# A NEW APPROACH FOR MULTI OBJECTIVE TRANSSHIPMENT PROBLEM UNDER FUZZY ENVIRIONMENT 

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Article History: Received: 02.03.2023
Revised: 08.04.2023
Accepted: 18.05.2023


#### Abstract

Transshipment problems are a specialized form of transportation problem where items are conveyed from an origin to a destination through certain intermediate points (sources/destinations), with changing modes of transportation or consolidating or deconsolidating shipments. In the period of e-commerce, these issues have found a broad range of applications. Knowledge of variables like demand, supply, related costs, time, etc. is necessary for formulating transshipment problems. But there are many kinds of uncertainties that arise when formulating the transshipment problem mathematically because of things like a lack of precise information, hesitations when defining parameters, unattainable information, or whether conditions. The relevant parameters can be represented as neutrosophic trapezoidal fuzzy numbers, which is a cooperative way to deal with this kind of uncertainty. In order to solve the neutrosophic fuzzy multi objective transshipment problem ( MOTrP ) in the best possible way, a new algorithm is provided in this study. This proposed method is simple to establish and take less time. The feasibility and application of the proposed problem are demonstrated using a numerical example. Finally, discuss our conclusions as well as our study's intended outcomes.


Keywords Transshipment problem; Multi objective transshipment problem, neutrosophic environment; neutrosophic fuzzy multi objective transshipment problem.

[^0]DOI: 10.31838/ecb/2023.12.s2.535

## 1. Introduction

The benefits of transport to society are substantial and essential in the globalization of the international economic significant capital investments in its material infrastructure. Transportation recognizes the fundamental concerns, implement important policy reforms, and address the problems of decentralized decision-making, transportation governance must be rationalized. Transportation problems are a specific sorts of linear programming problems that involves moving goods from an origin to a destination. In cruel time, goods may be conveyed to their target or via a number of intermediate points. Such problems are referred to as transshipment problems. The conceptualization of transshipment problem (Trp) established by Orden (1956) [9]. The problem of transshipment can be transformed into a transportation problem. In the TrP , minimize the total transportation cost for single objective function. However, the TrPs are not made as single objective function in real world situation. The Tr P that integrates with numerous objective functions which is known as multi-objective transshipment problem ( MOTrP ). In the MOTrP, a unique kind of multi-objective linear programming issue where the objectives dispute with one another. All transportation characteristics, such as supply, demand, and transportation cost, are assumed to be precise in classical TP. However, due to insufficient knowledge and unpredictability in multiple potential suppliers and environments, these parameters are imprecise in real-life circumstances. Uncertainty can be attributed to various of uncontrollable factor such as, i) When a new delivery of an item is scheduled, the decision maker has no idea about the transportation cost. As a result, considerable ambiguity regarding transportation costs may arise. ii) Due to intense rivalry, today's market is continually in flux. As a result, the demand for new things is completely unpredictable. Many researchers investigated the MOTrP in fuzzy environment, which was developed by Zadeh [12], to deal quantitatively with such uncertain information. Nagoor Gani et al. solved the transportation problem using fuzzy numbers in [7] and the transshipment problem in a fuzzy environment in [8]. Das et al [2] proposed a solution procedure of the multi objective transportation problem (MOTP), where all the parameters are expressed in terms of triangular fuzzy number by the decision maker. Although fuzzy set theory (FST) is particularly beneficial when addressing ambiguity, it is unable to handle some circumstances where it is difficult to describe membership degree with a single value. In 1986, Atanassov [1] presented intuitionistic fuzzy set (IFS) as an extension of FST to address the lack of knowledge about non-membership degrees. IFS assigns two grades-membership and non-membership-to each variable in a set, with the aggregate of these two grades never being greater than one. As a result, non-belongingness grade of a given variable is equal to 1 minus the belongingness grade. The IFS has flourished in decision-making applications [3]. In addition, several authors have used IFS to solve various problems [4]. Smarandache[11] recently studied the neutrosophic set, which is a generalized variant of FST and IFS. It provides a more general structure and a very acceptable format for addressing the difficulties addressed. The study of neutral cognition is referred to as "neutosophy," and the neutrality is the key difference between fuzzy and intuitionistic fuzzy logic. The neutrosophic set depends on logic and presents aspects of the universe in three degrees. Namely, the truth degree, indeterminacy degree, and falsity degree of a multi-objective transportation model in a neutrosophic environment are all between [0, 1]. It contrasts from intuitionistic fuzzy sets, in which related ambiguity is determined by degrees of belongingness and nonbelongingness; here, uncertainty, or indeterminacy factor, is not dependent of truth and falsity values. Some attention has been developed for optimization elements [5] since its beginning by Smarandache[11]. In this research a novel algorithm is put forward to find the optimal solution of neutrosophic fuzzy multi objective transshipment problem. As far as, there is no research paper that takes the neutrosophic fuzzy multi objective transshipment problem into mind.

The following are the primary contributions of the proposed study:

1. Due to the fluctuation of the market scenario, all of the $\operatorname{TrP}$ parameters are considered single valued trapezoidal neutrosophic fuzzy numbers in our proposed MOTrP.
2. We have developed an algorithm which is based on making allocations in the cell with minimum cost of $r^{t h}$ objectives corresponding to the row/column of cell having maximum cost of $r^{t h}$ objective.

Section 2 summarises the fundamental terminology related to the neutrosophic and the trapezoidal single valued neutrosopic; Section 3 contains the mathematical model of the neutrosophic single valued multi objective transshipment problem (NSMOTrP) with a converting technique for NSMOTrP into the crisp model and mathematical model of conversion of NSMOTrP to NSMOTP; In section 4, provides algorithm for proposed method; in Section 5, the benefits of the study have been discussed; in section 6, the numerical example is solved to illustrate the algorithm. The problem's solution is compared to existing method. Conclusions of the study along with suggestions for future research are present in Section 7.

## 2. Preliminaries

The basic concepts are given in this section.

## Definition 2.1

Let $E$ be a universal set and $r \in E$. fuzzy set, $\widetilde{S}$ in $E$ is given by: $\tilde{S}=\left\{\left(r, \mu_{s}(r): r \in E\right\}\right.$ where $\mu_{s}: E \rightarrow[0,1]$ is called the membership function of $S \forall r \in E$.

## Definition 2.2

An intuitionistic set $S^{1}$ in a universal set $E$ such that $r \in E$ is given by: $\tilde{S}=\left\{\left\langle r, \mu_{s}(r), \gamma_{s}(r)\right\rangle: r \in E\right\}$ where $\mu_{s}(r), \gamma_{s}(r) \rightarrow[0,1]$ are the degree of membership and non membership function that satisfy: $0 \leq \mu_{s}(r)+$ $\gamma_{s}(r) \leq 1 \forall r \in E$.

## Definition 2.3

Neutrosophic set $S^{N S}$ in $E$ is given by: $S^{N S}=\left\{\left\langle r, T_{S}(r), I_{S}(r), F_{S}(r)\right\rangle: r \in E\right\}$ where $T_{A}, I_{A}, F_{A}: E \rightarrow[0,1]$ are called the truth, indeterminacy and falsity membership functions, that satisfy: $0^{-} \leq \mathrm{T}_{\mathrm{S}}(\mathrm{r})+\mathrm{I}_{\mathrm{S}}(\mathrm{r})+\mathrm{F}_{\mathrm{S}}(\mathrm{r}) \leq 3^{+}, \forall \mathrm{r} \in \mathrm{E}$.

## Definition 2.4

An Single Valued Neutrosophic set, $\mathrm{S}^{\mathrm{NS*}}$ in E is given by: $S^{N S *}=\left\{\left\langle r, \mathrm{~T}_{\mathrm{S}}(\mathrm{r}), \mathrm{I}_{\mathrm{S}}(\mathrm{r}), \mathrm{F}_{\mathrm{S}}(\mathrm{r})\right\rangle: \mathrm{r} \in \mathrm{E}\right\}$ where $T_{A}, I_{A}, F_{A}: E \rightarrow[0,1]$ are called the truth, indeterminacy and falsity membership functions, that satisfy:
$0 \leq \mathrm{T}_{\mathrm{S}}(\mathrm{r})+\mathrm{I}_{\mathrm{S}}(\mathrm{r})+\mathrm{F}_{\mathrm{S}}(\mathrm{r}) \leq 3, \forall \mathrm{r} \in \mathrm{E}$.

## Definition 2.5

A single valued trapezoidal neutrosophic number is of the form $a^{N S *}=\left\langle\left(a_{1}, b_{1}, c_{1}, d_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle$
Which is a special neutrosophic set on the real numbers R , where $\alpha_{a}^{N S *}, \beta_{a}^{N S^{*}}, \gamma_{a}^{N S *} \in[0,1]$. Then the membership functions of truth, indeterminacy, falsity are defined as follows:

- $\quad a^{N S *}=\left\langle\left(a_{1}, b_{1}, c_{1}, d_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle$ is called positive trapezoidal neutrosophic number if $a_{1} \geq 0$ and at least $d_{1}>0$. It is denoted by $a^{N S *}>0$.
- $\quad a^{N S *}=\left\langle\left(a_{1}, b_{1}, c_{1}, d_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle$ is called negative neutrosophic number if $a_{1} \leq 0$ and at least $d_{1}<0$. It is denoted by $a^{N S *}<0$.


## Definition 2.6

Let $a^{N S *}=\left\langle\left(a_{1}, b_{1}, c_{1}, d_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle$ and $b^{N S *}=\left\langle\left(a_{2}, b_{2}, c_{2}, d_{2}\right): \alpha_{b}^{N S *} \beta_{b}^{N S *} \gamma_{b}^{N S *}\right\rangle$ be two SVN and $\rho \neq 0$ be any real number then,

- Addition:
- $\quad a^{N S *}+b^{N S *}=\left\langle\left(a_{1}+a_{2}, b_{1}+b_{2}, c_{1}+c_{2}, d_{1}+d_{2}\right): \alpha_{a}^{N S *} \alpha_{b}^{N S *} \beta_{a}^{N S *} \beta_{b}^{N S *} \gamma_{a}^{N S *} \gamma_{b}^{N S *}\right\rangle$
- Subtraction:
- $\quad a^{N S *}-b^{N S *}=\left\langle\left(a_{1}-a_{2}, b_{1}-b_{2}, c_{1}-c_{2}, d_{1}-d_{2}\right): \alpha_{a}^{N S *} \alpha_{b}^{N S *} \beta_{a}^{N S *} \beta_{b}^{N S *} \gamma_{a}^{N S *} \gamma_{b}^{N S *}\right\rangle$
- Multiplication:
- Scalar multiplication:
$\rho a^{N S *}=\left\{\begin{array}{l}\left\langle\left(\rho a_{1}, \rho b_{1}, \rho c_{1}, \rho d_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle(\rho>0) \\ \left\langle\left(\rho d_{1}, \rho c_{1}, \rho b_{1}, \rho a_{1}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle((\rho<0)\end{array}\right.$
- Division:

$$
\frac{a^{N S *}}{b^{N S *}}= \begin{cases}\left\langle\left(\frac{a_{1}}{a_{2}}, \frac{b_{1}}{b_{2}}, \frac{c_{1}}{c_{2}}, \frac{d_{1}}{d_{2}}\right): \alpha_{a}^{N S *}\right. & \left.\alpha_{b}^{N S *} \beta_{a}^{N S *} \beta_{b}^{N S *} \gamma_{a}^{N S *} \gamma_{b}^{N S *}\right\rangle\left(d_{1}>0, d_{2}>0\right) \\ \left\langle\left(\frac{d_{1}}{d_{2}}, \frac{c_{1}}{c_{2}}, \frac{b_{1}}{b_{2}}, \frac{a_{1}}{a_{2}}\right): \alpha_{a}^{N * *}\right. & \left.\alpha_{b}^{N S *} \beta_{a}^{N S *} \beta_{b}^{N S *} \gamma_{a}^{N S *} \gamma_{b}^{N S *}\right\rangle\left(d_{1}<0, d_{2}>0\right) \\ \left\langle\left(\frac{d_{1}}{a_{2}}, \frac{c_{1}}{b_{2}}, \frac{b_{1}}{c_{2}}, \frac{a_{1}}{d_{2}}\right): \alpha_{a}^{N S *}\right. & \left.\alpha_{b}^{N S *} \beta_{a}^{N S *} \beta_{b}^{N S *} \gamma_{a}^{N S *} \gamma_{b}^{N S *}\right\rangle\left(d_{1}<0, d_{2}<0\right)\end{cases}
$$

- Inverse:

$$
a^{-N S *}=\left\langle\left(\frac{1}{d_{1}}, \frac{1}{c_{1}}, \frac{1}{b_{1}}, \frac{1}{a_{1}}\right): \alpha_{a}^{N S *} \beta_{a}^{N S *} \gamma_{a}^{N S *}\right\rangle\left(a^{N S *} \neq 0\right)
$$

## Definition 2.7

$\mathrm{S}\left(a^{N S *}\right)=\frac{1}{16}(a+b+c+d) *\left(2+\alpha_{a}^{N S *}-\beta_{a}^{N S *}-\gamma_{a}^{N S *}\right)$ which is score function used to compare two single valued trapezoidal nutrosophic numbers.

## 3. Mathematical Formulation of Multi Objective Transshipment Problem

Many practical issues do not lend themselves to Transshipment problem with a single objective function. We choose many goal functions into the Transshipment problem to handle this difficulty, and this is referred to as Multi objective transshipment problem. Mathematical formulation of this problem may be written as:

## Model 3.1

Minimize $\tilde{A}_{K}(e)=\sum_{l=1}^{p+q} \sum_{m=1, j \neq i}^{p+q} s_{l m}^{k} e_{l m}$
Subject to $\quad \sum_{m=1, m \neq l}^{p+q} e_{m l}-\sum_{m=1, m \neq l}^{p+q} e_{l m}=d_{l}, \quad l=1,2,3,4 \cdots, p$

$$
\sum_{l=1, l \neq m}^{p+q} x_{l m}-\sum_{l=1, l \neq m}^{p+q} e_{l m}=f_{m}, \quad m=p+1, p+2, p+3, \ldots, p+q
$$

Where $e_{l m} \geq 0$ and $l, m$ are equal to $1,2,3, \ldots p+q, m \neq l$.
Here, if $\sum_{l=1}^{p} d_{l}=\sum_{m=1}^{q} f_{m}$, then the above problem is balanced otherwise unbalanced.
Due to insufficient information of multiple potential suppliers and environments, transportation factors (cost, supply, and demand) are not precise in real scenario. We explore the MOTrP in the neutrosophic environment to deal quantitatively with such imprecise information. The multi objective transshipment problem with single valued trapezoidal nutrosophic parameters are treated is here as a neutrosophic fuzzy multi objective transshipment problem. The mathematical formulation is defined as follows

## Model 3.2

Minimize $A_{k}^{N S *}=\sum_{l=1}^{m+n} \sum_{m=1, m \neq l}^{p+q} s_{l m}^{k^{N S *}} e_{i j}^{N S *}$
Subject to $\quad \sum_{m=1, m \neq l}^{p+q} e_{l m}^{N S *}-\sum_{m=1, m \neq l}^{p+q} e_{m l}^{N S *}=d_{l}^{N S *}, l=1,2,3,4 \cdots, p$ $\sum_{l=1, l \neq m}^{p+q} e_{l m}^{N S *}-\sum_{l=1, l \neq m}^{p+q} e_{m l}^{N S *}=f_{m}^{N S^{*}}, m=p+1, p+2, p+3, \ldots, p+q$
Where $e_{l m}^{N S *} \geq 0, l$, are equal to $1,2,3, \ldots p+q, m \neq l$.
Where $d_{l}^{N S *}, f_{m}^{N S *}, s_{l m}^{k^{N S *}}$ and $k^{t h}$ objective function are represented in SVNS trapezoidal number.
The single valued neutrosophic multi objective transshipment problem is converted to neutrosophic fuzzy multi objective transportation problem form as:
Minimize $A_{k}^{N S *}=\sum_{l=1}^{p+q} \sum_{m=1, m \neq l}^{p+q} s_{l m}^{k^{N S *}} e_{l m}^{N S *}$
Subject to $\quad \sum_{m=1}^{p+q} e_{l m}^{N S *}=d_{l}^{N S *}+T^{N S *}, l=1,2,3,4 \cdots, p$

$$
\sum_{l=1}^{p+q} e_{m l}^{N S *}=T^{N S *} \quad m=p+1, p+2, p+3, \ldots, p+q
$$

$$
\sum_{l=1}^{p+q} e_{m l}^{N S *}=T^{N S *} \quad j=1,2,3, \ldots, m
$$

$$
\sum_{l=1}^{p+q} e_{m l}^{N S *}=f_{m}^{N S *}+T^{N S *} \quad m=p+1, p+2, p+3, \ldots, p+q
$$

Where $e_{l m}^{N S *} \geq 0, l, m=1,2,3, \ldots p+q, m \neq l$.
A standard neutrosophic fuzzy multi objective transportation problem with ( $\mathrm{p}+\mathrm{q}$ ) sources and $(\mathrm{p}+\mathrm{q})$ destinations is represented by the above mathematical model. At each origin and destination, T represents a buffer stock. T should be large sufficient to handle all transshipments, as we anticipate that mass of items can be transshipped at
each point. It is obvious that the quantity of goods transshipped any given place cannot go beyond the quantity generated or received, hence $T^{N S *}=\sum_{l=1}^{p} d_{l}^{N S *}$ or $\sum_{m=1}^{q} s_{m}^{N S *}$

## 4. Proposed Method

The following new approach is an extension of [6] which is to illustrate the neutrosophic fuzzy multi objective transshipment problem
Step 4.1: first we check the balance of the given problem. Suppose this given problem is unbalanced then we add the dummy row or column to the initial table with zero cost.
Step 4.2: Convert the balanced neutrosophic fuzzy multi objective transshipment problem into neutrosophic fuzzy multi objective transportation problem by adding buffer stock to the transshipment points. Shown in table

| DESTINATION $\rightarrow$ SOURCE $\downarrow$ | $M_{1}$ | $M_{2}$ | $\ldots$ | $M_{p}$ | $C_{p+1}$ | $\ldots$ | $C_{p+q}$ | $\operatorname{SUPPLY}\left(d_{l}^{\text {NS* }}\right.$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | $s_{11}^{1^{N S *}}$ $\begin{gathered} \vdots \\ s_{11}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} s_{12}^{1^{N S *}} \\ \vdots \\ s_{12}^{k^{N S *}} \end{gathered}$ | $\ldots$ | $\begin{gathered} s_{1 p}^{1^{N S *}} \\ \vdots \\ s_{1 p}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} s_{1 p+1}^{1^{N S *}} \\ \vdots \\ s_{1 p+1}^{k^{N S *}} \end{gathered}$ | ... | $\begin{gathered} S_{1, p+q}^{1^{N S *}} \\ \vdots \\ s_{1, p+q}^{k^{N S *}} \end{gathered}$ | $d_{1}^{N S *}+T^{N S *}$ |
| ! | : | ! | ! | ! | ! | ... | : | ! |
| $M_{p}$ | $\begin{gathered} s_{p 1}^{1_{1}^{N S *}} \\ \vdots \\ s_{p 1}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} S_{p 2}^{1 N S *} \\ \vdots \\ s_{p 2}^{k^{N S *}} \\ \hline \end{gathered}$ | $\ldots$ | $\begin{gathered} s_{p p}^{1^{N S *}} \\ \vdots \\ s_{p p}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{aligned} & s_{p p+1}^{1^{N S *}} \vdots \\ & s_{m m+1}^{k^{N S *}} \end{aligned}$ | ... | $\begin{gathered} s_{p, p+q}^{1^{N S *}} \\ \vdots \\ s_{p, p+q}^{k^{N S *}} \\ \hline \end{gathered}$ | $d_{p}^{N S *}+T^{N S *}$ |
| : | : | ! | ! | : | ! | $\vdots$ | ! | : |
| $C_{p+1}$ | $\begin{gathered} s_{p+1,1}^{k^{N S *}} \\ \vdots \\ s_{p+1,1}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{p+1,2}^{1^{N S *}} \\ \vdots \\ s_{p+1,2}^{k^{N S *}} \end{gathered}$ |  | $\begin{gathered} s_{m+1 m}^{1^{N S *}} \\ \vdots \\ S_{m+1 m}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{1 m+1}^{1^{N S *}} \\ \vdots \\ s_{m+1 m+1}^{k^{N S *}} \end{gathered}$ | ... | $\begin{gathered} s_{m+1 m+n}^{1^{N S *}} \\ \vdots \\ s_{m+1 m+n}^{k^{N S *}} \end{gathered}$ | $T^{N S *}$ |
| ! | : | : | : | ; | : | : | : | : |
| $C_{p+q}$ | $\begin{gathered} S_{p+q, 1}^{1^{N S *}} \\ \vdots \\ s_{p+q, 1}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{p+q, 2}^{1^{N S *}} \\ \vdots \\ s_{p+q, 2}^{k^{N S *}} \end{gathered}$ | $\ldots$ | $\begin{gathered} s_{p+q, p}^{k^{N S *}} \\ \vdots \\ s_{p+q, p}^{k^{N S *}} \end{gathered}$ | $s_{p+q, p+1}^{1^{1 S *}}$ <br> $\vdots$ $s_{p+q, p+1}^{k^{N S *}}$ | $\ldots$ | $s_{p+q, p+q}^{1^{N S *}}$ <br> $\vdots$ $s_{p+q, p+q}^{N^{N S *}}$ | $T^{\text {NS* }}$ |
| DEMAND $\left(f_{m}^{N S *}\right)$ | $T^{\text {NS* }}$ | $T^{N S *}$ | $\ldots$ | $T^{N S *}$ | $\begin{aligned} & f_{p+1}^{N S *} \\ & +T^{N S *} \\ & \hline \end{aligned}$ | ... | $\begin{aligned} & f_{p+q}^{N S *} \\ & +T^{N S *} \end{aligned}$ |  |

TABLE 4.1 Multi objective transportation

Step 4.3: observe the largest row $\operatorname{cost}\left(\rho^{N S *}\right)$ as $\rho_{l}^{r^{N S *}}=\max \left(c_{l m}^{r^{N S *}}\right)$, for fixed $1,1 \leq m \leq p+q$ and
$1 \leq r \leq k$, also find the maximum column $\operatorname{cost}\left(\delta^{N S^{*}}\right)$ as $\delta_{m}^{r^{N * *}}=\max \left(c_{l m}^{r^{N S *}}\right)$,for fixed $\mathrm{m}, 1 \leq m \leq p+q$ and $1 \leq r \leq k$, where $\rho^{N S *}=\left\{\rho_{1}^{1^{N S *}} \cdots, \rho_{1}^{k^{N S *}} ; \ldots ; \rho_{p}^{1^{N S *}}, \ldots \rho_{p}^{k^{N S *}} ; \ldots ; \rho_{p+q}^{1^{N S *}}, \cdots \rho_{p+q}^{k^{N S *}}\right\}$ and $\delta^{N S *}=\left\{\delta_{1}^{1^{N S *}}, \cdots \delta_{1}^{k^{N S *}} ; \cdots ; \delta_{p}^{1^{N S *}}, \cdots \delta_{p}^{k^{N S *}} ; \cdots ; \delta_{p+q,}^{1^{N S *}} \cdots \delta_{p+q}^{k^{N S *}}\right\}$ which are shown in table 4.2

| DESTINATION $\rightarrow$ SOURCE $\downarrow$ | $M_{1}$ | $M_{2}$ | $\ldots$ | $M_{m}$ | $C_{m+1}$ | ... | $C_{m+n}$ | $\operatorname{SUPPLY}\left(d_{l}^{\text {NS* }}\right)$ | $\rho^{N S *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | $\begin{gathered} s_{11}^{1^{N S *}} \\ \vdots \\ s_{11}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{12}^{1^{N S *}} \\ \vdots \\ s_{12}^{k^{N S *}} \\ \hline \end{gathered}$ | ... | $s_{1 p}^{1^{N S *}}$ $\begin{gathered} \vdots \\ s_{1 p}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} s_{1, p+1}^{1^{N S *}} \\ \vdots \\ s_{1, p+1}^{k^{N S *}} \\ \hline \end{gathered}$ | ... | $\begin{gathered} S_{1, p+q}^{1^{N S *}} \\ \vdots \\ s_{1, p+q}^{k^{N S *}} \end{gathered}$ | $d_{1}^{N S *}+T^{N S *}$ | $\rho_{1}^{1^{N S *}}$ $\rho_{1}^{k^{N S *}}$ |
| : | ! | : | : | ! | ! | ... | : | : | : |
| $M_{p}$ | $\begin{gathered} S_{p 1}^{1^{N S *}} \\ \vdots \\ s_{p 1}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} s_{p 2}^{1^{N S *}} \\ \vdots \\ s_{p 2}^{k^{N S *}} \\ \hline \end{gathered}$ | $\ldots$ | $\begin{gathered} S_{p p}^{1^{N S^{*}}} \\ \vdots \\ s_{p p}^{k^{N S *}} \\ \hline \end{gathered}$ | $\begin{gathered} S_{p, p+1}^{1^{N S *}} \\ \vdots \\ s_{p, p+1}^{k^{N S *}} \\ \hline \end{gathered}$ | $\ldots$ | $\begin{gathered} s_{p, p+q}^{1^{N S *}} \\ \vdots \\ s_{p, p+q}^{N^{N S *}} \\ \hline \end{gathered}$ | $d_{p}^{N S *}+T^{N S *}$ | $\begin{gathered} \rho_{p}^{1^{N S *}} \\ \vdots \\ \rho_{p}^{k^{N S *}} \\ \hline \end{gathered}$ |
| ! | ! | : | ... | : |  | ... | : |  | : |
| $C_{p+1}$ | $s_{p+1,1}^{k^{N S *}}$ $s_{p+1,1}^{k^{N S *}}$ | $\begin{gathered} S_{p+1,2}^{1^{N S *}} \\ \vdots \\ s_{p+1,2}^{k^{N S *}} \end{gathered}$ | ... | $\begin{gathered} s_{p+1, p}^{1^{N S *}} \\ \vdots \\ s_{p+1, p}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{1, p+1}^{1^{N S *}} \\ \vdots \\ s_{p+1, p+1}^{k^{N S *}} \\ \hline \end{gathered}$ | ... | $\begin{gathered} s_{p+1, p+q}^{1^{N S *}} \\ \vdots \\ s_{p+1, p+q}^{k^{N S *}} \end{gathered}$ | $T^{\text {NS* }}$ | $\begin{gathered} \rho_{p+1}^{1^{N S *}} \\ \vdots \\ \rho_{p+1}^{k^{N S *}} \end{gathered}$ |
| : | : | : | ... | ! | ! | ... |  | : | ... |
| $C_{p+q}$ | $\begin{gathered} s_{p+q, 1}^{1^{N S *}} \\ \vdots \\ s_{p+q, 1}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{p+q, 2}^{\mathbf{N S N}^{N+}} \\ \vdots \\ s_{p+q, 2}^{k^{N S *}} \end{gathered}$ | ... | $\begin{gathered} s_{p+q, p}^{k^{N S *}} \\ \vdots \\ s_{p+q, p}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} s_{1, p+1}^{1^{N S *}} \\ \vdots \\ s_{p+q, p+1}^{k^{N S *}} \\ \hline \end{gathered}$ | ... | $\begin{gathered} s_{p+q, p+q}^{1^{N S *}} \\ \vdots \\ s_{p+q, p+q}^{k^{N S *}} \end{gathered}$ | $T^{N S *}$ | $\begin{gathered} \rho_{p+q}^{1^{N S *}} \\ \vdots \\ \rho_{p+q}^{k^{N S *}} \\ \hline \end{gathered}$ |
| DEMAND $\left(f_{m}^{N S *}\right)$ | $T^{N S *}$ | $T^{N S *}$ | $\ldots$ | $T^{\text {NS* }}$ | $\begin{aligned} & f_{p+1}^{N S *} \\ & +T^{N S *} \end{aligned}$ | $\ldots$ | $\begin{aligned} & f_{p+q}^{N S *} \\ & +T^{N S *} \end{aligned}$ |  |  |
| $\delta^{N S *}$ | $\begin{gathered} \delta_{1}^{1^{N S *}} \\ \vdots \\ \delta_{1}^{k^{N S *}} \end{gathered}$ | $\begin{gathered} \delta_{2}^{1^{N S *}} \\ \vdots \\ \delta_{2}^{k^{N S *}} \end{gathered}$ | $\ldots$ | $\begin{gathered} \delta_{p}^{1^{N S *}} \\ \vdots \\ \delta_{p}^{k^{N S *}} \end{gathered}$ | $\delta_{p+1}^{1^{N S *}}$ $\delta_{p+1}^{k^{N S *}}$ |  | $\begin{gathered} \delta_{p+q}^{1^{N S *}} \\ \vdots \\ \delta_{p+q}^{k^{N S *}} \end{gathered}$ |  |  |

TABLE 4.2 maximum and minimum value
Where $T^{N S *}=\sum_{l=1}^{p} d_{l}^{N S *}$ or $\sum_{m=1}^{q} f_{m}^{N S *}$

Step 4.4: choose

$$
\begin{aligned}
& B^{N S *}=\max \\
& \left\{\rho_{1}^{1^{N S *}} \cdots, \rho_{1}^{k^{N S *}} ; \ldots ; \rho_{p}^{1^{N S *}}, \ldots \rho_{p}^{k^{N S *}} ; \ldots ; \rho_{p+q}^{1^{N S *}}, \cdots \rho_{p+q}^{k^{N S *}}, \delta_{1}^{1^{N S *}}, \cdots \delta_{1}^{k^{N S *}} ; \cdots ; \delta_{p}^{1^{N S *}}, \cdots \delta_{p}^{k^{N S *}} ; \cdots ; \delta_{p+q}^{1^{N S *}} \cdots \delta_{p+q}^{k^{N S *}}\right\}
\end{aligned}
$$

Step 4.5: choose cell (C) that has $B^{N S *}$ as an objective value. If more than one cell (C) exists, choose the one with the highest cost for the other objectives.

Step 4.6: Select the cell that contains the corresponding row or column of the cell selected in Step 5. If a tie arises, choose the one with the highest potential allocation.

Step 4.7: Allocating the maximum possible cost to the cell chosen in Step 6, and cross out the row or column where supply and demand are satisfied.

Step 4.8: Repeat the procedures Steps 4-7 for the remaining sources and destinations until the total supply or demand criteria are not met.

## Advantage of our proposed study

1. This proposed method is simple to establish and takes less time.
2. All the transportation variables are included in our proposed algorithm as single valued trapezoidal neutrosophic numbers, which are not considered in the existing methods.
3. Both in crisp and fuzzy environment, this algorithm provides better solution.

## Numerical example

Consider the following neutrosophic fuzzy multi objective transshipment problem involving two objectives (Transportation cost and labor cost), 2 pure sources, 1 Transshipment point, 3 pure destinations. The supply value of the sources $M_{1}$, and $M_{2}$ are $\langle(8,16,24,32) ; 0.5,0.6,0.2\rangle$ units, and $\langle(16,32,48,64) ; 0.1,0.3,0.2\rangle$ units. The demand values of destinations $\quad C_{1}, C_{2}$, and $C_{3}$
are $\langle(12,24,36,48) ; 0.6,0.1,0.9\rangle$ units, $\langle(4,8,12,16) ; 0.6,0.7,0.2\rangle$ units, $\langle(14,28,42,56) ; 0.3,0.5,0.2\rangle$ respectively[1 $0]$. Aim of this objective minimizing the transportation costs. The allocation of the per-unit transportation costs between various sources and destinations is shown in Table 6.1

| त $\stackrel{\rightharpoonup}{2}$ ज | $$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta$ |  |  | $\begin{array}{ll}  & \hat{\overline{0}} \\ & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |
| $v^{m}$ |  |  |  |  |


| V |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $v$ |  |  |  |  |
|  | $\Sigma$ | $\Sigma$ | $\Sigma$ |  |

TABLE 6.2 Multi objective transportation problem
Where $M_{1}, M_{2}$ - Pure sources, $C_{1}, C_{2}, C_{3}$ - Pure destinations, $V_{1}$ - Transits point. Here Total supply $\left(\sum_{i=1}^{m} a_{i}^{N S *}\right) \quad=\langle(24,48,72,96) ; 0.5,0.5,0.5\rangle$ is not equal to Total demand $\left(\sum_{j=1}^{n} b_{j}^{N S *}\right)=\langle(30,60,90,120) ; 0.5,0.4,0.5\rangle$. So that we add the dummy row with $\langle(0,0,0,0) ; 0,0,0\rangle$ cost and $\langle(6,12,18,24) ; 0.5,0.4,0.5\rangle$ unit supply.
Step 2: Next convert the balanced neutrosophic fuzzy multi objective transshipment problem into balanced neutrosophic fuzzy multi objective transportation problem by adding the buffer stock to the transshipment points ( $M_{3}, C_{4}$ )
buffer stock $=T^{N S *}=\sum_{i=1}^{m} a_{i}^{N S *}$ or $\sum_{j=1}^{n} b_{j}^{N S *}=\langle(30,60,90,120) ; 0.5,0.4,0.5\rangle \quad$ The following table represent the balanced single valued multi objective transportation table

| $\begin{aligned} & \grave{2} \\ & \stackrel{2}{\beth} \\ & \stackrel{n}{n} \\ & \hline \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\nabla$ |  |  | $\begin{array}{ll}  & \hat{30} \\ & 0 \\ \hat{0} 0 & 0 \\ 00 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{array}{ll}  & \hat{0} 0 \\ & 00 \\ \hat{0} 0 & 0 \\ 00 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |
| $v^{m}$ |  |  |  | $\begin{array}{ll}  & \hat{\bar{\circ}} \\ & 0 \\ & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |


| $\cdots$ |  |  |  |  |  |  | 10 000 $0-0$ 000 00 | $\begin{aligned} & 10 \\ & 000 \\ & 000 \\ & 000 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v$ |  |  |  |  |  |  | $\begin{aligned} & 10 \\ & 0.0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 10 \\ & 00 \\ & 00 \\ & 000 \\ & 000 \end{aligned}$ |  |
|  | $\Sigma$ |  | $\Sigma^{N}$ |  | 5 |  | $\Sigma^{m}$ |  | T |

TABLE 6.3 Minimum and maximum value
Step 3:Find the maximum value in each row and column and the maximum value represent in below table Step 4: choose the maximum value $B^{N S *}$ among column $\left(\delta_{j}^{r}\right)$ and row $\left(\rho_{i}^{k}\right)$ in table 6.3
$B^{N S *}=\langle(70,140,210,280) ; 0.5,0.4,0.5\rangle$
Step 5: cell $(2,5)$ has $B^{N S *}$ as one of the objective value.
Step 6: the cell $(4,4)$ has the minimum value among all the cells in corresponding row and column of the cell $(2,5)$. Allocate $\min \{\langle(30,60,90,120) ; 0.5,0.4,0.5\rangle,\langle(6,12,18,24) ; 0.5,0.4,0.5\rangle\}$ in the cell $(2,5)$. Repeat the procedures of Steps 3-6 till the optimum solution is obtained. The optimum solution is given us follows

TABLE 6.4 Optimum Solution

| $\begin{aligned} & \grave{2} \\ & \stackrel{\rightharpoonup}{訁} \\ & \stackrel{n}{n} \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |
| $\cdots$ |  |  |  | $\begin{aligned} & \hat{0} 0 \\ & \hat{0} 0 \\ & 000 \\ & 000 \\ & 000 \\ & 000 \end{aligned}$ |  |


| $\cdots$ |  |  |  | $\begin{array}{ll} \hat{30} & \hat{0} 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \stackrel{\lambda}{2} \\ & \stackrel{2}{2} \\ & \stackrel{\rightharpoonup}{n} \end{aligned}$ |  |  | $\begin{array}{ll}\hat{\circ} \\ \text { N } \\ 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0\end{array}$ |  |  |  |
| $\lambda$ |  |  | $\begin{array}{ll}  & \hat{\bar{\sigma}} \\ & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{array}{ll}  & \hat{2} \\ & 0 \\ & 0 \\ 0 & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |  |
| $v^{m}$ |  |  |  | $\begin{array}{ll}  & \hat{2} 0 \\ & 0 \\ & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |  |
| $\cdots$ |  |  |  | $\begin{array}{ll}  & \hat{\sigma} 0 \\ & \\ & 0 \\ & 0 \\ 0 & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \end{array}$ |  |  |
| $v$ |  |  |  | $\begin{array}{ll}  & \hat{\sigma} 0 \\ & \\ & 0 \\ 0 & 0 \\ \hat{0} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |  |
| $\begin{aligned} & \uparrow \\ & \stackrel{0}{0} \\ & 0 \\ & \tilde{E} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\Sigma$ | $\Sigma^{N}$ | $\Delta$ | $\Sigma^{m}$ | $\begin{aligned} & \text { ت̈ } \\ & \text { ت̈ } \\ & \text { In } \end{aligned}$ | $\vdots$ |


| $\cdots$ |  |  |  | $\begin{aligned} & \hat{3} 0 \\ & 00 \\ & 000 \\ & 000 \\ & 000 \\ & 000 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| v |  |  |  | $\begin{aligned} & \hat{0} 0 \\ & 00 \\ & 000 \\ & 000 \\ & 000 \\ & 000 \end{aligned}$ |  |
|  | $\Sigma$ | $\Sigma$ | $\Sigma$ | $\Sigma^{m}$ |  |

The objective values are $z_{1}^{\text {NS }}{ }^{*}=\langle(1,2,3,4) ; 0.3,0.4,0.3\rangle \times\langle(12,24,36,48) ; 0.5,0.4,0.5\rangle+$ $\langle(2,4,6,8) ; 0.2,0.4,0.2\rangle \times\langle(4,8,12,16) ; 0.5,0.4,0.5\rangle+\langle(8,14,20,22) ; 0.9,0.1,0.8\rangle \times\langle(4,8,12,16) ; 0.5,0.4,0.5\rangle+$ $\langle(4,6,8,14) ; 0.9,0.8,0.1\rangle \times\langle(6,12,18,24) ; 0.5,0.4,0.5\rangle+\langle(8,14,20,22) ; 0.9,0.1,0.8\rangle \times\langle(4,8,12,16) ; 0.5,0.4,0.5\rangle$

```
    ={(108,376,804,1360);0.1,0.8,0.8\rangle by using score function the crisp value is 82.75.
Z2
\langle(6,12,18,24);0.5,0.4,0.5\rangle+\langle(9,15,22,26);0.9,0.1,0.8\rangle\times\langle(4,8,12,16);0.5,0.4,0.5\rangle+
\langle(5,10,11,24);0.1,0.05,0.05\rangle \times \langle(12,24,36,48);0.5,0.4,0.5\rangle + \langle(4,5,6,9); 0.9,0.1,0.8\rangle\times
<(4,8,12,16); 0.5,0.4,0.5\rangle
```

$=\langle(152,508,936,2152) ; 0.1,0.8,0.8\rangle$ by using score function the crisp value is 117.13
The numerical problem was taken from［10］．The numerical problem was solved using the algorithm that was proposed．The obtained solution was compared to［10］．The effectiveness of the proposed algorithm is shown in the table below．

Table 6．5 Comparison table

| Objective Values | Proposed Method | Existing Method |
| :---: | :--- | :--- |
| $z_{1}^{N S *}$ | 82.75. | 85.5 |
| $z_{2}^{N S *}$ | 117.13 | 117.09 |

## 5．Conclusion

In order to solve the multi－objective transshipment problem in a neutrosophic environment，a new method is proposed in this paper．The suggested technique is quite simple and effortless to comprehend．This method provides an enhanced solution in both crisp and fuzzy environment．All the demand，supply，and transportation costs are represented as single valued trapezoidal neutrosophic numbers．The hypothesis of suggested technique is elementary and effective to use in real world phenomena，such as management，transshipment system，and plenty of other network enhancement problems．Here，we illustrate the proposed algorithm with numerical example and compared with existing method．The proposed algorithm provides more effective solution than the existing method．Therefore，as future research this method can be easily used to solve large scale MOTrP and better decisions can be made in less time．So，it can be more suitable to apply for each decision maker． Furthermore，will make an effort to modify this proposed method for the interval valued neutosophic set，etc．，

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