# Integrable and NonIntegrable Model of Quintic Nonlinear Equation

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## Abstract

The dynamics of an alpha-helical protein chain with quintic nonlinearity are described in this article by identifying a perfectly integrable model. Using Hirota's bilinearisation technique, we analyse the dynamics in the continuum limit and study the solitonic feature of the system by developing multisoliton solutions to the resultant completely integrable two-dimensional protein system. In the Inhomogeneous sitiuation, we use a peturbation technique to investigate the stability of solitons and we find the center of mass and velocity of the soliton.

Keywords: Alpha-helical protein, Soliton, Center of mass, Velocity.

## 1. Introduction

The alpha-helical structure of protein is one of the siginificant types of subordinate reections of proteins. HNCO includes three chains of hydrogen-bonded peptide groups accompanying lateral groups arranged in the form "...H-N-C=O...H-N-C=O...H-N-C=O...". where C=O denotes the amide-1 bond and dotted lines indicate hydrogen bonds. The amide-1 vibrations of atoms in peptide groups perform the most important role in energy transfers in alpha-helical proteins. According to Davydov [1-3] nonlinear self-trapping may aid in the process of energy transfer laterally in almost one-dimensional chains of protein molecules over the development of solitons stirring without energy loss. The dynamical equillibirium between the diffusion due to resonant collabortion of intra-peptide charged particle vibrations and the nonlinearity offered the interaction of these vibrations with the constrained movements of the symmetry positon of the peptide groups results in a soliton along the hydrogen bonding spines in alpha-helical proteins. In this contex, Davydov demonstrated that the fully integrable NLS equation which holds N-Soliton solutions controls the dynamics of alpha-helical proteins. Several physicists have modified Davydov's original model to better accurately represent the dynamics of energy transfer across alpha-helical proteins at the classical and quantum levels [4-22]. Analytical and numerical analyses are included in these work, similarly the effects of higher order interactions and excitations as well as inter spine copling on the dynamics of an alpha-helical protein molecule have been examined at both the discrete and continuum levels. Also, inhomogeneous alpha-helical protein structures play an important role in the energy transfer mechanism. The defects caused due to the presence of additional molecules such as drugs in specific sites of the sequence and the presence of abasic site like nonpolar mimic of thymine leads to inhomogeneity in alpha-helical proteins [23]. In this study, we investigate the quintic type non-linearity in the two-dimensional alpha-helical protein system. We recommend a model including quintic type non-linearity in the hamiltonian. This paper is encircled as follows section 2 deals with the model hamiltonian for two dimensional alpha-helical proteins is introduced and the equations of motion are constructed. The integrable system of nonlinear partial differential equation is recast into a system of bilinear equation through Hirota's bilinearization technique the one, two and the three-soliton solutions are given in section 3. Section 4 explains the inhomogeneous effect of quintic nonlinearity. The impact of nonlinear type inhomogeneities on the transfer of energy through alpha-helical protein the centre of mass and velocity is covered in section 5. Section 6 represents a concluding report on the workdone.

## 2. Model Hamiltonian and Equation of motion

The interaction between the neighbouring chain of alpha-helical protein has also been considered in this model resulting in a 2D model. The Hamiltonian model can be used to simulate it

$$H_{2D} = H_{ex} + H_{ph} + H_{ph-ex} \qquad (1)$$

In Eqn. (1) $H_{ex}$ denotes the exchange hamiltonian, which represents internal molecular excitations, $H_{ph}$ denotes the phonon hamiltonian's contribution, which corresponds to the displacement of unit cells from their equillibirium position and $H_{ph-ex}$ denotes the hamiltonian for the coupling between the internal molecular excitations and the displacement. The contribution resulting from unperturbed excitons might thus be written as,

$$H_{ex} = \sum_{\alpha,\beta} \{\phi^{\dagger}_{\alpha,\beta} E_0 \phi_{\alpha,\beta} + \phi^{\dagger}_{\alpha,\beta} E_1 \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} - \sum_{\rho=1,-1} J_1 (\phi^{\dagger}_{\alpha,\beta} \phi_{\alpha+\rho,\beta} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+\rho,\beta} + \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta+\rho} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta+\rho}) - J_2 (\phi^{\dagger}_{\alpha,\beta} \phi_{\alpha+\rho,\beta+\rho} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+\rho,\beta+\rho} + \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha+\rho,\beta-\rho} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+\rho,\beta-\rho})\} \quad (2)$$

Where $\alpha, \beta$ are the indices along the x and y axes. $\phi^{\dagger}_{\alpha,\beta}$ creates and $\phi_{\alpha,\beta}$ (annihilates) an excitation on a site $(\alpha, \beta)$. The constant value $E_0$ represents the energy of single exciton of given site. The bouncing exciton between near by sites is represented by $J_1$and $J_2$. In each unit cell $E_1$ stands for the two exciton energies that correspond to the higher order excitations of the molecules.

$$H_{ph} = \sum_{\alpha,\beta} \{\frac{\hat{p}^2_{\alpha,\beta}}{2M} + \frac{k}{2}[(v_{\alpha,\beta} - v_{\alpha-1,\beta})^2 + (v_{\alpha,\beta} - v_{\alpha,\beta-1})^2]\} \quad (3)$$

The parameters M and k in Eqn.(3) denote the mass of a peptide unit and the elastic constant. $v_{\alpha,\beta}$ is the operator for the longitudinal displacement of the peptide group parallel to the helical axis from it's equllibirium position. $\hat{p}_{\alpha,\beta}$ is the momentum conjugate of $v_{\alpha,\beta}$. The interaction between excitons and phonons takes the form

$$H_{ph-ex} = \sum_{\alpha,\beta} \{\phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} [\chi_1 (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} \chi_2 (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta-1} - v_{\alpha,\beta-1})\} \quad (4)$$

where $\chi_1$ and $\chi_2$ represents the change in energy of the amide I bond caused by the streching of the helix between two neighbouring unit cells of dipole-dipole type and quadrupole-quadrupole type of excitation respectively. The Hamiltonian for collective excitations of coherent states in the Davydov ansatz [3] for 2D lattice is written as,

$$H_{2D} = \sum_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} E_0 \phi_{\alpha,\beta} + \phi^{\dagger}_{\alpha,\beta} E_1 \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} - J_1 (\phi^{\dagger}_{\alpha,\beta} \phi_{\alpha+1,\beta} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+1,\beta} + \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta+1} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta+1}) - J_2 (\phi^{\dagger}_{\alpha,\beta} \phi_{\alpha+1,\beta+1} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+1,\beta+1} + \phi_{\alpha+1,\beta-1} \phi^{\dagger}_{\alpha,\beta} + \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha+1,\beta-1}) + \frac{\hat{p}^2_{\alpha,\beta}}{2M} + \frac{k}{2}[(v_{\alpha,\beta} - v_{\alpha-1,\beta})^2 + (v_{\alpha,\beta} - v_{\alpha,\beta-1})^2] + \chi_1 (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) + \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} \phi^{\dagger}_{\alpha,\beta} \phi_{\alpha,\beta} \chi_2 (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) \quad (5)$$

is the deformation energy of the spines. we drive the equation of motion for the dynamical variables $\phi_{\alpha,\beta}, v_{\alpha,\beta}, p_{\alpha,\beta}$ which can be represented by

$$i\hbar\frac{d\Phi_{\alpha,\beta}}{dt} = [\Phi_{\alpha,\beta}, H] \tag{6}$$

where $\Phi_{\alpha,\beta}$ stands for anyone of the above variables complying the commutation relations $[\phi_{\alpha,\beta}, \phi^{\dagger}_{\alpha,\beta}] = 1$ and the conjugate variable $[v_{\alpha,\beta}, p_{\alpha,\beta}] = i\hbar$. The normalization condition for the function leads to the equality $\Sigma_{\alpha,\beta}|\phi_{\alpha,\beta}|^2 = 1$ . Therefore $|\phi_{\alpha,\beta}|^2$ characterizes the probability for exciting $\alpha$ and $\beta$ molecule in the chain. By applying the hamiltonian Eqn. (5) in Eqn. (6), the equations of motion for the variables $\phi_{\alpha,\beta}, v_{\alpha,\beta}$ and $p_{\alpha,\beta}$ can be written as,

$$\begin{aligned} i\hbar\frac{d\phi_{\alpha,\beta}}{dt} = {} & E_0\phi_{\alpha,\beta} + 2E_1\phi^2_{\alpha,\beta}\phi^{\dagger}_{\alpha,\beta} - J_1[\phi_{\alpha+1,\beta} + \phi_{\alpha-1,\beta} + \phi_{\alpha,\beta+1} \\ & +\phi_{\alpha,\beta-1}] - J_2[\phi_{\alpha+1,\beta+1} + \phi_{\alpha-1,\beta-1} + \phi_{\alpha+1,\beta-1} + \\ & \phi_{\alpha-1,\beta+1}] + \phi_{\alpha,\beta}\chi_1[v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}] \\ & -2\phi^{\dagger}_{\alpha,\beta}\phi^2_{\alpha,\beta}\chi_2[v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}] \end{aligned} \tag{7}$$

$$\begin{aligned} M\frac{d^2v_{\alpha,\beta}}{dt^2} = {} & -k(4v_{\alpha,\beta} - v_{\alpha+1,\beta} - v_{\alpha-1,\beta} - v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) + \chi_1 \\ & [|\phi_{\alpha+1,\beta}|^2 - |\phi_{\alpha-1,\beta}|^2 + |\phi_{\alpha,\beta+1}|^2 - |\phi_{\alpha,\beta-1}|] + \\ & \chi_2[|\phi_{\alpha+1,\beta}|^4 - |\phi_{\alpha-1,\beta}|^4 + |\phi_{\alpha,\beta+1}|^4 - |\phi_{\alpha,\beta-1}|]^4 \end{aligned} \tag{8}$$

Eqn. (7) and (8) represent the discrete dynamics of higher dimensional alpha-helical. Because these equations are difficult we use the Taylor's expansion to find the continuum limit and the resulting equations are,

$$\begin{aligned} i\hbar\phi_t = {} & -[(4(J_1 + J_2) - E_0)\phi + 2E_1|\phi|^2\phi + \epsilon[2\chi_1(v_x + v_y)\phi + 4\chi_2(v_x + \\ & v_y)|\phi|^2\phi] - \epsilon^2[(J_1 + 2J_2)(\phi_{xx} + \phi_{yy}) - 4J_2\phi_{xy}] \end{aligned} \tag{9}$$

$$Mv_{tt} = K\epsilon^2[v_{xx} + v_{yy}] + 2(\chi_1 + \chi_2)\epsilon[(|\phi|^2)_x + (|\phi|)_y] \tag{10}$$

Introducing the wave variable $\xi = k_1x + k_2y - ct$ in Eqns. (9) and (10) and solving Eqn. (10), we get $u_\xi = 2(\chi_1 + \chi_2)A|\phi|^2$ and using it in Eqn. (9) we get,

$$i\phi_t + a_1\phi + a_2(\phi_{xx} + \phi_{yy}) + a_3\phi_{xy} - a_4|\phi|^2\phi - a_5|\phi|^4\phi = 0 \tag{11}$$

where,

$$a_1 = [\tfrac{4(J_1+J_2)-E_0}{\hbar}],\ a_2 = [\tfrac{\epsilon^2(J_1+2J_2)}{\hbar}],\ a_3 = \tfrac{2\epsilon^2 4J_2}{\hbar},\ a_4 = [\tfrac{4A\epsilon\chi_1(\chi_1+\chi_2)+2E_1}{\hbar}],$$

$$a_5 = [\tfrac{8\epsilon A\chi_2(\chi_1+\chi_2)}{\hbar}], \text{ and } A = \frac{\epsilon^2(k_1+k_2)}{Mc^2 - k\epsilon^2(k_1^2+k_2^2)}.$$

After making the transformation $\phi = \phi exp(-ia_1t)$ we get,

$$i\phi_t + a_2(\phi_{xx} + \phi_{yy}) + a_3\phi_{xy} - a_4|\phi|^2\phi - a_5|\phi|^4\phi = 0 \tag{12}$$

For further transformation $t \to \frac{t}{a_2}$, After making the Galilean transformation $X = (c - t), t \to T, y \to Y$ in Eqn. (12) we get,

$$i\phi_t - i\phi_X + \phi_{XX} + \phi_{YY} - 2\phi_{XY} - 2|\phi|^2\phi - 4|\phi|^4\phi = 0 \tag{13}$$

In higher dimensions, Eqn. (13) represents the dynamics of an extended Davydov model of alpha-helical proteins. It has entirely integrable NLS type equation (2+1) dimensions. It can be solved using the Bilinearization technique for multisoliton solutions and the details are described in the next section.

## 3. Bilinearization Procedure

To use the Hirota's bilinearization procedure we convert Eqn. (13) with the new dependent variables g and f as [24]

$$\phi = \frac{g}{f} \tag{14}$$

Where g(X,Y,T) is a complex function and f(X,Y,T) is a real function. substituting Eqn. (14) in (13) and making use of the properties of the Hirota operator D defined as,

$$D_X^l D_Y^m D_T^n f.g = (\frac{\partial}{\partial X} - \frac{\partial}{\partial X'})^l (\frac{\partial}{\partial Y} - \frac{\partial}{\partial Y'})^m.$$

$$(\frac{\partial}{\partial T} - \frac{\partial}{\partial T'})^n f(X,Y,T).g(X',Y',T')|X = X', Y = Y', T = T'. \tag{15}$$

We write Eqn. (13) in the bilinear form

$$[iD_T - iD_X - D_X^2 - D_Y^2 + 2D_X D_Y](g.f) = 0 \tag{16}$$

$$[D_X^2 + D_Y^2 - 2D_X D_Y](f^2.f^2) + 2(g.g^*)(f.f) + 4(g^2.g^{*2}) = 0 \tag{17}$$

For finding soliton solutions, we expand the functions g and f in power series given by,

$$g = \gamma g^{(1)} + \gamma^3 g^{(3)} + \ldots.. \tag{18}$$

$$f = 1 + \gamma^2 f^{(2)} + \gamma^4 f^{(4)} + \ldots... \tag{19}$$

where $\gamma$ is an arbitrary small parameter. The solutions can be found by substituting Eqns. (18) and (19) in Eqns. (16) and (17) and collecting terms proportional to different powers of $\gamma$ and solving the resultant equation.

### 3.1. One-soliton solution

To obtain the one-soliton solution, we choose

$$g = \gamma g^{(1)} \tag{20}$$

$$f = 1 + \gamma^{(2)} f^{(2)} \tag{21}$$

substituting Eqns. (20) and (21) in Eqns. (16) and (17) and collecting terms proportional to different powers of $\gamma$ we obtain the following set of equations:

$$\gamma^{(1)}: [iD_T - iD_X - D_X^2 - D_Y^2 + 2D_X D_Y] g^{(1)} = 0 \quad (22)$$

$$\gamma^2: [D_X^2 + D_Y^2 - 2D_X D_Y](4f^{(2)} + 2g^{(1)}g^{*(1)}) = 0 \quad (23)$$

$$\gamma^3: [iD_T - iD_X - D_X^2 - D_Y^2 + 2D_X Y](g^{(1)}.f^{(2)}) = 0 \quad (24)$$

$$\gamma^4: [D_X^2 + D_Y^2 - 2D_X D_Y](6f.^{(2)}.f^{(2)} + 4f^{(2)}g^{(1)}g^{(*1)} + g^{(1)2}. g^{(1)*2}) = 0 \quad (25)$$

we ensure that the solutions are compatible with the system of Eqns. (22), (23), (24) and (25) are

$$g_1^{(1)} = exp(\eta_1), f^{(2)} = aexp(\eta_1 + \eta_1^*) \quad (26)$$

where,

$$\eta_1 = k_1 X + k'_1 Y + \omega_1 T + \eta_1^{(0)}, \omega_1 = k_1 - 2i(k_1 - k'_1)^{(2)} \quad (27)$$

and

$$a = \frac{1}{2}\left[\frac{1}{((k_1 + k_1^*) - (k'_1 + k'_1{}^*))^2}\right] \quad (28)$$

In Eqns. (27) and (28), $\eta_1^{(0)}, k_1$ and $k'_1$ are complex constants. As a result we express the one-soliton solution as,

$$\phi = \frac{1}{2} sech\left[\frac{1}{2}(\eta_1 + \eta_1^{(*)} + \eta_1^{(0)})\right]. exp\left[\frac{1}{2}(\eta_1 - \eta_1^{(*)} + \eta_1^{(0)}\right] \quad (29)$$

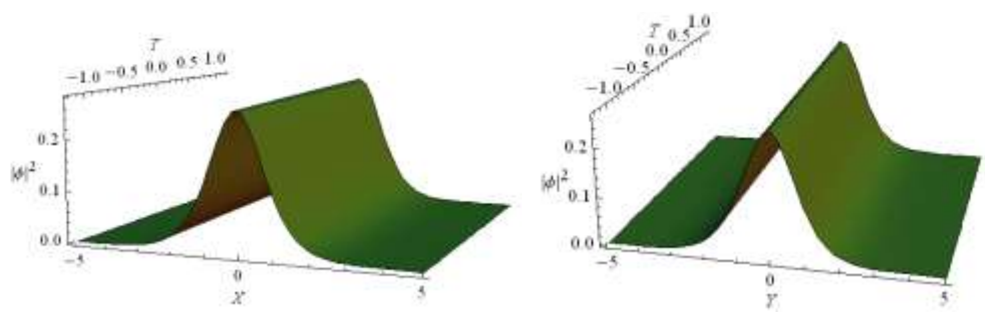



Figure 1: Soliton moving in the (a) X-direction (b) Y-direction

Fig. (1a) shows the soliton moving in the X-direction with $k_1 = 0.9 + i, k'_1 = 1 + i$ and $y = 0.05$. Fig.(1b) represents the soliton along y-direction with $k_1 = 1.5 + i, k'_1 = 0.7 + i$ and $X = 0.05$.

### 3.2. Two-soliton solution

In a similar way, the two-soliton solution can be generated by terminating the series as

$$g = \gamma g^{(1)} + \gamma^3 g^{(3)}, f = 1 + \gamma^2 f^{(2)} + \gamma^4 f^{(4)} \tag{30}$$

Solving the resulting linear partial differential equations, we obtain

$$g = [exp(\eta_1) + exp(\eta_2) + H_{12}exp(\eta_1 + \eta_2 + \eta_1^*) + H_{21}exp(\eta_1 + \eta_2 + \eta_2^*)] \tag{31}$$

and

$$f = 1 + \Sigma_{\alpha,\beta} G_{\alpha,\beta} exp(\eta_\alpha + \eta_\beta^*) + G exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^*) \tag{32}$$

with

$$\eta_2 = k_2 X + k'_2 Y + \omega_2 T + \eta_2^{(0)}, \omega_2 = k_2 - 2i(k_2 - k'_2)^2 \tag{33}$$

$$H_{\alpha,\beta} = \frac{1}{2(k_\alpha + k_\alpha^* + k_\beta - k'_\alpha - k'_\alpha - k'_\beta)^2} \tag{34}$$

$$G_{\alpha,\beta} = \frac{1}{2(k_\alpha + k_\beta^* - k'_\alpha - k'^*_\beta)^2}, G = \frac{1}{2(\xi_1 - \xi_2)^2} \tag{35}$$

$$\xi_1 = (k_1 + k_1^* + k_2 + k_2^*) \tag{36}$$

$$\xi_2 = (k'_1 + k_1'^* + k'_2 + k_2'^*), \alpha, \beta = 1,2, \ldots \tag{37}$$

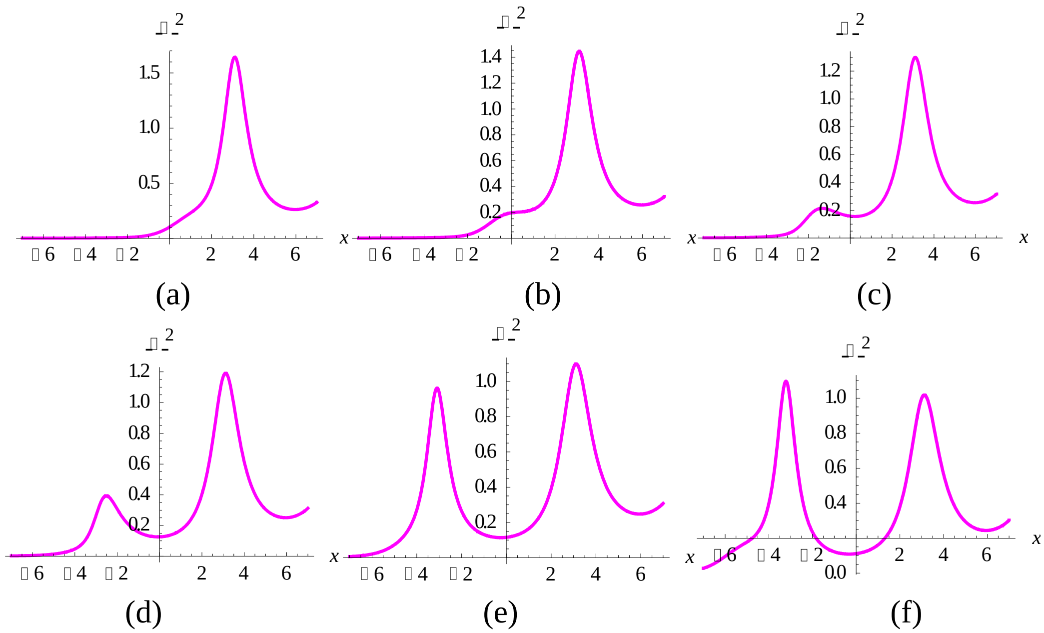


Figure 2: Two-soliton interaction at time $(a)T = 0.5, (b)T = 1, (c)T = 1.5,$ $(d)T = 2, (e)T = 2.5$ and (f) $T = 3$

In Eqns. (33)- (37), $\eta_2^{(0)}$. $k_2$ and $k'_2$ are all complex constant. Then using Eqn. (14) the two-soliton solution is found. Fig (2) depicits the two-soliton interaction at $k_1 = 0.1 + i$,

$k'_1 = 0.01 + i, k_2 = 1.01 + i, k'_2 = 0.01 + i$ and $y = 0.05$ $[(a)T = 0.5, (b)\ T = 1, (c)\ T = 1.5, (d)T = 2, (e)T = 2.5$ and $T = 3]$.We have illustrated the energy sharing features of two solitons in fig.(2). It is obvious that the amplitude and phases of two-solitons alter as a result of their interaction. But the total energy of each soliton is found to be preserved.

**3.3. Three-soliton solution**

Proceeding further to find the three-soliton solution, we terminate the series as,

$$g = \gamma g(1) + \gamma^3 g^{(3)} + \gamma^5 g^{(5)} \tag{38}$$

$$f = 1 + \gamma^2 f^{(2)} + \gamma^4 f^{(4)} + \gamma^6 f^{(6)} \tag{39}$$

solving the resulting linear partial differential equations, we obtain,

$$\begin{aligned} g = {} & exp\eta_1 + exp\eta_2 + exp\eta_3 + H_{121} exp(\eta_1 + \eta_2 + \eta_1^*) + H_{122} exp \\ & (\eta_1 + \eta_2 + \eta_2^*) + H_{123} exp(\eta_1 + \eta_2 + \eta_3^*) + H_{131} exp(\eta_1 + \eta_3 + \eta_1^*) \\ & + H_{132} exp(\eta_1 + \eta_3 + \eta_2^*) + H_{133} exp(\eta_1 + \eta_3 + \eta_3^*) + H_{231} exp \\ & (\eta_2 + \eta_3 + \eta_1^*) + H_{232} exp(\eta_2 + \eta_3 + \eta_2^*) + H_{233} exp(\eta_2 + \eta_3 + \eta_3^*) \\ & + A_1 exp(\eta_1 + \eta_2 + \eta_3 + \eta_1^* + \eta_2^*) + A_2 exp(\eta_1 + \eta_2 + \eta_3 + \eta_1^* + \eta_3^*) \\ & + A_3 exp(\eta_1 + \eta_2 + \eta_3 + \eta_2^* + \eta_3^*) \end{aligned} \tag{40}$$

and

$$\begin{aligned} f = {} & 1 +^3 \Sigma_{\alpha,\beta=1} G_{\alpha,\beta} exp(\eta_\alpha + \eta_\beta^*) + G_{122} exp(\eta_1 + \eta_2 + \eta_1^* + \eta_2^*) + G_{133} exp \\ & (\eta_1 + \eta_3 + \eta_1^* + \eta_3^*) + G_{233} exp(\eta_2 + \eta_3 + \eta_2^* + \eta_3^*) + G_{123} exp(\eta_1 + \eta_2 + \eta_1^* + \\ & \eta_3^*) + G_{132} exp(\eta_1 + \eta_3 + \eta_1^* + \eta_2^*) + G_{213} exp(\eta_1 + \eta_2 + \eta_2^* + \eta_3^*) + G_{231} exp \\ & (\eta_2 + \eta_3 + \eta_1^* + \eta_2^*) + G_{312} exp(\eta_1 + \eta_3 + \eta_2^* + \eta_3^*) + G_{321} exp(\eta_2 + \eta_3 + \eta_1^* + \\ & \eta_3^*) + G_2 exp(\eta_1 + \eta_2 + \eta_3 + \eta_1^* + \eta_2^* + \eta_3^*) \end{aligned} \tag{41}$$

$$H_{\alpha\beta\gamma} = [\frac{1}{2(k_\alpha + k^*{}_\alpha + k_\beta - k'{}_\alpha - k'^*_\alpha - k'{}_\beta)^2}] \tag{42}$$

$$G_{\alpha\beta} = [\frac{1}{2(k_\alpha + k_\beta^* - k'{}_\alpha - k'^*_\beta)^2}], G_{\alpha\beta\gamma} = [\frac{1}{2(\gamma_1 - \gamma_2)^2}] \tag{43}$$

$$A_1 = [\frac{1}{2(k_\alpha + k_\beta + k_\gamma + k_\alpha^* + k_\beta^* - k'{}_\alpha - k'{}_\beta - k'{}_\gamma - k'^*_\alpha - k'^*_\beta)^2}] \tag{44}$$

$$A_2 = [\frac{1}{2(k_\alpha + k_\beta + k_\gamma + k_\alpha^* + k_\gamma^* - k'{}_\alpha - k'{}_\beta - k'{}_\gamma - k'^*_\alpha - k'^*_\gamma)^2}] \tag{45}$$

$$A_3 = [\frac{1}{2(k_\alpha + k_\beta + k_\gamma + k_\beta^* + k_\gamma^* - k'{}_\alpha - k'{}_\beta - k'{}_\gamma - k'^*_\beta - k'^*_\gamma)^2}] \tag{46}$$

$$G_2 = [\frac{1}{2(\xi_3 - \xi_4)^2}], \alpha, \beta, \gamma = 1,2,3, \ldots \tag{47}$$

$$\gamma_1 = (k_\alpha + k_\beta + k_\beta^* + k_\gamma^*), \gamma_2 = (k'_\alpha + k'_\beta + k'^*_\beta + k'^*_\gamma) \quad (48)$$

$$\xi_3 = (k_1 + k_2 + k_3 + k_1^* + k_2^* + k_3^*), \xi_4 = (k'_1 + k'_2 + k'_3 + k'^*_1 + k'^*_2 + k'^*_3) \quad (49)$$

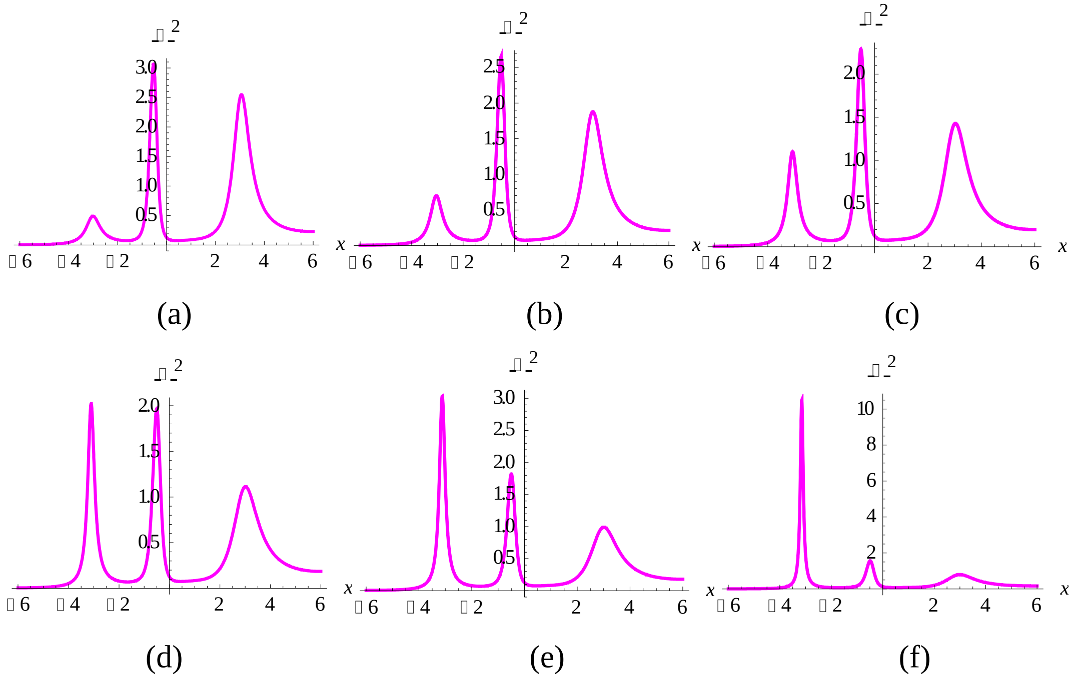


Figure 3: Three-soliton interaction at time $(a)T = 0.01 (b)T = 0.03, (c)T = 0.05,$ $(d)T = 0.07, (e)T = 0.08$ and (f) $T = 0.1$

Fig. (3) shows the three-soliton interaction at $k_1 = 0.3 + i, k'_1 = 0.1 + i, k_2 = 1.01 + i, k'_2 = 0.01 + i, k_3 = -6.2 + i$ and $k'_3 = 4.5 + i[(a) = T = 0.01, (b)T = 0.03, (c)T = 0.05, (d)T = 0.07, (e)T = 0.08$ and (f) $T = 0.1]$.The three-soliton example exhibits the same interaction features as the two-soliton case .

## 4. Inhomogeneous alpha-helical proteins system

We consider a two-dimensional inhomogeneous alpha-helical protein system. The energy is associated with including the charged particle resonance between proximal chains and the adjacent atoms in the same spine.

$$\begin{aligned} H = \sum_{\alpha,\beta} \{ & \phi^\dagger_{\alpha,\beta} E_0 \phi_{\alpha,\beta} + \phi^\dagger_{\alpha,\beta} E_1 \phi_{\alpha,\beta} \phi^\dagger_{\alpha,\beta} \phi_{\alpha,\beta} - J_1 F_{\alpha,\beta} (\phi^\dagger_{\alpha,\beta} \phi_{\alpha+1,\beta} + \phi_{\alpha,\beta} \\ & \phi^\dagger_{\alpha+1,\beta} + \phi^\dagger_{\alpha,\beta} \phi_{\alpha,\beta+1} + \phi_{\alpha,\beta} \phi^\dagger_{\alpha,\beta+1}) - J_2 F_{\alpha,\beta} (\phi^\dagger_{\alpha,\beta} \phi_{\alpha+1,\beta+1} + \phi_{\alpha,\beta} \\ & \phi^\dagger_{\alpha+1,\beta+1} + \phi_{\alpha+1,\beta-1} \phi^\dagger_{\alpha,\beta} + \phi_{\alpha,\beta} \phi^\dagger_{\alpha+1,\beta-1}) + \frac{\hat{P}^2_{\alpha,\beta}}{2M} + \frac{K}{2} [(v_{\alpha,\beta} - v_{\alpha-1,\beta})^2 \\ & + (v_{\alpha,\beta} - v_{\alpha,\beta-1})^2] + \chi_1 \phi^\dagger_{\alpha,\beta} \phi_{\alpha,\beta} (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) \\ & + \chi_2 \phi_{\alpha,\beta} \phi^\dagger_{\alpha,\beta} \phi_{\alpha,\beta} \phi^\dagger_{\alpha,\beta} (v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}) \} \end{aligned} \quad (50)$$

The function $F_{\alpha,\beta}$ characterize the variation of interactions along the hydrogen bonding spine due to inhomogeneities. Having consructed the Hamiltonian for the inhomogeneous alpha-helical protein molecules the corresponding dyamical equation can be obtained by deriving the associated Hamiltonian's equations of motion

$$i\hbar\frac{\phi_{\alpha,\beta}}{dt} = E_0\phi_{\alpha,\beta} + 2E_1\phi^{\dagger}_{\alpha,\beta}\phi^2_{\alpha,\beta} - J_1[F_{\alpha,\beta}\phi_{\alpha+1,\beta} + F_{\alpha-1,\beta}\phi_{\alpha-1,\beta} + F_{\alpha,\beta}\phi_{\alpha,\beta+1} + F_{\alpha,\beta-1}\phi_{\alpha,\beta-1}] - J_2[F_{\alpha,\beta}\phi_{\alpha+1,\beta+1} + F_{\alpha-1,\beta}\phi_{\alpha-1,\beta-1} + F_{\alpha,\beta}\phi_{\alpha+1,\beta-1} + F_{\alpha-1,\beta+1}\phi_{\alpha-1,\beta+1}] + \chi_1\phi_{\alpha,\beta}[v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}] + 2\chi_2\phi^{\dagger}_{\alpha,\beta}\phi^2_{\alpha,\beta}[v_{\alpha+1,\beta} - v_{\alpha-1,\beta} + v_{\alpha,\beta+1} - v_{\alpha,\beta-1}] \quad (51)$$

$$M\frac{d^2v_{\alpha,\beta}}{dt^2} = -K[4v_{\alpha,\beta} - v_{\alpha-1,\beta} - v_{\alpha+1,\beta} - v_{\alpha,\beta-1} - v_{\alpha,\beta+1}] + \chi_1[|\phi_{\alpha+1,\beta}|^2 - |\phi_{\alpha-1,\beta}|^2 + |\phi_{\alpha,\beta+1}|^2 - |\phi_{\alpha,\beta-1}|^2] + \chi_2[|\phi_{\alpha+1,\beta}|^4 - |\phi_{\alpha-1,\beta}|^4 + |\phi_{\alpha,\beta+1}|^4 - |\phi_{\alpha,\beta-1}|^4] \quad (52)$$

The discrete form of higher dimensional alpha-helical proteins is denoted in Eqns. (51) and (52). Eqns. (51) and (52) are hard to solve because of their non-linearity and compactness, it is appropriate to apply Taylor's series expansions to make continium solutions.

$$i\hbar\phi_t = -[4F(J_1 + J_2) - E_0]\phi + 2E_1|\phi|^2\phi + \varepsilon[2\chi_1(v_x + v_y)\phi + 4\chi_2(v_x + v_y)|\phi|^2\phi + F_x(J_1 + 2J_2)\phi + J_1F_y\phi] - \varepsilon^2[\tfrac{1}{2}J_1(F_{xx} + F_{yy}\phi + J_2(F_{xx} + F_{yy})\phi + F_x(J_1 + 2J_2)\phi_x + F(J_1 + 2J_2)\phi_{xx} + F_y(J_1 + 2J_2)\phi_y + F(J_1 + 2J_2)\phi_{yy} + 4J_2F\phi_{xy} + 2J_2F_{xy}\phi] \quad (53)$$

$$Mv_{tt} = k\epsilon^2[v_{xx} + v_{yy}] + 2\epsilon(\chi_1 + \chi_2)[(|\phi|^2)_x + (|\phi|^2)_y] \quad (54)$$

Introducing the wave variables $\xi = k_1x + k_2y - ct$ in Eqns. (53) and (54) and solving Eqn. (54) we get $u_{\xi\xi} = 2(\chi_1 + \chi_2)A|\phi|^2$ and using it in Eqn. (53) we get,

$$i\phi_t + b_1\phi + b_2(\phi_x + \phi_y) + b_3(\phi_{xx} + \phi_{yy}) + b_4\phi_{xy} - b_5\phi|\phi|^2 - b_6\phi|\phi|^4 = 0 \quad (55)$$

Where,

$$b_1 = \frac{4F(J_1+J_2) - E_0 + \varepsilon F_x(J_1+2J_2) + J_1\varepsilon F - \varepsilon^2\frac{1}{2}(J_1+J_2)(F_{xx}+F_{yy}) + 2J_2\varepsilon^2F_{xy}}{\hbar},$$

$$b_2 = \frac{\varepsilon^2(J_1+2J_2)(F_x+F_y)}{\hbar},\ b_3 = \frac{\varepsilon^2 F(J_1+2J_2)}{\hbar},\ b_4 = \frac{4J_2\varepsilon^2 F_x}{\hbar},\ b_5 = \frac{4\chi_1\varepsilon(\chi_1+\chi_2)A+2E_1}{\hbar},$$

$b_6 = \frac{8\chi_2\varepsilon(\chi_1+\chi_2)A}{\hbar}$, and the value of $A = \frac{\varepsilon(k_1+k_2)}{Mc^2-k\varepsilon^2(k_2^2+k_1^2)}$.

The quintic inhomogeneous alpha-helical proteins are described in Eqn. (55). The perturbation process offers an operative method to construct the solitary wave solution.

## 5. Effect of inhomogeneity

Many effective techniques for getting clear drifting and solitary wave solution of nonlinear growth of equation have been putforth in recent years [25-27]. There are many methods used to solve for nonlinear equations. From these methods, we have chosen SC method to make the soliton solution. Implementing above method, we write $\phi = u + iv$ in Eqn. (55) and seperate the real and imaginary parts to get,

$$-v_t + b_1 u + b_2(u_x + u_y) + b_3(u_{xx} + u_{yy}) + b_4 u_{xy} - b_5(u^3 + uv^2)$$
$$-b_6(u^5 + 2u^3v^2 + uv^4) = 0 \quad (56)$$

$$-u_t + b_1 v + b_2(v_x + v_y) + b_3(v_{xx} + v_{yy}) + b_4 v_{xy} - b_5(v^3 + uv^2)$$
$$-b_6(v^5 + 2v^3v^2 + u^4 v) = 0 \quad (57)$$

In Eqn.(56) and (57), applying wave variable $\xi = x + y - ct$ we abtain,

$$-cv_\xi + b_1 u + b_2(u_\xi + u_\xi) + b_3(u_{\xi\xi} + u_{\xi\xi}) + b_4 u_{\xi\xi} - b_5(u^3 + uv^2)$$
$$-b_6(u^5 + 2u^3v2 + uv^4) = 0 \quad (58)$$

$$-cu_\xi + b_1 v + b_2(v_\xi + v_\xi) + b_3(v_{\xi\xi} + v_{\xi\xi}) + b_4 v_{\xi\xi} - b_5(u^3 + uv^2)$$
$$-b_6(u^5 + 2u^3v2 + uv^4) = 0 \quad (59)$$

where,

$$b_1' = \frac{4F(J_1+J_2)-E_0+\varepsilon F_x(J_1+2J_2)+J_1\varepsilon F-\varepsilon^2\frac{1}{2}(J_1+J_2)(F_{xx}+F_{yy})+2J_2\varepsilon^2 F_{xy}}{\hbar},$$

$$b_2' = \frac{\varepsilon^2(J_1+2J_2)(F_x+F_y)}{\hbar},\ b_3' = \frac{\varepsilon^2 F(J_1+2J_2)}{\hbar},\ b_4' = \frac{4J_2\varepsilon^2 F_x}{\hbar},\ b_5' = \frac{4\chi_1\varepsilon(\chi_1+\chi_2)A+2E_1}{\hbar},$$

$$b_6' = \frac{8\chi_2\varepsilon(\chi_1+\chi_2)A}{\hbar}$$

Assume that Eqns.(58) and (59) confess the results as follows:

$$u(x,y,t) = \lambda_1 cos^{\beta_1}(\mu\xi), v(x,y,t) = \lambda_2 cos\beta_2(\mu\xi) \quad (60)$$

Where $\lambda_1$ and $\lambda_2$ are the parameter constants. In Eqn.(60) we assume $\beta_1$ and $\beta_2 = -1$ and we balance the linear higher order derivative term with the nonlinear term in eqns. (58) and (59). We get the system of algebraic equations.

$$cos^2(\mu\xi).sin(\mu\xi) = 2b_2\mu\lambda_1 + c\mu\lambda_2 \tag{61}$$

$$cos^{-1}(\mu\xi) = b_1\lambda_1 - 2b_3\lambda_1\mu^2 - b_4\lambda_1\mu^2 \tag{62}$$

$$cos^{-3}(\mu\xi) = 4b_3\lambda_1\mu^2 - 2b_4\lambda_1\mu^2 - b_5\lambda_1^3 - b_5\lambda_1\lambda_2^2 \tag{63}$$

$$cos^{-5}(\mu\xi) = -b_6\lambda_1^5 + 2b_6\lambda_1^3\lambda_2^2 - b_6\lambda_1\lambda_2^4 \tag{64}$$

$$cos^2(\mu\xi).sin(\mu\xi) = 2a_2\mu\lambda_1 - c\mu\lambda_2 \tag{65}$$

$$cos^{-1}(\mu\xi) = b_1\lambda_2 - 2b_3\lambda_2\mu^2 - b_4\lambda_2\mu^2 \tag{66}$$

$$cos^{-3}(\mu\xi) = 4b_3\lambda_2\mu^2 - 2b_4\lambda_2\mu^2 - b_5\lambda_2^3 - b_5\lambda_2\lambda_1^2 \tag{67}$$

$$cos^{-5}(\mu\xi) = -b_6\lambda_2^5 + 2b_6\lambda_1^2\lambda_2^3 - b_6\lambda_2\lambda_1^4 \tag{68}$$

Using symbolic computation to solve the system of algebraic equation we obtain,

$$\mu = -a_2 + \sqrt{b_2^2 - 2b_1b_3 - b_1b_4} \tag{69}$$

$$\lambda = \frac{i\sqrt{b_5}}{\sqrt{2}\sqrt{b_6}} \tag{70}$$

hence the solution of Eqn. (55) becomes,

$$u(x,y,t) = \lambda_1 sec[\mu(x+y-ct)] \tag{71}$$

$$v(x,y,t) = \lambda_2 sec[\mu(x+y-ct)] \tag{72}$$

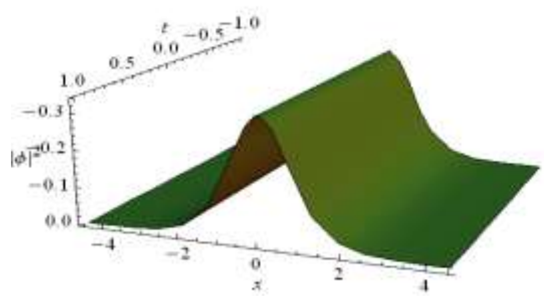



Figure4: unperturbed soliton with $J_1 = 0.5$, $J_2 = 1.5$, $\chi_1 = 1$, $\chi_2 = 0.5$, $A = 1$, $\xi = 1$, $h = 1$, $E_0 = 1$, $E_1 = 1$ and $F = 1$

In many physical system, it is practicable to comprehend non-linear wave events using systematic soliton solutions inhomogeneity. The coefficients of equation predicted to depend on the function representing the inhomogeneity and its derivatives if the process is inhomogeneous and its derivatives F(x) point is determine by the solutions Eqns. (71) and (72) when F(x)=1 the structure remains unchanged as shown in fig. (4). It display a soliton for the choice of parameters $J_1 = 0.5$, $J_2 = 1.5$, $\chi_1 = 1$, $\chi_2 = 0.5$, $A = 1$, $\xi = 1$, $h = 1$, $E_0 = 1$, $E_1 = 1$ and $F = 1$ as interpretative and informative cases we present cubic, biquadratic and localized type of inhomogeneities [28].

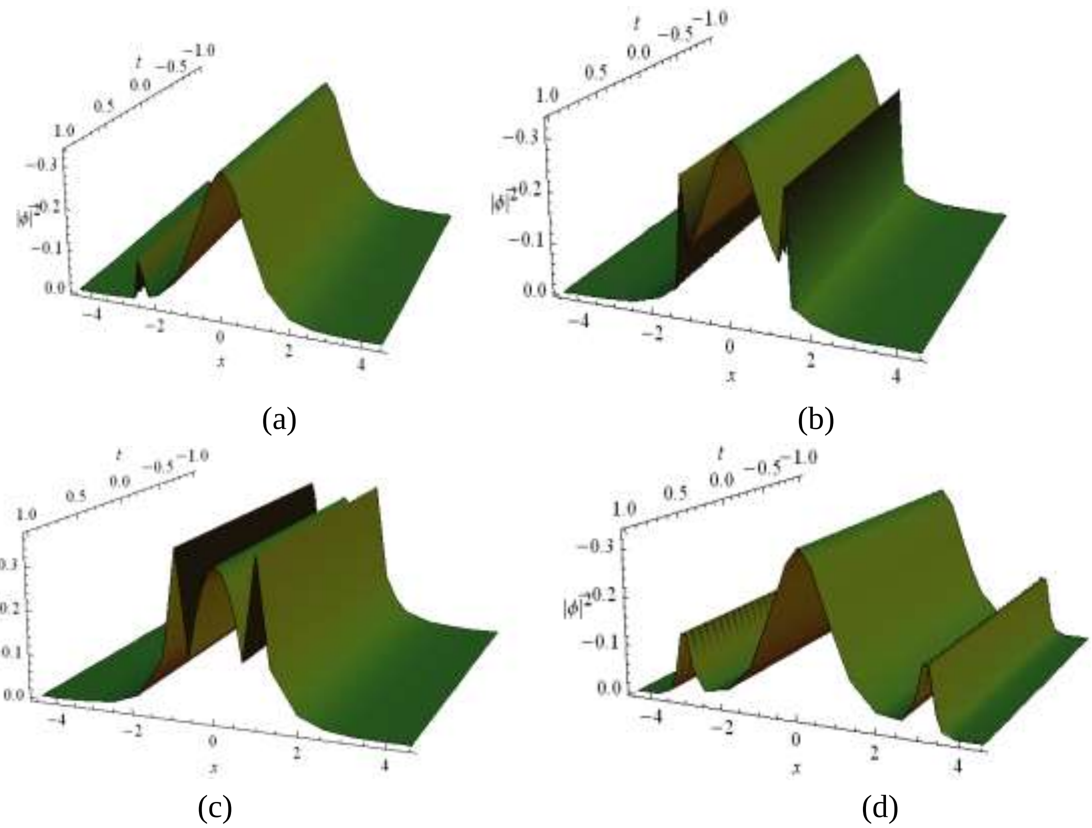
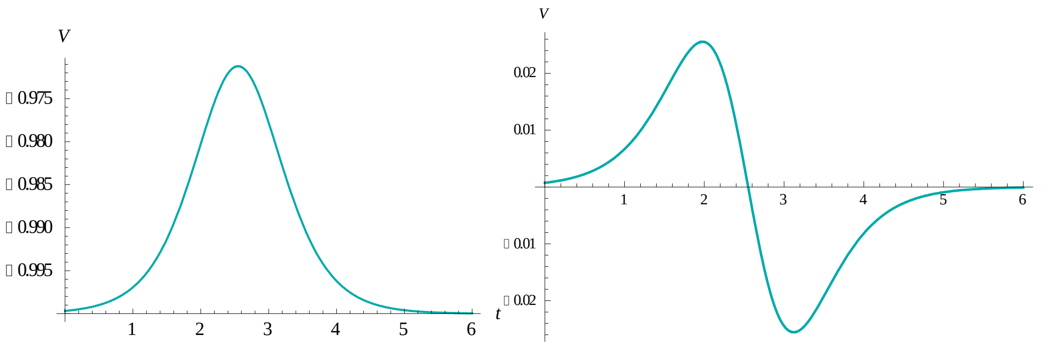



Figure 5: The graphical illustration of $|\phi|^2$ with (a) cubic inhomogeneity (b) biquadratic inhomogeneity (c) periodic inhomogeneity (d) localized inhomogeneity

We consider the cubic inhomogeneity of the form $F(x) = 1 + q_1x^3 + q_2x^2$ and plot the stuctures in fig. (5a). This plot shows that the soliton's figure is fixed and solid, the value of $q_1 = 1.018$ and $q_2 = 2.49$. Where $q_1$ and $q_2$ value increase the soliton becomes deformed and the unstable modes have generated and expanded as a result of biquadratic inhomogeneity $F(x) = 1 + q_3x^4 + q_4x^2$. The values of $-0.67$ to $0.59$ are less than $q_3$ and $q_4$ respectively, the progressive formation of soliton is illustrates in fig. (5b). It demonstrate the soliton's split and consequent instability. In fig. (5c) the equation with periodic inhomogeneity $F(x) = 1 + q_5 sin(x)$ was shown the values of $q_5 = 1.3299$ from the plot, it is found that the value of $q_5$ increases, the distortion produces a soliton of smaller amplitude with periodic variability in the localized region. For the case of localized inhomogeneity we use the distortion function $F(x) = 1 + q_6 sech(x)$ in fig. (5d) this inhomogeneity distorts the localized region for the value $q_6 = 0.865$.




(a) (b)

Figure 6: (a) Center of mass of the soliton (b) Velocity of soliton

Fig. (6a) illustrates a soliton's center of mass changes over time. The center of mass in protein lattices almost constantly remains in the same position. In parameter range, it suggest that the basic solitons for the protein lattices are nonlinearly stable. Velocity of soliton in a protein lattices is given in fig. (6b). The figure represents the periodic difference in the velocity of a soliton before expanding to a constant speed. The variables are $J_1 = 3$, $J_2 = 3$, $\chi_1 = 1$, $\chi_1 = 4$, $A = -100$, $\epsilon = 1$, $h = 1$, $E_0 = 0.1$ and $E_1 = 0.1$.

## 6. Modulation Instability of cubic-quintic NLS Equation

Modulation Instability in several fields of physics, a plane wave may shatter into filamenta at enormous intensities. It has been suggested that it might be responsible for the energy localization mechanisms that cause DNA molecules and hydrogen-bonded crystals to create large amplitude nonlinear excitations. MI arises from the interaction of nonlinearity diffraction or dispersive processes. Because of the symmetry-breaking nature of the instability, a small perturbation on top of a background with constant amplitude grows exponentially leading to beam breakup in either space or time. In some ways, MI is thought of as a precursor region. We use a linear stability analysis to look into the evolution of weak perturbation for inhomogeneities. We begin with the perturbed Eqn. (55) to conduct the linear stability analysis. We consider plane wave solution with constant amplitude[29]

$$\sigma(x, y, t) = u_0 exp[i(q_1 x + q_2 y - \omega t)] \tag{73}$$

$\omega$ is the frequency, $u_0$ is the amplitude, $q_1$ and $q_2$ are the wave numbers. Substitute Eqn. (73) we get the amplitude dependent relationship.

$$\omega = -a_1 - ia_2(k_1 + k_2) + a_3(k_1^2 + k_2^2) - a_4 k_1 k_2 + a_5 u_0^2 + a_6 u_0^4 \tag{74}$$

refered to as dispersion relation. Considering perturbed plane wave solutions of the type, we now investigate the linear stability of Eqn. (55)

$$\sigma(x,y,t) = (u_0 + \varepsilon\sigma_1)exp[i(q_1x + q_2y - \omega t) + \varepsilon\sigma_2(x,y,t)] \tag{75}$$

where the small parameter is $\varepsilon$ and

$$\sigma_1(x,y,t) = aexp[i\beta(x,y,t)] \tag{76}$$

$$\sigma_2(x,y,t) = bexp[i\beta(x,y,t)] \tag{77}$$

using $\beta(x,y,t) = Kx + Ky - \Omega t$ the dispersion relation relating the wave numbers K and frequency $\Omega$ is given by

$$\Omega^2 u_0 + \Omega(Ru_0 + S) + RS = 0 \tag{78}$$

where,

$$R = [\Omega a + \omega a + a_1 a - Ku_0 a_2 b - u_0 bKa_2 - a_3 aK^2 - 2a_3 k_1 aK$$
$$-a_3 k_1^2 a - a_3 aK^2 - 2a_3 k_2 aK - k_2^2 aa_3 - a_4 aK^2 - a_4 k_2 Ka$$
$$-a_4 k_1 Ka - a_4 k_1 k_2 a - 3u_0^2 - 5u_0^4 a] \tag{79}$$

$$S = [u_0 b\Omega + ka_2 a + k_1 aa_2 + ak + k_2 a - 2a_3 u_0 k_1 bk - a_3 u_0 bk^2$$
$$-2a_3 k_2 u_0 bk - a_3 bu_0 k^2 - a_4 k_1 u_0 bk - a_4 u_0 k^2 b - a_4 k_2$$
$$u_0 bk] \tag{80}$$

From the quadratic Eqn. (80) the dispersion relation for linearised disturbance can be expressed as,

$$\Omega = \frac{1}{2}(a_1 + 4K^2 a_3 + 2K^2 a_4 + 4Ka_3 k_1 + 2Ka_4 k_1 + a_3 k_1^2 + 2Ka_3 k_2$$
$$+2Ka_4 k_2 + a_4 k_1 k_2 - 2Ka_3^2 k_2^3 + 3u_0^2 + 5u_0^4) \pm (a_1^2 - 16K^2 a_2^2$$
$$+2a_1 a_3 k_1^2 + a_3^2 k_1^4 - 4Kk_2 a_1 a_3 + 2a_1 a_4 k_1 k_2 - 4Ka_3^2 k_1^2 k_2 + 2a_3 a_4$$
$$k_1^3 k_2 + 4K^2 a_3^2 k_2^2 - 4Ka_3 a_4 k_1 k_2^2 + a_4^2 k_1^2 k_2^2 - 4Ka_1 a_3^2 k_2^3 - 4Ka_3^3$$
$$k_1^2 k_2^3 + 8K^2 a_3^3 k_2^4 - 4Ka_3^2 a_4 k_1 k_2^4 + 4K^2 a_3^4 k_2^6 + 6a_1 u_0^2 + 6a_3 k_1^2 u_0^2$$
$$-12Ka_3 k_2 u_0^2 + 6a_4 k_1 k_2 u_0^2 - 12Ka_3^2 k_2^3 u_0^2 + 9u_0^4 + 16a_1 u_0^4 + 10k_1^2$$
$$a_3 u_0^4 - 20Ka_3 k_2 u_0^4 + 10a_4 k_1 k_2 u_0^4 - 20Ka_3^2 k_2^3 u_0^4 + 30u_0^6 + 25u_0^8) \tag{81}$$

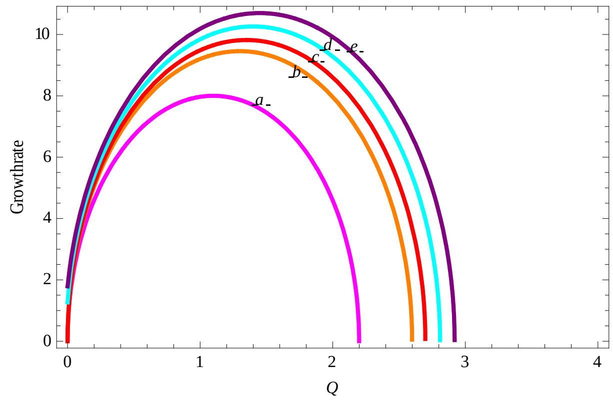



Figure 7: Growth rate Vs wave number $\Omega$ for unperturbed

The imaginary part $\Omega$ can resolve stability of non-linear alpha-helical protein chain. The relationship mentioned above demonstrates the square are bigger than zero. For any value of the wave number K, the eigen value $\Omega$ is real, and the solution is modulationally stable. When it is less than zero $\Omega$ becomes complex. In this scenario the perturbation grows exponentially with time the excited alpha-helical protein system is unstable and soliton production is supported. The growth rate curve in fig. (7) is depicted by setting up $q_2$ and variying $q_1$. The values are $u_0 = 6$, $J_1 = 2$ and $J_2 = 3$ with (a) $k_1 = -1$ and $k_2 = -1.2$, (b) $k_1 = -1.4$ and $k_2 = -1.2$, (c) $k_1 = -1.5$ and $k_2 = -1.2$, (d) $k_1 = -1.6$ and $k_2 = -1.2$, (e) $k_1 = -1.7$ and $k_2 = -1.2$. The plot shows that the growth rate depends on $q_1$ and $q_2$ for constant $J_1$ and $J_2$. As $k_2$ increases the growth rate and the band width shrinks, and the maximum gain decreases. Inhomogeneities can have a impact on solitons development. We study whether nonlinear lattice inhomogeneities affects the stability of soliton.

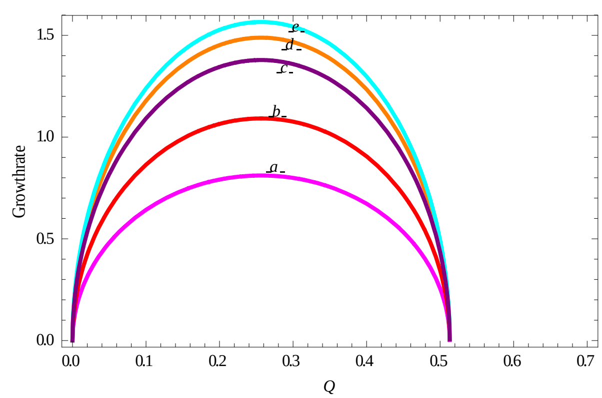



Figure 8: Growth rate Vs wave number $\Omega$ for periodic inhomogeneity

For various values of the periodic inhomogeneity parameters, Fig. (8) depicits the growth rate of the periodic inhomogeneity. Consider the alpha-helical protein lattice with periodic inhomogeneity $F(x) = 1 + q_1 sin(x)$. In the absence of such inhomogeneity, the system allows for the steady propagation of solitary waves. The growth rate for periodic inhomogeneities with $q_1 = 1.7, 1.5, 1.2$ and $0.5$ is shown in fig. (8.a,b,c,d). As the inhomogeneities increase, the growth rate decreases and the band width shrinks. At $q_1 = 1.75$, modulation instability becomes high and beyond this value it decreases and the probability of solitory wave formation is suppressed. The effect of cubic $F(x) = 1 + q_2 x^3 + q_3 x^2$, biquadratic $F(x) = 1 + q_4 x^4 + q_5 x^2$ and localized inhomogeneities $F(x) = 1 + q_6 tanh(x)$ are similar to the above results. The threshold values are given by $q_2 = 0.7$ and $q_3 = 0.002$ for cubic inhomogeneity. For biquadratic inhomogeneity $q_4 = 0.8$ and $q_5 = 0.09$ and for localized inhomogeneity $q_6 = 0.97$. At $q_4 = 0.8$ and $q_6 = 0.97$ Modulation Instability becomes high and beyond this value it decreases and the probability of solitary wave formation is suppressed. In this condition, the energy is no longer distributed equally along the protein chain, instead, the motion of certain particles has an amplitude that is significantly greater than the amplitude of the orginal wave.

## 7. Conclusion

In this paper, we suggest a hamiltonian model that incorprates molecular excitations using the exciton, phonon and phonon-exciton modes. We analyze the dynamics of the corresponding model by applying Hamiltonian's equation of motion to create the quintic equation. A two-dimensional integrable NLS type equation is to govern the dynamics for some set of parameters.We use the Hirota bilinearization technique to construct the one, two and three soliton solution. Using a peturbation technique, we analyzed the cubic, biquadratic, periodic, and localized types. This result suggest that the soliton splits when the amount of inhomogeneity exceeds a limitting value. The modulation instability conditions for the quintic inhomogeneous nonlinear schrodinger equation have also been discussed. Then the velocity and center of mass are also analyzed.

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