6)$	(m^2+m+6, m^2+m+6)
Number of edges	$2m$	m	$2m$	$\frac{m(m-1)}{2}$

Theorem 2.1. Let G be the line graph of Jahangir graph $J_{n,m}$ for $n, m \geq 3$. Then $DK^{\alpha}(G) = m^2[2n.3^{\alpha} + 2.3^{\alpha} + (m+1)^{\alpha}.n + n^2 2^{\alpha} + n.2^{\alpha} - n.2^{\alpha+1} - 2^{\alpha+1}] + m[2(m+1)^{\alpha} - 3n.2^{\alpha} - 3m.2^{\alpha+1}]$

Proof:

Let G be the line graph of $J_{n,m}$ for $n, m \geq 3$. We observe that G has total $m(n+1)$ −vertices and $\frac{m^2+2mn+3m}{2}$ edges. Out of $m(n+1)$ vertices, there are $2m$ vertices are of degree 3, m vertices are of degree $m+1$ and $(n-2)m$ vertices are of degree 2.

Therefore, the complement of G, denoted by $\bar{G}$ contains $2m$ vertices are of degree $m(n+1)-4$, m −vertices are of degree $mn-2$ and $m(m-2)$ vertices are of degree $m(n+1)-3$.

Hence, $DK^{\alpha}(G) = m^2[2n.3^{\alpha} + 2.3^{\alpha} + (m+1)^{\alpha}.n + n^2 2^{\alpha} + n.2^{\alpha} - n.2^{\alpha+1} - 2^{\alpha+1}] + m[2(m+1)^{\alpha} - 3n.2^{\alpha} - 3m.2^{\alpha+1}]$

Theorem 2.2. Let G denote the line graph of Jahangir graph $J_{3,m}$ for $m \geq 3$. Then

(1) $ND_1(G) = m[m^3 + 2m^2 + 13m + 56]$

(2) $ND_2(G) = \frac{m}{2}[m^5 - 3m^4 + 19m^3 + 41m^2 + 156m + 74]$

(3)$HND_1(G) = 2m[m^5 + 2m^4 + 7m^3 + 33m^2 + 84m + 189]$

(4)$HND_2(G) = 2m[m^9 + 3m^8 + 13m^7 + 25m^6 + 271m^5 + 333m^4 + 2244m^3 + 3078m^2 + 14760m + 31104]$

(5) $MND(G) = 2m[|(m-1)(m-2)|]$

(6)$QND(G) = 2m[|(m^4 - 5m^2 + 2m + 10)|]$

(7) $F_1ND(G) = m[m^5 + 3m^4 + 14m^3 + 32m^2 + 156m - 650]$

Proof:

Let G be the line graph of Jahangir graph $J_{n,m}$ for $m \geq 3$.

Then it is clear that the order of G is $m(n+1)$ and size of G is $\frac{m^2+2m+3m}{2}$.

Now we partition the edges of G into edges of four types based on $(D_G(u), D_G(v))$, the degrees of the end vertices of each edge in Table 2. Using formulae (1.1)-(1.8) to this information in Table 1., we obtained the required result

Table 3. The edge partition of the line graph $J_{4,m}$ for $m \geq 3$.

$(D_{L(J_{4,m})}(u), D_{L(J_{4,m})}(v))$, where $uv \in E(L(J_{4,m}))$	(7,7)	$7, m + 9)$	$(m + 9, m + 9)$	$(m + 9, m^2 + m + 6)$	$((m^2 + m + 6, m^2 + m + 6)$
Number of edges	m	$2m$	m	$2m$	$\frac{m(n-1)}{2}$

The Jahangir graph $J_{4,5}$ and its line graph is shown in Figure 3

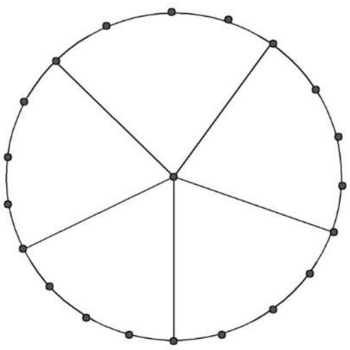
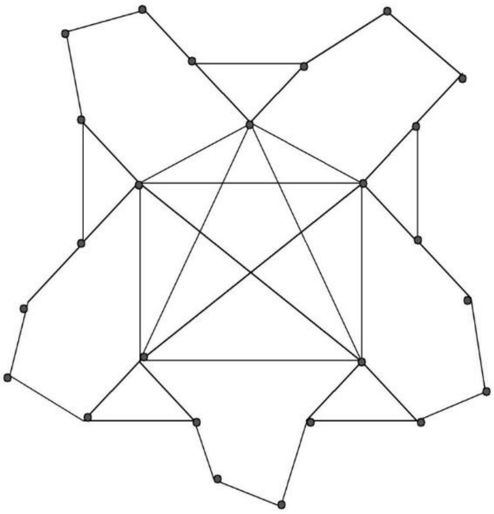


Figure 3 The graph $J_{4,5}$



The line graph of $J_{4,5}$

Theorem 2.3. Let G be the line graph of Jahangir graph $J_{4,m}$ for $m \geq 3$. Then

(1) $ND_1(G) = m[m^3 + 2m^2 + 13m + 88]$

(2) $ND_2(G) = \frac{m}{2}[m^5 + m^4 + 15m^3 + 39m^2 + 112m + 530]$

(3) $HND_1(G) = 2m[2m^4 + 6m^3 + 49m^2 + 122m + 794]$

(4) $HND_2(G) = m[\frac{m^9}{2} + \frac{3}{2}m^8 + 18m^7 + 25m^6 + \frac{393}{2}m^5 + \frac{689}{2}m^4 + 1164m^3 + 2921m^2 + 5004m + 7279]$

(5) $MND(G) = 2m[|(m^2 + m - 1)|]$

(6) $QND(G) = 2m[|(m^4 - 5m^2 + 4m + 13)|]$

(7) $F_1ND(G) = m[m^5 + 3m^4 + 15m^3 + 31m^2 + 156m - 620]$

Proof:
Let G be the line graph of Jahangir graph $J_{4,m}$ for $m \geq 3$.
Then it is clear that $m(n+1)$ and size of G is $\frac{m^2+2mn+3n}{2}$ are the number of vertices and edges of G, respectively.

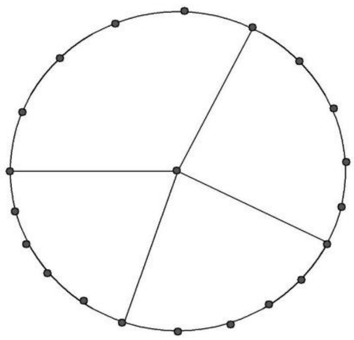
Now we partition size of G into edges of five types based on

$(D_G(u), D_G(v))$ the degrees of the end vertices of each edge given in Table 3. Using formulae (1.1)-(1.8) to this information in Table 3., we obtained the required result

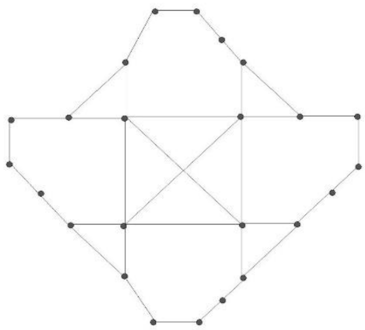
Table 4. The edge partition of the line graph $J_{5,m}$ for $m \geq 3$.

$(D_{L(J_{5,m})}(u), D_{L(J_{5,m})}(v))$, where $uv \in E(L(J_{5,m}))$	(6,7)	$7, m + 9)$	$(m + 9, m + 9)$	$(m + 9, m^2 + m + 6)$	$((m^2 + m + 6, m^2 + m + 6)$
Number of edges	$2m$	$2m$	m	$2m$	$\frac{m(n-1)}{2}$

The Jahangir graph $J_{5,4}$ and its line graph is shown in Figure 4



The graph $J_{5,4}$



The line graph of $J_{5,4}$ Figure 4

Theorem 2.4. Let G be the line graph of Jahangir graph $J_{5,m}$ for $m \geq 3$. Then

(1) $ND_1(G) = m[m^3 + 2m^2 + 13m + 100]$

(2) $ND_2(G) = \frac{m}{2}[m^5 + m^4 + 15m^3 + 41m^2 + 148m + 542]$

(3) $HND_1(G) = m[3m^4 + 19m^3 + 67m^2 + 244m + 1426]$

(4) $HND_2(G) = \frac{m}{2}[m^9 + 3m^8 + 36m^7 + 50m^6 + 253m^5 + 429m^4 + 2328m^3 + 6042m^2 + 1008m + 8282]$

(5) $MND(G) = 2m^2[|(m+1)|]$

(6) $QND(G) = 2m[|(m^4 + 7m^2 + 4m + 14)|]$

(7) $F_1ND(G) = m[m^5 + 3m^4 + 15m^3 + 33m^2 + 146m + 810]$

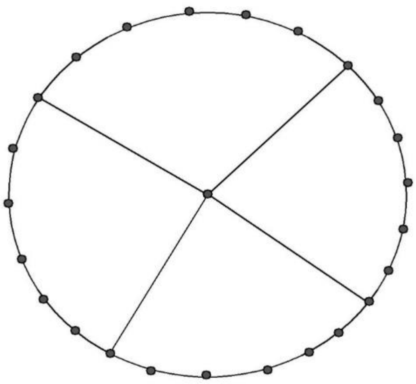
Proof:
Let G be the line graph of Jahangir graph $J_{5,m}$ for $m \geq 3$.
Then there are total $m(n+1)$ vertices and $\frac{m^2+2mn+3n}{2}$ edges in G.

Now we partition size of G into edges of five types based on $(D_G(u), D_G(v))$ the degrees of the end vertices of each edge given in Table 4. Using formulae (1.1)-(1.8) to this information in Table 4., we obtained the required result

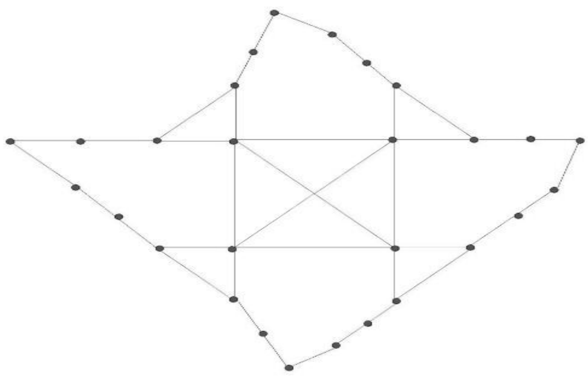
Table 5. The edge partition of the line graph $J_{m,n}$ for $m \geq 3$ and n=6.

$(D_{L(J_{m,n})}(u), D_{L(J_{m,n})}(v))$, where $uv \in E(L(J_{m,n}))$	(6,6)	$7, m + 9)$	$(m + 9, m + 9)$	$(m + 9, m^2 + m + 6)$	$((m^2 + m + 6, m^2 + m + 6)$
Number of edges	$2m$	$2m$	m	$2m$	$\frac{m(n-1)}{2}$

The Jahangir graph $J_{6,4}$ and its line graph is shown in Figure 5



The graph $J_{6,4}$



The line graph of $J_{6,4}$ **Figure 5**

Theorem 2.5. Let G be the line graph of Jahangir graph $J_{n,m}$ for $m \geq 3$ and $n \geq 6$. Then

(1) $ND_1(G) = m[m^3 + 2m^2 + 13m + 12n + 40]$

(2) $ND_2(G) = \frac{m}{2}[m^5 + m^4 + 15m^3 + 41m^2 + 148m + 36n + 362]$

(3) $HND_1(G) = 2m[m^5 + 2m^4 + 15m^3 + 33m^2 + 152m + 72n + 691]$

(4) $HND_2(G) = \frac{m}{2}[m^9 + 3m^8 + 36m^7 + 50m^6 + 253m^5 + 429m^4 + 2328m^3 + 6042m^2 + 1008m + 1296n - 2339]$

(5) $MND(G) = 2m^2[|(m+1)|]$

(6) $QND(G) = 2m[|(m^4 + 7m^2 + 4m + 14)|]$

(7) $F_1ND(G) = m[m^5 + 3m^4 + 15m^3 + 33m^2 + 146m + 72n + 450]$

Proof:

Let G be the line graph of Jahangir graph $J_{n,m}$ for $m \geq 3$and $n \geq 6$.

Then G contains $m(n+1)$ number of vertices and $\frac{m^2+2mn+3n}{2}$ number of edges.

Now we partition the size of G into edges of six types based on $(D_G(u), D_G(v))$ the degrees of the end vertices of each edge given in Table 5. Using formulae (1.1)-(1.8) to this information in Table 5., we obtained the required result

References

1. G. Chartrand and P. Zhang, Introduction to graph Theory, Tata Mc Graw Hill Edition Private Limited, New Delhi,(2006).
2. P. Gutman and N. Trinajstic, Graph Theory and molecular orbitals, Total π- electron energy of alternant hydrocarbons. chem. Phys. Letter, 17, (1972), 535-538.
3. SR Jog, S.P. Hande, I. Gutman and S. Bozkurt, Derived graphs of Some graphs Kragujevac. J. Math., 36 (2012), 309-314.
4. V.R Kulli, Neighbourhood Dakshayani Indices, Int. J. of Mathematical Archive, 10(7),(2019), 23-31
5. V.R. Kulli, F1-Neighbourhood and Square neighbourhood Dakshayani indices ofsome nano Structures, Int. J. of Engineering Science and Research Technology (2019), 8, 126-138.
6. V.R Kull, Dakshayani Indices, Annals of pure and Applied Mathematics, (2018),139-146.
7. J. Xu, Topological Structure and analysis of interconnection networks, Kluwer Academic Publishers, (2001).
8. D.B. West, Introduction to Graph Theory, First Indian Reprint (2002).[10] A.M.Anto, P. Paul Hawkins, “Vertex Polynomial of Graphs with New Results”,
9. Global Journal of Pure and Applied Mathematics, Volume 15, Issue 4, pages 469-475,(2019).
10. A.M.Anto, P. Paul Hawkins, T. Shyla Isac Mary “Perfect Dominating Sets and Perfect Dominating Polynomial of a Path", International journal of advanced Science and Technology, Volume 28, Issue 16, pages 1226- 1236, (2019).
11. A. M. Anto, P. Paul Hawkins, T. Shyla Isac Mary “Perfect Dominating Sets and Perfect Dominating Polynomial of a Cycle", Advances in Mathematics:Scientific Journal, Volume-8, Issue 3, pages 538- 543, (2019).