COMMON FIXED POINT THEOREMS FOR HYBRID PAIRS OF MAPS IN FUZZY METRIC-LIKE SPACES BY DISTANCE ADJUSTMENT

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Abstract

The primary goal of this manuscript is to discover some common fixed point theorems of tangential mappings for hybrid pairs of both types of mappings (single and multi-valued) in fuzzy metric-like spaces and make the most of the varying distance function. These theorems extend and generalize a number of previously established findings. Additionally, we employed a few implicit function functions in this case to support our findings. To confirm the accuracy of the presumptions and the level of stimulus generalization of our primary conclusion, we also gave an illustrated case.

Keywords: Fuzzy metric space, Fuzzy metric-like space, the property (E.A), hybrid maps, altering distance.

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1. Introduction and Preliminaries

The concept of a fuzzy set, developed by Zadeh ([1]) in 1965 to capture the ambiguity in ordinary life, is the foundation of fuzzy mathematics. Numerous issues are frequently represented in mathematical programming as the optimization of suitable target functions that are outfitted with particular restrictions that are suggested by some concrete practical challenge because of its concrete scenario. There are several real-world issues that take into account multiple goals, and it is typically exceedingly challenging to find a workable solution that can achieve the optimum of all the goal functions. The usage of fuzzy sets (e.g., Turkoglu) provides a workable solution to such issues. In fact, the variety of applications has facilitated fuzzy logic's overall development (e.g. [2]). In actuality, the abundance of applications has facilitated the overall advancement of fuzzy mathematics. The study of fuzzy metric space has been done in a variety of methods, just like many other ideas (see, for example, [3,4]) . In order to obtain a Hausdorff topology on fuzzy metric spaces, George and Veeramani [5] modified the notion of fuzzy metric space first introduced by Kramosil and Michalek in [6] . This modification has recently found very successful applications in quantum particle physics, particularly in string theory and e1 theory (e.g. [7] and references cited therein). In metric and fuzzy metric spaces, various writers have recently demonstrated fixed and common fixed point theorems. We provide a few examples [2,8,9,10,11,12,13,14,15,16,17,18,19,20]. Before presenting our results, we collect relevant background materials as follows.

Definition 1.1. [21,22] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is said to be continuous t-norm if it satisfies the following conditions:

1. $*$ is commutative and associative;
2. $*$ is continuous;
3. $a * 1 = a$ for all $a \in [0,1]$;
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

For classical examples of continuous t-norm, we recall t-norms T_l, T_p and T_m defined as $T_l(a,b) = \max(a+b-1,0), T_p(a,b) = ab$ and $T_m(a,b) = \min(a,b)$ respectively.
A fuzzy metric space in the sense of George and Veeramani [5] is defined as follows:

Definition 1.2 [5] The 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$:

1. $M(x, y, t) > 0$;
2. $M(x, y, t) = 1 \; \forall t > 0$ iff $x = y$;
3. $M(x, y, t) = M(y, x, t)$;
4. $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;
5. $M(x, y, .) : (0, \infty) \rightarrow [0,1]$ is continuous.

It is worth pointing out that due to (GV-1) and (GV-2), $0 < M(x, y, t) < 1$ for all $t > 0$ provided $x \neq y$, (cf. [23]). In what follows, fuzzy metric spaces in the sense of George and Veeramani will be called GV-fuzzy metric spaces. It is known that, for all $x, y \in X$, $M(x, y, .)$ is non decreasing function. Several examples of fuzzy metric spaces can be found in George and Veeramani [5], Sapena [24], Gregori et al. [25] and Roldan et al. [26].

Remark 1.3. [27] The function $M(x, y, t)$ is often interpreted as the nearness between x and y with respect to t.

Remark 1.4. [28] For every $x, y \in X$, the mapping $M(x, y, .)$ is nondecreasing on $(0, \infty)$.

Definition 1.5. [29] The 3-tuple $(X, F, *)$ is a fuzzy metric like space if X is an arbitrary set $*$ is continouos norm and F is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$:

1. (FML-1) $F(x, y, t) > 0$;
2. (FML-2) If $F(x, y, t) = 1 \; \forall t > 0$ then $x = y$;
3. (FML-3) $F(x, y, t) = F(y, x, t)$;
4. (FML-4) $F(x, z, t + s) \geq F(x, y, t) * F(y, z, s)$;
5. (FML-5) $F(x, y, .) : (0, \infty) \rightarrow [0,1]$ is continuous.

Here F (endowed with $*$) is called a fuzzy metric like on X.

Remark 1.6. A fuzzy metric-like space satisfies all of the conditions of a fuzzy metric space except that $F(x,x,t)$ may be less than 1 for all $t>0$ and for some (or may be for all) $x \in X$. Also, every fuzzy metric space is fuzzy metric-like space with unit self fuzzy distance, that is, with $F(x,x,t)=1$ for all $t>0$ and for all $x \in X$.
Note that, the axiom (GV-2) in Definition 1.2 gives the idea that when $x=y$ the degree of nearness of x and y is perfect, or simply 1, and then $M(x,x,t)=1$ for each $x \in X$ and for each $t>0$. While in fuzzy metric-like space, $F(x,x,t)$ may be less than 1, that is, the concept of fuzzy metric-like is applicable when the degree of nearness of x and y is not perfect for the case $x=y$.

Example 1.7. If $X=[0,1]$, then the triplet $(X,F,*_l)$ is a fuzzy metric-like space, where the fuzzy set F is defined by

$$F(x,y,t)=\begin{cases} 1, & If\ x=y=0, \\ \dfrac{x+y}{2}, & otherwise \end{cases}$$

for all $t>0$.
Using the following propositions, several examples of fuzzy metric-like spaces can be obtained.

Proposition 1.8. [29] Let (X,σ) be any metric-like space (for the related definitions we refer to Harandi [30]. Then the triplet $(X,F,*_p)$ is a fuzzy metric-like space, where the fuzzy set F is given by

$$F(x,y,t)=\frac{kt^n}{kt^n+m\sigma(x,y)}$$

for all $x,y \in X,\ t>0$, where $k \in \mathbb{R},\ m>0$ and $n \geq 1$.

Remark 1.9. [29] Proposition 1.8. shows that every metric-like space induces a fuzzy metric-like spaces. For $k=n=m=1$ the induced fuzzy metric-like space $(X,F_\sigma,*_p)$ is called the standard fuzzy metric-like space, where

$$F_\sigma(x,y,t)=\frac{t}{t+\sigma(x,y)}$$

for all $x,y \in X,\ t>0$.

Proposition 1.10. [29] Let (X,σ) be any metric-like space. Then the triplet $(X,F,*_p)$ is a fuzzy metric-like space, where the fuzzy set F is defined by

$$F(x,y,t)=e^{-\frac{\sigma(x,y)}{t^n}}$$

for all $x,y \in X,\ t>0$, where $n \geq 1$.

Example 1.11. Let $X=N$. Define $*$ by $a*b=ab$ and the fuzzy set F in $X^2 \times (0,\infty)$ by

$$F(x,y,t)=\frac{1}{e^{\max\{x,y\}/t}}$$

for all $x,y \in X, t>0$. Then since $\sigma(x,y)=\max(x,y)$ for all $x,y \in X$ is a fuzzy metric-like on X (see [30]) therefore by Proposition 1.10 $(X,F,*)$ is a fuzzy metric-like space, but not a fuzzy metric space, as $F(x,x,t)=\frac{1}{e^{x/t}} \neq 1$ for all $x>0$ and $t>0$.

Example 1.12. ([29])Let $X=[0,1]$. Define $*$ by $a*b=ab$ and the fuzzy set F in $X^2 \times (0,\infty)$ by

$$F(x,y,t)=\begin{cases} \dfrac{x}{y^3} & if\ x \leq y \\ \dfrac{y}{x^3} & if\ y \leq x, \end{cases}$$

for all $x,y \in X, t>0$. Then $(X,F,*)$ is a fuzzy metric-like space.

We point out that the Propositions 1.8 and 1.10 are also hold even if we employ the minimum t −norm $*_m$ rather than product t −norm $*_p$ (see [29]).

Poposition 1.13. Let (X, σ) be the bounded metric-like space, that is there exists $K > 0$ such that $\sigma(x, y) \leq K$ for all $x, y \in X$. Then the triplet $(X, F, *_l)$ is a fuzzy metric-like space, where the fuzzy set F is defined by

$$F(x, y, t) = 1 - \frac{\sigma(x, y)}{K + t}$$

for all $x, y \in X,\ t > 0$.

Proof. The proofs of the properties (FML1)-(FML5) are obvious. For (FML4), let $x, y, z \in X,\ t > 0$, then since $\sigma(x, y) + \sigma(y, z) \geq \sigma(x, z)$, we have

$$1 - \frac{\sigma(x, y) + \sigma(y, z)}{K + t} \leq 1 - \frac{\sigma(x, z)}{K + t}.$$

It follows from the above inequality that

$$max\left\{1 - \frac{\sigma(x, y) + \sigma(y, z)}{K + t}, 0\right\} \leq 1 - \frac{\sigma(x, z)}{K + t}.$$

which implies that (FML4) holds.

Definition 1.14. [31] Let $CB(X)$ be the set of all nonempty closed bounded subsets of X. Then for every $A, B, C \in CB(X)$ and $t > 0$,

$$F(A, B, t) = \min\{\min F(a, B, t), \min F(A, b, t)\}$$

where $F(C, y, t) = \max\{F(z, y, t) : z \in C\}$.

Remark 1.15. [32] Obviously $F(A, B, t) = F(a, B, t)$ whenever $a \in A$ and $F(A, B, t) = 1$ iff $A = B$. Obviously, $1 = F(A, B, t) \leq F(a, B, t)$ for all $a \in A$.

We now discuss the completeness of fuzzy metric-like spaces as well as convergent and Cauchy sequences in such spaces.

Definition 1.16. [29] Let $(X, F, *)$ be a fuzzy metric-like space and $\{x_n\}$ be a sequence in X. Then

1. $\{x_n\}$ is said to be convergent to $x \in X$ and x is called a limit of $\{x_n\}$ if for all $t > 0$,
$$\lim_{n\to\infty} F(x_n, x, t) = F(x, x, t)$$
2. $\{x_n\}$ is said to be Cauchy if, for all $t > 0$ and each $p \geq 1$, the limit $\lim_{n\to\infty} F(x_{n+p}, x_n, t)$ exists.
3. $(X, F, *)$ is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to some $x \in X$ such that $\lim_{n\to\infty} F(x_n, x, t) = F(x, x, t) = \lim_{n\to\infty} F(x_{n+p}, x_n, t)$ for all $t > 0$ and each $p \geq 1$.

R**emark 1.17.** [29] A convergent sequence's limit does not necessarily have to be unique or a Cauchy sequence in a space with fuzzy metrics.

Definition 1.18. [34] Let $CL(X)$ be the set of all nonempty closed subsets of a metric space (X, d) and $S: Y \subset X \to CL(X)$. Then the map $f: Y \to X$ is said to be S-weakly commuting at $x \in X$ if $ffx \in Sfx$ provided that $fx \in Y$ for all $x \in Y$.

Definition 1.19. Two pairs (f, S) and (g, T) of self mappings of a fuzzy metric-like space $(X, F, *)$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $(\forall\ t > 0)$

$$\lim_{n\to\infty} F(fx_n, u, t) = \lim_{n\to\infty} F(Sx_n, u, t) = \lim_{n\to\infty} F(gy_n, u, t) = \lim_{n\to\infty} F(Ty_n, u, t) = 1.$$

for some $u \in X$.

Definition 1.20. Let $f, g: X \to X$ and $S, T: X \to CB(X)$ of fuzzy metric-like space $(X, M, *)$. Then the hybrid pair of mappings (f, S) and (g, T) are said to satisfy the common property (E.A) if there exist two sequences $\{xn\}$ and $\{yn\}$ in X, some $u \in X$ and $A, B \in CB(X)$ such that

$$\lim_{n\to\infty} Sx_n = A,\ \ \lim_{n\to\infty} Gy_n = B,\ \ \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A \cap B.$$

for some $u \in X$.

Definition 1.21. [8] Let (X, d) be metric space, $f, g: X \in X$ and $S, T: X \to CL(X)$. Then the hybrid pair (f, S) is said to be g-tangential at $u \in X$ if there exist two sequences $\{x_n\},\ \{y_n\}$ in X such that $\lim_{n\to\infty} Ty_n \in CL(X)$ and

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A = \lim_{n\to\infty} Sx_n.$$

Remark 1.22. [8] If the hybrid pair of mappings (f,S) and (g,T) satisfies the common property (E.A), then (f,S) is g-tangential whereas (g,T) is f-tangential but not conversely (in general).

Definition 1.23. [34] Let (X,d) be metric space, if $f,g:Y\subset X\in X$ and $S,T:Y\to CL(X)$, then the hybrid pair (f,S) is said to be g-tangential at $u\in Y$ with respect to T if there exist two sequences $\{x_n\}$, $\{y_n\}$ and $A\in CL(X)$ in Y such that $\lim_{n\to\infty} Ty_n\in CL(X)$ and

$$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gy_n=u\in A=\lim_{n\to\infty} Sx_n.$$

Remark 1.24. [34] The hybrid pairs of mappings (f,S) and (g,T) satisfy the common property (E.A) if and only if (f,S) is g-tangential with respect to G and (g,S) is f-tangential with respect to S but the converse is not necessary true. Notice that the common (E.A) property reduces to E.A property (cf. [35]) if we restrict to a single pair.

Definition 1.25. [34] A map $f:Y\subset X\to X$ is said to be coincidentally idempotent w.r.t. a mapping $S:Y\to CL(X)$ if f is idempotent at the coincidence points of (f,S), i.e., $ffx=fx$ for all $x\in X$ with $fx\subset Sx$ provided that $fx\in Y$.

2. Implicit Relations

Motivated by Ali and Imdad , we define an implicit function as follows:
Let Φ be the set of all functions $\phi(t_1,t_2,\cdots,t_6):[0,1]^6\to[0,1]$, satisfying the following conditions:

1. (ϕ_1) ϕ is non increasing in $3^{rd},4^{th},5^{th},6^{th}$;
2. (ϕ_2) if $\phi(u,0,0,u,u,0)\ge 0$ or
3. (ϕ_3) $\phi(u,0,u,0,0,u)\ge 0,\ \forall\ u\in[0,1]$ implies $u\ge 0$.

The following examples satisfy (ϕ_1), (ϕ_2), and (ϕ_3).
Example 2.1. Define $\phi(t_1,t_2,\cdots,t_6):[0,1]^6\to[0,1]$ as

$$\phi(t_1,t_2,\cdots,t_6)=t_1-\alpha\min\{t_2,t_3,t_4,t_5,t_6\},\quad \text{where } \alpha>1.$$

Example 2.2. Define $\phi(t_1,t_2,\cdots,t_6):[0,1]^6\to[0,1]$ as

$$\phi(t_1,t_2,\cdots,t_6)=t_1^3-c_1\min\{t_2^2,t_3^2,t_4^2\}-c_2\min\{t_3t_6,t_4t_5\},$$

where $c_1,c_2,c_3>0,\quad c_1+c_2>1,\quad c_1\ge 1$.
Example 2.3. Define $\phi(t_1,t_2,\cdots,t_6):[0,1]^6\to[0,1]$ as

$$\phi(t_1,t_2,\cdots,t_6)=t_1^3-a\min\{t_1^2t_2,t_1t_3t_4,t_5^2t_6,t_5t_6^2\},$$

where $a>1$.
Example 2.4. Define $\phi(t_1,t_2,\cdots,t_6):[0,1]^6\to[0,1]$ as

$$\phi(t_1,t_2,\cdots,t_6)=t_1-a_1t_2-a_2t_3-a_3t_4-a_4t_5-a_5t_6,$$

where $a_1,a_2,a_3,a_4,a_5>0,\ a_2+a_5\ge 1,\ a_3+a_4\ge 1$ and $a_1+a_4+a_5\ge 1$.
It is simple to generate a number of additional instances that meet the conditions of the preceding implicit function.

3. Common fixed points in fuzzy metric-like spaces

Firstly, we rewrite Definition 1.21,1.23 and 1.25.

Definition 3.1. Let $(X,F,*)$ be fuzzy metric-like space, $f,g:X\in X$ and $S,T:X\to CL(X)$. Then the hybrid pair (f,S) is said to be g-tangential at $u\in X$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ty_n\in CL(X)$ and

$$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gy_n=u\in A=\lim_{n\to\infty} Sx_n.$$

Definition 3.2. Let $(X,F,*)$ be fuzzy metric-like space, if $f,g:Y\subset X\in X$ and $S,T:Y\to CL(X)$, then the hybrid pair (f,S) is said to be g-tangential at $u\in Y$ with respect to T if there exist two sequences $\{x_n\}$ and $\{y_n\}$ and $A\in CL(X)$ in Y such that $\lim_{n\to\infty} Ty_n\in CL(X)$ and

$$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gy_n=u\in A=\lim_{n\to\infty} Sx_n.$$

Definition 3.3. Let $(X,F,*)$ be fuzzy metric-like space. A map $f:Y\subset X\to X$ is said to be coincidentally idempotent w.r.t. a mapping $S:Y\to CL(X)$ if f is idempotent at the coincidence points of (f,S), i.e., $ffx=fx$ for all $x\in X$ with $fx\in Sx$ provided that $fx\in Y$.

Remark 3.4. If the hybrid pair of mappings (f,S) and (g,T) satisfy the common property (E.A), then (f,S) is g-tangential with respect to T whereas (g,T) is f-tangential with respect to S but the converse is not necessarily true.

Definition 3.5. A function $\varphi:[0,1]\to[0,1]$ is called an altering distance function if it satisfies the followings;

1. (a) φ is strictly decreasing and continuous;
2. (b) $\varphi(\lambda) = 0$ if and only if $\lambda = 1$

Now, we prove our main theorem as follows.

Theorem 3.6. Let $f, g: Y \subset X \to X$ be two mappings from a subset Y of a fuzzy metric-like space $(X, F, *)$ into X and $S, T: Y \to CL(X)$ which satisfy the following conditions:

(φ_{11}) the hybrid pair (f, S) is g-tangential at $u \in X$ with respect to T (or the hybrid pair (g, T) is f-tangential at $u \in X$ with respect to S),

(φ_{22}) there exists $\phi \in \Phi$ and φ is an altering distance function for all $x, y \in X$, such that

$$\phi(\varphi(F(Sx,Ty,t)), \varphi(F(fx,gy,t)), \varphi(F(fx,Sx,t)), \varphi(F(gy,Ty,t)), \varphi(F(fx,Ty,t)), \varphi(F(gy,Sx,t))) \geq 0,$$

for all $x, y \in X$.

Then

I. the hybrid pair (f, S) have a coincidence point $v \in Y$ provided that $f(Y)$ is a closed subset of X;

II. the hybrid pair (g, T) have a coincidence point $w \in Y$ provided that $g(Y)$ is a closed subset of X;

III. the hybrid pair (f, S) have a common fixed point provided that f is S-weakly commuting at $v \in X$, $ffv = fv$ and $fv \in Y$;

IV. the hybrid pair (g, T) have a common fixed point provided that g is T-weakly commuting at $w \in Y$, $ggw = gw$ and $gw \in Y$;

V. f, g, S, T have a common fixed point provided that both (III) and (IV) are true.

Since the hybrid pair (f, S) is g-tangential at $u \in Y$ with respect to T if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Y and $A, B \in CL(X)$ such that $\lim_{n\to\infty} Ty_n = B$ and

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A = \lim_{n\to\infty} Sx_n.$$

Now, we proceed to show that $A = B$. To do this, consider

$$\phi(\varphi(F(Sx_n,Ty_n,t)), \varphi(F(fx_n,gy_n,t)), \varphi(F(fx_n,Sx_n,t)), \varphi(F(gy_n,Ty_n,t)), \varphi(F(fx_n,Ty_n,t)), \varphi(F(gy_n,Sx_n,t))) \geq 0$$

which on letting $n \to \infty$ gives rise

$$\phi\left(\varphi(F(A,B,t)), \varphi(F(u,u,t)), \varphi(F(u,A,t)), \varphi(F(u,B,t)), \varphi(F(u,B,t)), \varphi(F(u,A,t)\right) \geq 0$$

so that

$$\phi(\varphi(F(A,B,t)), 0, 0, \varphi(F(A,B,t)), \varphi(F(A,B,t)), 0)$$
$$\geq \phi(\varphi(F(A,B,t)), 0, \varphi(F(u,A,t)), \varphi(F(u,B,t)), \varphi(F(u,B,t)), \varphi(F(u,A,t)) \geq 0.$$

Owing to (ϕ_2) and (φ_{22}), we have $F(A, B, t) = 1$ so that $A = B$.

To prove (I), let $f(Y)$ is closed, then there exists some $v \in Y$ such that $u = fv$. Now, we show that $A = Sv$. To accomplish this, consider

$$\phi(\varphi(F(Sv,Ty_n,t)), \varphi(F(fv,gy_n,t)), \varphi(F(fv,Sv,t)), \varphi(F(gy_n,Ty_n,t)), \varphi(F(fv,Ty_n,t)), \varphi(F(gy_n,Sv,t))) \geq 0$$

which on letting $n \to \infty$ gives rise

$$\phi\left(\varphi(F(Sv,A,t)), \varphi(F(fv,u,t)), \varphi(F(fv,Sv,t)), \varphi(F(u,A,t)), \varphi(F(fv,A,t)), \varphi(F(u,Sv,t))\right) \geq 0$$

so that

$$\phi\left(\varphi(F(Sv,A,t)), 0, \varphi(F(A,Sv,t)), 0, 0, \varphi(F(A,Sv,t))\right)$$
$$\geq \phi\left(\varphi(F(Sv,A,t)), 0, \varphi(F(u,Sv,t)), \varphi(F(u,A,t)), \varphi(F(u,A,t)), \varphi(F(u,Sv,t))\right) \geq 0.$$

Owing to (ϕ_2) and (φ_{22}), this gets us $F(A, Sv, t) = 1$ which implies $A = Sv$. Then $fv \in Sv$ this proves (I). The proof of (II) is similar to that of (I). In order to prove (III), using the conditions given in (III), we have $ffv = fv$ and $ffv \in Sfv$ so that $u = fu \in Su$. The proof of (IV) is similar to that of (III) while (V) follows immediately.

In case the hybrid pair (g, T) is f-tangential at $u \in X$ with respect to S, a proof on the lines of the proceeding case can be outlined. This concludes the proof.

A series of multivalued mappings are involved in our following theorem.

Theorem 3.7. Let $\{S_n\}$, $n \in \mathbb{N}$ be a sequence of multi-valued mappings from a subset Y of a fuzzy metric-like space $(X, F, *)$ into $CL(X)$ and $f, g: Y \to X$ which satisfy the following conditions:

a) either the pair (f, S_k) is g-tangential at $u_k \in Y$ with respect to S_l (or the hybrid pair (g, S_l) is f-tangential at $u_l \in Y$ with respect to S_k where $k = 2n - 1$ and $l = 2n$ for all $n \in N$);

b) $\overline{\cup S_k(Y)} \subseteq g(Y)$ and $\overline{\cup S_l(Y)} \subseteq f(Y)$

c) there exists $\phi \in \Phi$ and φ is an altering distance function for all $x, y \in X$ such that

$$\phi(\varphi(F(S_kx,S_ly,t)),\varphi(F(fx,gy,t)),\varphi(F(fx,S_kx,t)),\varphi(F(gy,S_ly,t)),\varphi(F(fx,S_ly,t)),$$
$$\varphi(F(gy,S_kx,t)))\geq 0,$$

for all $x,y\in X$.

Then

I. (f,S_k) have a coincidence point $u_k\in Y$;
II. (g,S_l) have a coincidence point $u_l\in Y$;
III. (f,S_k) have a common fixed point provided that f is S_k-weakly commuting at u_k and f is coincidentally idempotent w.r.t. F_k;
IV. (f,S_l) have a common fixed point provided that f is S_l-weakly commuting at u_l and g is coincidentally idempotent w.r.t. F_l.

Proof. Since the hybrid pair (f,S_k) is g-tangential at $u_k\in Y$ with respect to S_l if there exist two sequences $\{x_{kn}\}$ and $\{y_{kn}\}$ in Y and $A_k,B_k\in CL(X)$ such that $\lim_{n\to\infty}F_ly_{kn}=B_k$ and

$$\lim_{n\to\infty}fx_{kn}=\lim_{n\to\infty}gy_{kn}=u_k\in A_k=\lim_{n\to\infty}S_kx_{kn}.$$

Now, we proceed to show that $A_k=B_k$. To do this, consider

$$\phi(\varphi(F(S_kx_{kn},S_ly_{kn},t)),\varphi(F(fx_{kn},gy_{kn},t)),\varphi(F(fx_{kn},S_kx_{kn},t)),\varphi(F(gy_{kn},S_ly_{kn},t)),$$
$$\varphi(F(fx_{kn},S_ly_{kn},t)),\varphi(F(gy_{kn},S_kx_{kn},t)))\geq 0$$

which on letting $n\to\infty$ gives rise

$$\phi\big(\varphi(F(A_k,B_k,t)),0,0,\varphi(F(u_k,B_k,t)),\varphi(F(u_k,B_k,t)),0\big)\geq 0$$

so that

$$\phi\big(\varphi(F(A_k,B_k,t)),0,0,\varphi(F(A_k,B_k,t)),\varphi(F(A_k,B_k,t)),0\big)$$
$$\geq\phi\Big(\varphi(F(A_k,B_k,t)),0,\varphi(F(u_k,A_k,t)),\varphi(F(u_k,B_k,t)),\varphi(F(u_k,B_k,t)),\varphi(F(u_k,A_k,t))\Big)\geq 0.$$

Owing to (ϕ_2) and (φ_{22}), we have $F(A_k,B_k,t)=1$ so that $A_k=B_k$.

As $u_k\in\overline{\cup S_l(Y)}$ and $\overline{\cup S_l(Y)}\subset f(Y)$, there exist $z_k\in Y$ such that $u_k=fz_k$.

Now, we show that $F_kz_k=A_k$. As

$$\phi(\varphi(F(S_kz_k,S_ly_{kn},t)),\varphi(F(fz_k,gy_{kn},t)),\varphi(F(fz_k,S_kz_k,t)),\varphi(F(gy_{kn},S_ly_{kn},t)),$$
$$\varphi(F(fz_k,S_ly_{kn},t)),\varphi(F(gy_{kn},S_kz_k,t)))\geq 0$$

which on letting $n\to\infty$ reduces to

$$\phi\big(\varphi(F(S_kz_k,A_k,t)),0,0,\varphi(F(u_k,A_k,t)),\varphi(F(u_k,A_k,t)),0\big)\geq 0$$

so that $S_kz_k=A_k$ which proves (I).

The remaining parts are easy to prove. This concludes the proof.

Remark 3.8. Theorem 3.8 is a generalization of Theorem 2 in [37].

One can derive the following corollary from Theorem 3.5 involving a hybrid pair of mappings (f,S) satisfying the property (E.A).

Corollary 3.9. Let $(X,F,*)$ be fuzzy metric-like space. If $f:Y\subset X\to X$ and $S:Y\to CL(X)$ be a pair of hybrid mappings satisfying the following conditions:

a) the pair (f,F) satisfy the property (E.A),
b) φ is an altering distance function for all $x,y\in Y$,

$$\varphi(F(fx,Sy,t))\geq\min\{\varphi(F(fx,fy,t)),\frac{\varphi(F(fx,Sx,t))+\varphi(F(fy,Fy,t))}{2},$$
$$\frac{\varphi(F(fx,Fy,t))+\varphi(F(fy,Sx,t))}{2}\},$$

or

$$\varphi(F(fx,Sy,t))\geq\alpha\min\{\varphi(F(fx,fy,t)),\varphi(F(fx,Sx,t)),\varphi(F(fy,Fy,t)),$$
$$\varphi(F(fx,Fy,t)),\varphi(F(fy,Sx,t))\},$$

where $\alpha>1$, or

$$\varphi(F(fx,Sy,t))\geq a_1\varphi(F(fx,fy,t))+a_2\varphi(F(fx,Sx,t))+a_3\varphi(F(fy,Fy,t))$$
$$+a_4\varphi(F(fx,Fy,t))+a_5\varphi(F(fy,Sx,t)),$$

where $a_1,a_2,a_3,a_4,a_5>0$, $a_3+a_4\geq 1$ and $a_1+a_4+a_5\geq 1$.

If $f(Y)$ is a closed subset of Y , then (f,S) have a common fixed point provided that f is S-weakly commuting at $v\in X$ and $ffv=fv$ for $v\in C(f,S)$.

Remark. 3.40.

I. Theorem 3.6 is a generalization of Theorem 2.8 in [34];
II. Corollary 3.9 is a generalization of Theorem 3.10 in [8].

4. **An illustrative example**

Now, we provide a case study to illustrate the viability of the assumptions and level of generality of our Theorem 3.4 relative to the vast majority of past findings established thus far with a few probable outliers.

Example 4.1. Let $(X, F, *)$ be a fuzzy metric-like space wherein $X = [0,1]$, $a * b = ab$ for all $a, b \in [0,1]$ with

$$F_\sigma(x, y, t) = \frac{t}{t + \sigma(x, y)}$$

where

$$\sigma(x, y) = \begin{cases} 2, & If\ x = y = 0, \\ 1, & otherwise \end{cases}$$

for all $t > 0,\ x, y \in X$. Define $\phi(t_1, t_2, \cdots, t_6): [0,1]^6 \to [0,1]$ as

$$\phi(t_1, t_2, \cdots, t_6) = t_1 - t_2.$$

and define the maps S, T, f, g on X as $Sx = \left[\frac{2x}{3}, 1\right]$, $Tx = [x^2, 1]$ and $fx = \frac{2x}{3}$, $gx = x^2$ for all $x, y \in X$. Define two sequences $\{x_n\} = \{\frac{1}{n}\}$, $\{yn\} = \{\frac{1}{2n}\}$, $n \in \mathbb{N}$ in X. As,

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = 0 \in [0,1] = \lim_{n\to\infty} Sx_n,$$

the hybrid pair (f, S) is g-tangential at $0 \in X$ with respect to T besides

$$\phi(\varphi(F(Sx, Ty, t)), \varphi(F(fx, gy, t)), \varphi(F(fx, Sx, t)), \varphi(F(gy, Ty, t)), \varphi(F(fx, Ty, t)), \varphi(F(gy, Sx, t))) = 0.$$

Thus, all the conditions of Theorem 3.5 are satisfied and 0 remains fixed under all the four involved maps.

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