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EXACT SOLUTION OF FUZZY FIXED CHARGE BULK TRANSPORTATION PROBLEM

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Abstract

Transportation problem (TP) is a well-versed problem in the field of Operation Research. It considers the least cost to ship goods from one source to the terminal. This problem is further extended to a Bulk transportation problem (BTP) which plays a vital role in the domain of socio-economic system where the bulk demand of each of the terminal can be fulfilled by only one source; though, each of the sources can fulfill the demand of any number of terminals. In the real world, the analysis of data for the bulk transportation is not so simple as the requirement and availability at the terminal and source, respectively are fuzzy parameter. Also, it has been observed that the fixed cost has not been included in the fuzzy BTP earlier in the literature to the best of my knowledge. So, in order to fulfill this gap, we have considered some additional cost which are involved during the shipment along with the bulk transportation cost to ship goods as fuzzy numbers. In the present work, a fuzzy bulk transportation problem (FBTP) is solved where the requirement at terminals, availability at sources, shipment cost and additional cost are trapezoidal fuzzy numbers. To optimize the total cost of shipment of the goods, a novel ranking system for trapezoidal fuzzy number is applied and an algorithm have been developed. A numerical problem is used as an integral aspect to demonstrate the efficiency of the suggested algorithm.

Keywords— bulk transportation problem, fuzzy BTP, fixed charge, trapezoidal fuzzy numbers, transportation problem.

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Introduction

Hitchcock [1] was the initial pioneer in the field of transportation problem which is also termed as classical transportation problem or normal transportation problem. The motive behind this problem was to transship homogeneous components from a source to the receiving node in the bare minimum cost. Many strategies were established by Szwarc [2], and Sharma and Swarup [3] in the process of finding the least cost or time depending upon the needs of the decision maker. This problem was further investigated by Purushotam, Prakash and Dhyani [4] and Prakash, Agarwal and Shah [5] with the goal to optimize the multiple objectives such as cost, time, etc. In later years, the usage of number of vehicles to transship the goods from one end to the other by the businessperson was increased, which result into less profit and more loss in transferring goods. It was noticed by Schell [6], for which the method for automatic computation to optimize the cost was developed by adding necessary conditions. This challenge was further discussed by Pandian and Anuradha [7] and Rautman, Reid and Ryder [8] through different approaches. The applicable need to minimize the multi objective transportation problem in conjunction with multiple indices was investigated and numerical model was formulated Elazeem, Mousa, El-Shorbagy, Elagan and Elnaga [9], and Latpate and Kurade [10] to fulfill the business demand. In inclusion of optimizing these problems, some properties on a fixed additional sum which was included in the cost of transshipment of material were discussed by Warren and George [11] when subsequent significant challenge occurred in the domain of programming. Further, an approximate algorithm to obtain optimal solution for the fixed charge transportation problem was proposed by Kuhn and Baumol [12]. Murty [13] applied the method of ranking the extreme points and Sandrock [14] used a low-tech algorithm to obtain a minimal solution for the small, fixed charge problem. Later, a new snap methodology was presented by Kowalski [15] to find a global minimum. In an attempt to solve this problem a Lagrangean relaxation approach was developed by Aguodo [16]. An approximate solution was shown by Balinski [17] to optimize the fixed charge transportation problem. In later years, due to uncertain social and economic environmental conditions such as weather, a problem arises while determining the proper data for the transportation model which was examined by Kaufmann and Gupta [18]. This problem was further studied by Li, Ida and Gen [19] by introducing the ranking fuzzy numbers in search of the nondominated points in the multi criteria transportation problem based on the level of optimism of the decision maker. A survey was conducted by Tuli, Chauhan and Sharma [20] for the deep understanding of fuzzy multi-objective linear programming problem. Later, Maliniand and Ananthanarayanan [21] employed a MODI method in fuzzy transportation problem by ranking the fuzzy numbers and converting into a crisp value. Vidya and Ganesan [22] used a new fuzzy arithmetic on parametric form of trapezoidal fuzzy numbers to obtain the efficient optimal solution of multi objective transportation problem.

In real world, the destination requires the demand of bulk goods to be satisfied by only one source which was the origin of a new type of transportation problem namely, Bulk transportation problem for which the optimal solution was first achieved by Maio and Roveda [23]. This problem was defined as the quantity of units needed by a terminal is satisfied by a single source, whereas a source can satisfy the demand of any number of destinations.

A survey was done and two new algorithm that is branch and bound and back tracking was developed by Foulds and Gibbons [24] to minimize the total time. Further, a survey was conducted by Singh, Chauhan and Tanwar [25] which gave a detailed view of the BTP, and a heuristic approach is used to determine the best optimal solution in minimum time was devised by Singh, Chauhan and Tanwar [26].

In some time, a need to solve a BTP with two or more than two objectives lead Prakash, Sharma and Singh [27] to employ extreme difference method to minimize the total cost and duration of transshipment by prioritizing the cost which helped the decision maker to decide which solution is in their favor. Further, modified VAM method to obtain more than one solution was applied by Singh, Chauhan and Tanwar [28] to give more flexibility to a decision maker.

Latha [29] employed a Lexi search exact algorithm based on pattern recognition technique to solve a mini-max combinatorial programming problem that involves multiple indices. Singh, Chauhan and Tanwar [30] used an extended version of VAM to maximize the profit by minimizing the duration when two or more vehicles are involved in the transport of goods in BTP. Further, Singh, Chauhan and Tanwar [31] aided the industrialist by minimizing the cost and time and maximizing a profit when a particular source generate more than one type of goods or uses more than one type of vehicle to transport good in BTP. Chauhan and Khanna [32] further examined the expanded version of BTP problem to calculate the cost time trade off pair. Chauhan, Tuli, Sindhwani and Khanna [33] deduced a novel algorithm to determine the best cost time trade off pair for the BTP with more than two modes which can be used in every unavoidable circumstances.

In day-to-day life there are several situations which lead to ambiguity in data, lack of proof, inaccurate judgement, undesirable environmental conditions and many more in BTP which lead to the introduction of fuzzy numbers in this problem. This was first considered by Tanwar and Chauhan [34] with a goal to minimize the duration of shipment with fuzzy parameters. Further, Chauhan, Khanna, Sindhwani, Saxena and Anand [35], put forward an algorithm to optimize the fuzzy cost and fuzzy time for a fully fuzzy bi-criteria BTP.

In later years, Kaushal and Arora [36] noticed a need to introduce a BTP which include some additional cost mainly known as set up or fixed cost which was earlier used only in normal transportation problem and obtained the best solution by applying the Lexi-search algorithm. Kaushal and Arora [37] further extended this problem in a scenario where the requirement of only some terminals is fulfilled out of the given number of terminals by finding the optimal solution.

In this paper the BTP in which some additional cost that is fixed cost has been considered with an objective to minimize the total cost in fuzzy environment. It has been observed that in literature several methods are employed to solve fuzzy transportation problem or fuzzy bulk transportation problem, but the fixed cost has not been included yet which is applicable in real world. Inspired by this void in the literature a new ranking methodology has been developed to solve and obtain best possible solution.

The layout of this paper is as follows: Some basic definitions related to fuzzy sets, fuzzy numbers and ranking function for these fuzzy numbers has been introduced in Section-2. A mathematical formulation of fuzzy fixed charge BTP is shown in Section- 3. The method or algorithm to solve the problem is given in Section-4, which was applied in the numerical problem represented in Section-5. The conclusion for the given problem is offered by Section-6.

I. Preliminaries

Some important definitions are defined in this section.

A. Fuzzy Set

Let a universal set be denoted by Y such that the element of Y is y. A membership function is defined from a crisp subset B of Y to the unit interval, as

$$\tau_B : B \subseteq Y \to [0,1]$$

such that for each element $y \in B$, there exist a unique image in the unit interval which is denoted by membership grade. The collection of all ordered pair whose first element is $y \in B$ and second element is its image $\tau_B(y)$ is known as Fuzzy sets. It is denoted as,

$$\tilde{B} = \{(y, \tau_B(y)) : y \in B\}$$

B. Fuzzy Number

A fuzzy set $\tilde{B}$ is said to be a fuzzy number if the following properties hold:

1. A fuzzy set $\tilde{B}$ is normal, that is $\tau_B(y) = 1$ for atleast one $y \in B$.
2. A fuzzy set $\tilde{B}$ is convex, that is $\tilde{B}(\lambda a + (1-\lambda)b) \geq min\{(\tilde{B}(a), \tilde{B}(b))\}$ for all $a, b \in B$ and $\lambda \in [0,1]$.
3. The membership function τ_B is continuous.

C. Generalized Trapezoidal Fuzzy Number

If a membership function is defined as:

$$\tau_B(u) = \begin{cases} \dfrac{u - x_1}{x_2 - x_1}, & x_1 \leq u < x_2 \\ 1, & x_2 \leq u < x_3 \\ \dfrac{u - x_4}{x_3 - x_4}, & x_3 \leq u < x_4 \\ 0, & otherwise \end{cases}$$

then fuzzy number $\tilde{B} = (x_1, x_2, x_3, x_4)$ is said to be trapezoidal fuzzy number.

D. Arithmetic of Trapezoidal Fuzzy Number

If $\tilde{B}_1 = (x_1, x_2, x_3, x_4)$ and $\tilde{B}_2 = (z_1, z_2, z_3, z_4)$ are two trapezoidal fuzzy numbers then,

1. $\tilde{B}_1 + \tilde{B}_2 = (x_1, x_2, x_3, x_4) + (z_1, z_2, z_3, z_4) = (x_1 + z_1, x_2 + z_2, x_3 + z_3, x_4 + z_4)$
2. $\tilde{B}_1 - \tilde{B}_2 = (x_1, x_2, x_3, x_4) - (z_1, z_2, z_3, z_4) = (x_1 - z_4, x_2 - z_3, x_3 - z_4, x_4 - z_1)$

E. Ranking Function

A ranking function is used for trapezoidal fuzzy numbers $\tilde{B}_1 = (x_1, x_2, x_3, x_4)$ to reduce them into crisp value. Mathematically, it is defined as a map of fuzzy numbers into real number, that is $\breve{R}: F \to R$, where F is a set of trapezoidal fuzzy numbers. This function is given as follows:

$$\breve{R}(\tilde{B}_1) = \lim_{\vartheta \to 1} \frac{d}{d\vartheta}\left[[(x_2 - x_1) - (x_4 - x_3)]\vartheta^{\frac{1}{4}} + \left(\frac{x_1 + x_4}{4}\right)\vartheta^2\right]$$

II. Mathematical Formulation of the problem

Let us consider there are n terminals and m sources in the BTP. The demand of each terminal and the availability at each source is D(j) (j = 1, 2, 3, ...n) and S(i) (i = 1, 2, 3, ...m), respectively. The cost to ship bulk good from each source i to terminal j is C(i, j) (i = 1, 2, 3, ...m; j = 1, 2, 3, ...n). In this paper, there is an additional cost also known as fixed cost is also being charged on shipping goods. This cost does not depend on the quantity of goods being shipped. This cost is given by F(i,j) (i = 1, 2, 3, ...m; j = 1, 2, 3, ...n). If the requirement of any terminal is being fulfilled by any source the value of the variable X(i, j) (i = 1, 2, 3, ...m; j = 1, 2, 3, ...) is 1. In this scenario the additional cost is also incurred so the value of a variable Y(i, j) (i = 1, 2, 3, ...m; j = 1, 2, 3, ...n) is also 1. However, if the requirement of any terminal is not being fulfilled by the i^{th} source, then the value of the variable X(i, j) is 0 and since in this case, the transport of goods has not been done from the i^{th} source to the j^{th} terminal, so the value of the variable Y(i, j) is also 0. The objective of this transportation problem is to minimize the total cost occurred during the transportation which is given by Z. Under the scenario that each terminals necessities are to be met by a single source and a single source can supply to any numbers of terminals to fulfill their necessities, the mathematical model for the balanced fuzzy fixed cost bulk transportation problem is demonstrated to strengthen the profit by minimizing the total cost. Any non-negative value is taken by S(i), D(j), C(i, j), F(i,j).

Mathematical model of fuzzy BTP including fixed charge to minimize:

$$Z = \left\{\sum_{j=1}^{n}\sum_{i=1}^{m} C(i,j)X(i,j) + F(i,j)Y(i,j)\right\} \quad (1)$$

Subject to constraints:

$$\sum_{j=1}^{n} D(j)X(i,j) \leq S(i) \quad (i = 1, 2, 3, ...m) \quad (2)$$

$$\sum_{i=1}^{m} X(i,j) = 1 \quad (j = 1, 2, 3, ...n) \quad (3)$$

$$X(i,j) = 0 \text{ or } 1 \quad (i = 1, 2, 3, ...m; j = 1, 2, 3, ...n) \quad (4)$$

$$Y(i,j) = \begin{cases} 0, & x_{ij} = 0 \\ 1, & x_{ij} > 0 \end{cases} \quad (5)$$

Here,

Equation (1): This equation represents the sum of the bulk transportation cost and fixed cost occurred in the shipment of bulk goods to all the terminals. This is the objective function which is required to be optimized by satisfying all the constraints mentioned from (2) to (4).

Constraint (2): It states that the sum of the requirements of all the terminals is always less than or equal to the sum of availability at all the sources.

Constraint (3): It redefines the definition of bulk transportation mathematically that the necessity of each terminal is fulfilled by only one source.

Constraint (4) and (5): The need of these constraints to be introduced is to allocate a true value of binary number wherever the allocation is made.

III. ALGORITHM

In this paper we have considered fuzzy bulk transportation cost and fuzzy fixed cost for the shipment of goods from each source to each terminal which is denoted by (C(i, j),F(i,j)) where C(i, j) is the bulk cost of transporting goods and F(i,j) is the fixed cost which incurred on it in the process of shipment. To obtain the optimal solution the following steps are applied:

Step 1: Apply the ranking function to convert each S(i), D(j), C(i, j) and F(i,j) into crisp value.

Step 2: Subtract the minimum of all the fixed cost in the i^{th} row from every fixed cost in the respective row to obtain new fixed cost $F(\acute{i,j})$.

Step 3: Convert the cost-fixed cost matrix into a new cost matrix by adding $\frac{F(\acute{i,j})}{min(S(i),D(j))}$ to C(i, j) and use $Y(i,j) = \frac{X(i,j)}{min(S(i),D(j))}$ to relax the binary condition on Y(i, j).

Step 4: Meet the requirement of the first terminal by shipping through the source which charge the least depending on its availability otherwise ship through the source with next least bulk cost. In case of the tie in least cost, we may choose any cost depending on the availability of the source. Next, identify the minimum cost charged by the source to fulfill the requirement of the second terminal and ship through that source. Continue this process until the requirement of all the terminals has been satisfied.

The solution obtained here is initial feasible solution.

Step 5: Consider any two sets of assignments from the initial feasible solution.

Set 1: Source $\xi_s \longrightarrow$ Terminal Υ_s at cost ζ_s

Set 2: Source $\xi_{s+1} \longrightarrow$ Terminal Υ_{s+1} at cost ζ_{s+1}

For sets 1 and 2, let $p = \zeta_s + \zeta_{s+1}$

Now interchange the terminals of both these sets and determine the corresponding cost for them to get a new pair of sets:

Set 3: Source $\xi_s \longrightarrow$ Terminal Υ_{s+1} at cost $\zeta_s{}^\circ$

Set 4: Source $\xi_{s+1} \longrightarrow$ Terminal Υ_s at cost $\zeta_{s+1}{}^\circ$

For sets 3 and 4, let $q = \zeta_{s+1}{}^\circ + \zeta_s{}^\circ$

Case 1: If $p \leq q$, then this interchange will not result in the better optimal solution.

Go to Step 6.

Case 2: If $p > q$, then this interchange will result in the better optimal solution.

Go to Step 6.

Step 6: If the situation is like the scenario in Case 1 suggested in Step 5, that is p ≤ q, then there arise two cases:

Case 1.1: If $\zeta_s \geq \zeta_{s+1}$ then $(\xi_s = \xi_s, \Upsilon_s = \Upsilon_s, \zeta_s = \zeta_s)$ and $(\xi_{s+1} = \xi_{s+2}, \Upsilon_{s+1} = \Upsilon_{s+2}, \zeta_{s+1} = \zeta_{s+2})$

Go to Step 7.

Case 1.2: If $\zeta_s < \zeta_{s+1}$ then $(\xi_s = \xi_{s+1}, \Upsilon_s = \Upsilon_{s+1}, \zeta_s = \zeta_{s+1})$ and $(\xi_{s+1} = \xi_{s+2}, \Upsilon_{s+1} = \Upsilon_{s+2}, \zeta_{s+1} = \zeta_{s+2})$

[If s + 2 > n then s + 2 = 1, that is the first set in the assignment is taken.]

Go to Step 7.

If the situation is like the scenario in Case 2 suggested in Step 5, that is p > q, then there arise two cases:

Case 2.1: If $\zeta_s{}^\circ < \zeta_{s+1}{}^\circ$ then $(\xi_s = \xi_{s+1}, \Upsilon_s = \Upsilon_s, \zeta_s = \zeta_{s+1}{}^\circ)$ and $(\xi_{s+1} = \xi_{s+2}, \Upsilon_{s+1} = \Upsilon_{s+2}, \zeta_{s+1} = \zeta_{s+2})$

If $[D(\xi_s) \leq S^\circ(\Upsilon_{s+1})$ and $D(\xi_{s+1}) \leq S^\circ(\Upsilon_s)]$

where, S°(i) is the availability in the source after supplying due to partial solution.

Go to Step 7.

Case 2.2: If $\zeta_s{}^\circ \geq \zeta_{s+1}{}^\circ$ then $(\xi_s = \xi_s, \Upsilon_s = \Upsilon_{s+1}, \zeta_s = \zeta_s{}^\circ)$ and $(\xi_{s+1} = \xi_{s+2}, \Upsilon_{s+1} = \Upsilon_{s+2}, \zeta_{s+1} = \zeta_{s+2})$

If $[D(\xi_s) \leq S^\circ(\Upsilon_{s+1})$ and $D(\xi_{s+1}) \leq S^\circ(\Upsilon_s)]$

Go to Step 7.

Step 7: If the first set of allocation does not change for $(n-1)$ iterations, the process comes to an end.

Go to step 8.

Otherwise, go to step 5.

Step 8: The cost corresponding to the final allocation is the optimal solution. Now, stop this process.

The total cost is obtained by adding all the corresponding costs where the allocation has been made.

IV. NUMERICAL PROBLEM

The fuzzy fixed cost bulk transportation matrix is given in table I with each entry as (C(i,j), F(i,j)) such that C(i,j) and F(i,j) are the fuzzy bulk cost and fuzzy fixed cost, respectively which occurred during the transportation of bulk goods from source i to the terminal j. In this matrix there are 5 destinations and 3 sources, that is D = {1, 2, 3, 4, 5} and S= {1, 2, 3}, respectively.

TABLE I. REPRESENTATION OF COST-FIXED COST OF FUZZY BTP

((3, 4, 6, 7), (0, 1, 4, 5))	((4, 5, 7, 8), (1, 2, 3, 6))	((3, 7, 10, 20), (1, 3, 4, 8))	((5, 8, 9, 10), (2, 5, 7, 14))	((1, 2, 4, 5), (0, 0.5, 1.5, 2))
((2, 5, 8, 11), (3, 5, 8, 16))	((0, 1, 3, 4), (1, 3, 4, 8))	((5, 9, 14, 28), (1, 2, 4, 5))	((4, 5, 6, 9), (0, 1, 4, 5))	((2, 3, 5, 6), (5, 7, 12, 20))
((2, 5, 4, 7), (9, 11, 20, 24))	((3, 5, 13, 22), (5, 7, 8, 12))	((1, 4, 8, 10), (0, 3, 9, 12))	((3, 7, 18, 28), (1, 2, 4, 5))	((13, 17, 30, 45), (3, 4, 5, 7))

The availability at each source and requirement at each terminal is given in table II and table III.

TABLE II. AVAILABILITY AT EACH SOURCE

S =	1	2	3
	(5, 6, 8, 9)	(6, 7, 9, 10)	(8, 9, 11, 12)

TABLE III. REQUIREMENT AT EACH TERMINAL

D =	1	2	3	4	5
	(1, 2, 4, 5)	(3, 4, 6, 7)	(2, 3, 5, 6)	(0, 1, 3, 4)	(4, 5, 7, 8)

Defuzzification is done for the table 1, 2 and 3 by applying step 1, to get the crisp value for cost, fixed cost, availability, and requirement which is shown in table 4, 5, and 6.

TABLE IV. DEFUZZIFICATION OF COST-FIXED COST OF FUZZY BTP

(5, 2.5)	(6, 3)	(10, 4)	(8, 7)	(3, 1)
(6.5, 8)	(2, 4)	(14, 3)	(6, 2.5)	(4, 11)
(4.5, 16)	(10.75, 8)	(5.75, 6)	(14, 3)	(26.25, 4.75)

TABLE V. DEFUZZIFICATION OF AVAILABILITY AT EACH SOURCE

S =	1	2	3
	7	8	10

TABLE VI. DEFUZZIFICATION OF REQUIREMENT AT EACH TERMINAL

D =	1	2	3	4	5
	3	5	4	2	6

Further, the table VII represents the cost and new fixed cost $F(\acute{\iota}, \jmath)$ which is obtained by applying step 2 in table IV.

TABLE VII. REDUCED COST-FIXED COST OF FUZZY BTP

(5, 1.5)	(6, 2)	(10, 3)	(8, 6)	(3, 0)
(6.5, 5.5)	(2, 1.5)	(14, 0.5)	(6, 0)	(4, 8.5)
(4.5, 13)	(10.75, 5)	(5.75, 3)	(14, 0)	(26.25, 1.75)

The table VII is reduced to a single cost matrix as shown in table VIII by applying the formula given in step 3 using the availability and requirement given in table V and VI, respectively.

TABLE VIII. COST MATRIX OF FUZZY BTP

5.5	6.4	10.75	11	3
8.333	2.3	14.125	6	5.416
8.833	11.75	6.5	14	26.5416

The initial feasible solution is obtained by using the procedure given in step 4 and is shown in table IX where S is the source and D is the terminal.

TABLE IX. INITIAL FEASIBLE SOLUTION

S	D	New Cost
1	1	5.5
2	2	2.3
3	3	6.5
2	4	6
3	5	26.5416

The above given table shows that the 1st source supply to the 1st terminal, the 2nd source fulfills the demand of the 2nd terminal, the 3rd terminal is assigned to the 3rd source, the 2nd source supply to the 4th terminal and the 3rd source fulfill the demand of the 5th terminal.

The total cost on allocating to the X(1, 1), X(2, 2), X(3, 3), X(2, 4) and X(3, 5) position is C(1, 1) + F(1, 1) + C(2, 2) + F(2, 2) + C(3, 3) + F(3, 3) + C(2, 4) + F(2, 4) + C(3, 5) + F(3, 5) = 5 + 2.5 + 2 + 4 + 5.75 + 6 + 6 + 2.5 + 26.25 + 4.75 = 64.75.

In order to obtain a best solution, we apply the algorithm from step 5 to step 8 by considering one pair of source and terminal and the next pair of source and terminal as the second pair.

We begin by considering a first pair $(\xi_s, \Upsilon_s) = (1, 1)$ with cost $\zeta_s = 5.5$ and second pair $(\xi_{s+1}, \Upsilon_{s+1}) = (2, 2)$ with cost $\zeta_{s+1} = 2.3$ such that $p = \zeta_s + \zeta_{s+1} = 7.5$.

On interchanging the terminals of both the sets we get a new pairs $(\xi_s, \Upsilon_{s+1}) = (1, 2)$ with cost $\zeta_s{}^{\circ} = 6.4$ and second pair $(\xi_{s+1}, \Upsilon_s) = (2, 1)$ with cost $\zeta_{s+1}{}^{\circ} = 8.333$ such that $q = \zeta_s{}^{\circ} + \zeta_{s+1}{}^{\circ} = 14.733$ which results into $p \leq q$, that means this interchange will not result into the better solution. Thus, go to step 6.

Consider next set of pairs:

$(\xi_s = 1, \Upsilon_s = 1, \zeta_s = 5.5)$ and $(\xi_{s+1} = 3, \Upsilon_{s+1} = 3, \zeta_{s+1} = 6.5)$ such that $p = \zeta_s + \zeta_{s+1} = 12$.

On interchanging the terminals of both the sets we get a new pairs $(\xi_s = 1, \Upsilon_{s+1} = 3, \zeta_s{}^{\circ} = 10.75)$ and $(\xi_{s+1} = 3, \Upsilon_s = 1, \zeta_{s+1}{}^{\circ} = 8.833)$ such that $q = \zeta_s{}^{\circ} + \zeta_{s+1}{}^{\circ} = 19.583$ which results into $p \leq q$, that means this interchange will also not result into the best solution.

On applying the same procedure for the next set of pairs:

$(\xi_s = 1, \Upsilon_s = 1, \zeta_s = 5.5)$ and $(\xi_{s+1} = 2, \Upsilon_{s+1} = 4, \zeta_{s+1} = 6)$ such that $p = \zeta_s + \zeta_{s+1} = 11.5$.

On interchanging we get a new pairs $(\xi_s = 1, \Upsilon_{s+1} = 4, \zeta_s{}^{\circ} = 11)$ and $(\xi_{s+1} = 2, \Upsilon_s = 1, \zeta_{s+1}{}^{\circ} = 8.333)$ such that $q = \zeta_s{}^{\circ} + \zeta_{s+1}{}^{\circ} = 19.333$.

Thus, $p \leq q$, that means this interchange will also not affect the result in a better way.

Hence, we apply the procedure for the next set of pairs:

$(\xi_s = 1, \Upsilon_s = 1, \zeta_s = 5.5)$ and $(\xi_{s+1} = 3, \Upsilon_{s+1} = 5, \zeta_{s+1} = 26.5416)$ such that $p = \zeta_s + \zeta_{s+1} = 32.0416$.

On interchanging, we get a new pairs $(\xi_s = 1, \Upsilon_{s+1} = 5, \zeta_s{}^{\circ} = 3)$ and $(\xi_{s+1} = 3, \Upsilon_s = 1, \zeta_{s+1}{}^{\circ} = 8.833)$ such that $q = \zeta_s{}^{\circ} + \zeta_{s+1}{}^{\circ} = 11.833$.

Thus, $p > q$, that means this interchange will provide us with the best optimal solution.

Therefore, the algorithm is terminated.

In this manner, the algorithm is applied for all the other set of pairs, and we noticed that in this problem there is only one interchange which will result into the best optimal solution.

TABLE X. HEURISTIC OPTIMAL SOLUTION

Changed Source	S	D	New Cost	Cost after interchange
3	1	1	5.5	3
-	2	2	2.3	-
-	3	3	6.5	-
-	2	4	6	-
1	3	5	26.5416	8.833

The table X shows that on allocating to the cells X(3, 1), X(2, 2), X(3, 3), X(2, 4), X(1, 5) the best optimal solution is obtained.

Thus, the total cost so obtained is C(3, 1) + F(3, 1) + C(2, 2) + F(2, 2) + C(3, 3) + F(3, 3) + C(2, 4) + F(2, 4) + C(1, 5) + F(1, 5) = 3 + 1 + 2 + 4 + 5.75 + 6 + 6 + 2.5 + 4.5 + 16 = 50.75.

V. CONCLUSION

In real life while transporting bulk goods from one source to terminal, the availability at each source, the requirement at each destination and bulk cost of shipping goods are fuzzy parameters. It has been observed that there is always a hidden cost during the shipment of bulk goods. Therefore, the fixed cost is taken with bulk transportation cost as the fuzzy parameter which has not been studied previously in the literature to the best of my understanding. We have developed a novel ranking system to defuzzify every fuzzy parameter and an algorithm has been introduced. Further, these defuzzied values of fixed cost is reduced and added to the defuzzied bulk cost to obtain a single cost matrix such that it is simpler to optimize the total cost. On applying the proposed algorithm, it has been observed that the total optimal cost for shipping goods is 50.75. In our case, we are unable to compare our result due to unavailability of this type of fuzzy bulk transportation problem with fuzzy fixed cost in the literature. This method of solving supports the decision maker in obtaining the optimum value of the cost which helps the businessperson to reduce his total transportation cost of bulk goods.

REFERENCES

[1] F.L. Hitchcock, "The distribution of a product from several sources too numerous locations", Journal of Mathematics and Physics, vol. 20, pp. 224-230, 1941.

[2] W. Szwarc, "Some remarks on the time transportation problems", Naval Research Logistics Quarterly, vol. 18, pp. 473–485, 1971.

[3] J.K. Sharma, and K. Swarup, "Time minimizing transportation problem", In: Proceedings of Indian Academy of Sciences-Mathematical Sciences India, vol. 86 A, pp. 513-518, 1977.

[4] S. Purushotam, S. Prakash and P. Dhyani, "A transportation problem with minimization of duration and total cost of transportation as high and low priority objectives respectively", Bulletin of the Technical University of Istanbul, vol. 37, pp. 1-11, 1984.

[5] S. Prakash, A.K. Agarwal, and S. Shah, "Nondominated solutions of cost-time trade-off transportation and assignment problems", Opsearch, vol. 25, pp. 126-131, 1988.

[6] E. D. Schell, "Distribution of a product of several properties", In: Proceedings of 2nd Symposium in Linear Programming DCS/ controller H.Q.U.S. Air Force Washington D.C, pp. 615–642, 1955.

[7] P. Pandian, and G. Anuradha, "A new approach for solving solid transportation problems", Applied Mathematical Sciences, vol. 4, pp. 3603-3610, 2010.

[8] C. A. Rautman, R. A. Reid, and E. E. Ryder, "Scheduling the disposal of nuclear waste material in a geologic repository using the transportation model", Operations Research, vol. 41, pp. 459-469, 1993.

[9] M. A. Elazeem, A. A. Mousa, M. El-Shorbagy, S. K. Elagan, Y. A. Elnaga, "Detecting all non-dominated points for Multi-Objective Multi-Index transportation problems", Sustainability, vol. 13, pp. 1372- 1390, 2021.

[10] R. Latpate, and S. S. Kurade, "Multi-objective multi-index transportation model for crude oil using Fuzzy NSGA-II", IEEE Transactions on Intelligent Transportation System, vol. 23, pp. 1347-1356, 2022.

[11] M. H. Warren and B. D. George, "The Fixed Charge Problem", Naval Research Logistics Quarterly, vol. 15, pp. 413- 424, 1968.

[12] H. W. Kuhn and W. J. Baumol, "An approximative algorithm for the fixed-charges transportation problem", Naval Research Logistics Quarterly, vol. 9, pp. 1–15, 1962.

[13] K. G. Murty, "Solving the fixed charge problem by ranking the extreme points", Oper Research, vol. 16, pp. 268-279, 1968.

[14] K. Sandrock, "A simple algorithm for solving small, fixed charge transportation problem", The Journal of the Operational Research Society, vol. 39, pp. 467-475, 1988.

[15] K. Kowalski, "On the structure of the fixed charge transportation problem", International Journal of Mathematical Education in Science and Technology, vol. 36, pp. 879-888, 2004.

[16] J. S. Aguado, "Fixed charge transportation problems: a new heuristic approach based on Lagrangean relaxation and the solving of core problems", Ann Oper Res, vol. 172, pp. 45-69, 2009.

[17] M. L. Balinski, "Fixed Cost Transportation Problems", Naval Research Logistic Quarterly, vol. 8, pp. 41- 54, 1961.

[18] A. Kaufmann, and M. M. Gupta, "Fuzzy Mathematical Models in Engineering and Management Science", Van Nostrand Reinhold, 1988.

[19] Y. Li, K. Ida, and M. Gen, "Evolutionary program for Multicriteria Solid Transportation Problem with Fuzzy Numbers", IEEE International Conference on System, Man and Cybernetics. Information Intelligence and System, pp. 1960 – 1965, 1996.

[20] R. Tuli, S. K. Chauhan, and S. Sharma, "A survey of fuzzy multi-objective linear programming problems", Proceedings of the International Conference on Emerging Trends in Engineering and Management, pp. 410 – 412, 2012.

[21] P. Maliniand, and D. M. Ananthanarayanan, "Solving fuzzy transportation problem using ranking of trapezoidal fuzzy numbers", International Journal of Mathematics Research, vol. 8, pp. 127 – 132, 2016.

[22] V. Vidhya, and K. Ganesan, "Efficient solution of multi objective fuzzy transportation problem", Journal of Physics National Conference on Mathematical Techniques and its Application, vol. 1000, 2018.

[23] A. D. Maio and C. Roveda, "An all zero–one algorithm for a certain class of transportation problems", Operations Research, vol. 19, pp. 1406 – 1418, 1971.

[24] L. R. Foulds, and P. B. Gibbons, "New Algorithms for The Bulk, Zero-One Time Mini-Max Transportation Model", NZOR, vol. 8, 1980.

[25] S. Singh, S. K. Chauhan and K. Tanwar, "A Survey on Bulk Transportation Problem", International Journal of Advanced Research, vol. 5, pp. 1240-1245, 2017.

[26] S. Singh, S. K. Chauhan and K. Tanwar, "A Simple Heuristic Algorithm to Solve the Bulk Transportation Problem", International Journal of Scientific & Engineering Research, vol. 8, 2017.

[27] S. Prakash, M. K. Sharma and A. Singh, "An Efficient Heuristic for Multi-Objective Bulk Transportation Problem", In: International Conference on computers and Industrial Engineering, IEEE, Troyes, France, pp. 1005 – 1009, 2009.

[28] S. Singh, S. K. Chauhan and K. Tanwar, "Efficient Solution of a Multi – Objective Bulk Transportation Problem", International Journal of Advanced Research, vol. 5, pp. 2337-2341, 2017.

[29] A. V. Latha, "Three-dimensional time minimization Bulk Transportation problem", International Journal of Research in Mathematics and Computation, vol. 1, pp. 26-40, 2013.

[30] S. Singh, S. K. Chauhan and K. Tanwar, "Time Minimizing Multi-Index Bulk Transportation Problem", Journal of Mathematics and Informatics, vol. 13, pp. 13-19, 2018.

[31] S. Singh, S. K. Chauhan and K. Tanwar, "A Bi-Criteria Multi-Index Bulk Transportation Problem", Annals of Pure and Applied Mathematics, vol. 16, pp. 479-485, 2018.

[32] S. K. Chauhan and P. Khanna, "Efficient Solution for Bi-Criteria Multi Index Bulk Transportation Problem", In: Proc. ICRITO'21, IEEE, Noida, India, pp. 1- 4, 2021.

[33] S. K. Chauhan, R. Tuli, N. Sindhwani, and P. Khanna, "Optimal Solutions of the Bulk Transportation Problem with Two Criteria and Two Modes of Transportation", In: Proc. International Mobile and Embedded Technology Conference (MECON), IEEE, Noida, India, pp. 471 – 474, 2022.

[34] K. Tanwar and S. K. Chauhan, "A Simple Algorithm for Fuzzy Time Minimizing Bulk Transportation Problem", 9th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), 2021

[35] S. K. Chauhan, P. Khanna, N. Sindhwani, K. Saxena and R. Anand, "Pareto Optimal Solution for Fully Fuzzy Bi-criteria Multi-index Bulk Transportation Problem" In: Conference Proceedings of Mobile Radio Communication and 5G Network, pp. 457-470, 2023.

[36] B. Kaushal and S. Arora, "Fixed charge bulk transportation problem", Operations Research and Optimization, pp. 315-331, 2018.

[37] B. Kaushal and S. Arora, "Extension of Fixed charge bulk transportation problem", Advanced Modeling and Optimization, vol. 19, pp. 199-212, 2017.