INTUITIONISTIC FUZZY ω - AUTOMATA AND ITS RELATIONSHIPS

A. Uma* and S. Jayalakshmi †
*Basic engineering, Mathematics,
Government polytechnic college, Tondiarpet, Chennai, Tamilnadu, India.
† Department of Mathematics,
Annamalai University, Annamalainagar - 608002, India.

Abstract
In this paper, Inspired by the Fuzzy ω language as fuzzy ω automata, a new approach is proposed as the relationships among the variety of Intuitionistic Fuzzy ω language as intuitionistic fuzzy ω automata.

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1.Introduction:

Automata theory is firmly similar to formal Language theory. Fuzzy automata has been widely used in diverse applications, such as lexical analysis, learning systems, control system, etc [3]. The fuzzy language and fuzzy automata was introduced for dealing with uncertainly in a system in 1960 by Santos[1]. Fuzzy automata can be classified into nondeterministic and deterministic fuzzy finite automata. Fuzzy automata depends on the membership value, which lies between 0 and 1 [3]. Fuzzy automata, Grammars, and Language[2] are leading to greater under standing of nondeterministic algorithms. Schutzenberger (1961) introduced the concept of weighted automaton. Weighted automaton is a classical automaton whose transitions are equipped with a weight over the semiring $(S, \cdot, +, 0, 1)$. The classical Schutzenberger theorem states that the class of recognizable languages is equal to the rational expression. Buchi (1962) introduced the concept of ω automaton which accepts infinite words. Buchi theorem states that the class of languages recognized by a Buchi automaton is equal to the rational expression.

Using the notion of intuitionistic fuzzy sets [8, 9] it is possible to obtain intuitionistic fuzzy language [6] by introducing the non-membership value to the strings of fuzzy language. This is a natural generalization of a fuzzy language as it is characterized by two functions expressing the degree of belongingness and the degree of non - belongingness. An intuitionistic fuzzy language is called intuitionistic fuzzy regular language, if its strings are regular having the finite membership and non - membership values between [0,1] in [7].

The paper is organized as follows. In section 2, we recall the definitions of Intuitionistic fuzzy automata. In section 3, we introduce the definition of intuitionistic fuzzy ω automata and intuitionistic fuzzy Buchi automata with example. In section 4, we introduce the theorem in Construction of intuitionistic fuzzy buchi automata from intuitionistic fuzzy language with example.

2. Preliminaries:

Definition 2.1. *Let a set 'E' be fixed. An intuitionistic fuzzy set 'A' in 'E' is an object having the form* $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in E\}$
where, the functions $\mu_A(x) : E \to [0,1]$ *and* $\gamma_A(x) : E \to [0,1]$ *define the degree of membership and the degree of nonmembership of the element* $x \in E$ *to the set 'A', the subset of 'E' respectively, and for every* $x \in E; 0 < \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.2. *An Intuitionistic fuzzy finite state machine is a triple* $IM = (P, X, \delta)$ *where,*

$*\ P$ *is a set of states.*

$*\ X$ *is a input alphabets.*

$* \ \delta = (\delta_1, \delta_2)$

is an intuitionistic fuzzy subset of $P \times X \times P, \ni \forall q, p \in P, \forall x, y \in X$.

$$\delta_1(p, \lambda, q) = \begin{cases} 1 \ if \ q = p \\ 0 \ if \ q \neq p \end{cases} \qquad \delta_2(p, \lambda, q) = \begin{cases} 0 \ if \ q = p \\ 1 \ if \ q \neq p \end{cases}$$

The function $\delta = (\delta_1, \delta_2)$ *is extended to the set of words* $w \in X^*$ *as follows.*

$$\delta_1(p, w, q) = \max_{r \in Q} \{ \min(\delta_1^*(p, w_1, r), \delta_1(r, a, q)) / w = w_1 a \forall w_1 \in X^*, a \in X\}$$

$$\delta_2(p, w, q) = \min_{r \in Q} \{ \max(\delta_2^*(p, w_1, r), \delta_2(r, a, q)) / w = w_1 a \forall w_1 \in X^*, a \in X\}.$$

Definition 2.3. *An Intuitionistic fuzzy automata is a five tuple* $IM = (P, X, \delta, P_0, F)$ *where,*

P *is a finite non - empty set of states,*

X *is a finite non - empty set of inputs,*

$\delta : P \times X \times P \to [0,1]$ *is called the intuitionistic fuzzy transition function.*

p_0 *: is called the initial state and*

F *: is called the set of final states.*

Definition 2.4. *Let* $IM = (P, X, \delta, p_0, F)$ *be an intuitionistic fuzzy automaton. Then the language* $L(IM) = \{(x, (\mu, \gamma)) | x \in X^*, \delta(p_0, w, q_f) = (\mu, \gamma), q_f \in F\}$ *is called the intuitionistic fuzzy regular language accepted by intuitionistic fuzzy automaton* IM.

3. KINDS OF INTUITIONISTIC FUZZY ω AUTOMATA

3.1 Definition

A (nondeterministic) intuitionistic fuzzy ω automaton is a 5 - tuple $K = (P, N, q, S, Acc)$, where

P is the finite set of states.

N is the finite set of input alphabets.

$q: P \times N \times P \to (\delta_1, \delta)$,where $0 < \delta_1 + \delta_2 \leq 1$ is the fuzzy transition function. $S: P \to (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of initial states.

Acc is the acceptance component.
An Intuitionistic fuzzy ω automaton, K is said to have deterministic transition, if S contains a single fuzzy state and for every state $p \in P$ and every symbol n$\in N$, there is at most one state $t \in N$ and one constant $c > 0 \in F$ such that (p, n, t, c) is a transition. That is, the intuitionistic fuzzy transition function is a partial intuitionistic fuzzy function. In otherwords, for all $n \in N,\ s,\ t,\ t' \in\ N,\ q(p, a, t) = (\delta_1, \delta_2)$,where $0 < \delta_1 + \delta_2 \leq 1$ implies that $t = t'$. It is deterministic if it has deterministic transition.
An intuitionistic fuzzy ω - automaton, K is said to be complete if the intuitionistic fuzzy transition function is complete. That is for every state $p \in P$ and every alphabet $n \in N$, there exists atleast one state $t \in P$ such that $q(p, n, t) = (\delta_1, \delta_2)$,where $0 < \delta_1 + \delta_2 \leq 1$.
If the intuitionistic fuzzy ω automaton, K have single initial state and the intuitionistic fuzzy transition function is complete and deterministic, then K is said to be a complete intuitionistic fuzzy deterministic automaton.
Let $N^{(j)}$ denote the set of infinite words (j - words) on N. The extension of the intuitionistic transition function $P \times N^{(j)} \times P \rightarrow (\delta_1, \delta_2)F$ is defined in the similar way.
A run of the intuitionistic fuzzy ω automaton K on j word $j = j_1 j_2 \cdots$ is j word $R = p_1 p_2 \cdots \in P^{(j)}$ such that $0 < S(p_1) \leq 1$ and $0 < q(p_i, (j_1, p_{i+1}) \leq 1$
for $i \geq 1$. The set of infinitely often states occuring in the run $\mathfrak{R}$ is denoted by $\inf(\mathfrak{R})$. That is
$inf\ \mathfrak{R}$ ={ $p \in P|$ there exists infinitely many i such that $p_i = p$}
A j - word is called accepted word by K, if the corresponding run $\mathfrak{R}$ is successful. The set of j - words recognized by P is a set, denoted by IL(K) , of labels of successful runs in K. A set X of j - words is recognizable if there exists a intuitionistic fuzzy Buchi automaton K such that $= IL(K)$.

3.2 Definition

An intuitionistic fuzzy Buchi automaton is a 5 - tuple K $= (P, N, q, S, \mathfrak{A})$, where P is the finite set of states.
N is the finite set of input alphabets
$S: P \rightarrow (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic fuzzy initial states.
$q: P \times N \times P \rightarrow (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic initial states.
where,

$\delta_1\left(q\left(p_i, w, p_j\right)\right) = \vee\left\{\delta_1(p_i, w_1, p_k) \wedge \delta_1\left(p_k, a, p_j\right)\right\}\ where, w = w_1 a_1$ for all $p_k \in P$

$\delta_2(q(p_i, w, p_j)) = \wedge\{\delta_2(p_i, w_1, p_k) \vee \delta_2(p_k, a, p_j)\}$ where, $w = w_1 a_1$ for all $p_k \in P$
$\mathfrak{A} : P \rightarrow (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic fuzzy final states.
A run in K is successful if it visit $\mathfrak{A}$ infinitely often, that is $inf\ (\mathfrak{R}) \neq \varphi$
The weight of the accepted word $w(\delta_1, \delta_2)$ is calculated as follows.

$w(\delta_1) = \vee \{\wedge \left\{\{S(p_1)\} \cup \{q(p_i, w, p_j)|i, j \geq 1\} \cup \left\{\frac{\mathfrak{A}(t)}{t} \in inf(\mathfrak{R}) \cap \mathfrak{A}\right\}\right\}\}$

$w(\delta_2) = \wedge \{\vee \left\{\{S(p_1)\} \cup \{q(p_i, w, p_j)|i, j \geq 1\} \cup \left\{\frac{\mathfrak{A}(t)}{t} \in inf(\mathfrak{R}) \cap \mathfrak{A}\right\}\right\}\}$

An intuitionistic fuzzy Buchi automaton is said to be deterministic intuitionistic fuzzy buchi automaton if it has deterministic transition.

Since, every word in IL(K) is the label of exactly one run, the weight of the accepted word $w(\delta_1, \delta_2)$ in a deterministic intuitionistic fuzzy Buchi automaton is calculated as follows

$w(\delta_1) = \wedge \{\{\{S(p_1)\} \cup \{q(p_i, w_j, p_k)|i, j, k \geq 1\} \cup \{\mathfrak{A}(t)/t \in inf(\mathfrak{R}) \cap \mathfrak{A}\}\}\}$

$w(\delta_2) = \vee \{\{\{S(p_1)\} \cup \{q(p_i, w_j, p_k)|i, j, k \geq 1\} \cup \{\mathfrak{A}(t)/t \in inf(\mathfrak{R}) \cap \mathfrak{A}\}\}\}$

Let $(K = (P, N, q, S, \mathfrak{A})$ be a intuitionistic fuzzy Buchi automata. A state $p \in P$ is said to be accessible if there exist a finite run R in $\mathfrak{A}$ ending with p. A state $p \in P$ is said to be coaccessible if there exist a infinite run R in K starting with p. A intuitionistic fuzzy Buchi automaton K is said to be trim if all the states of K are both accessable and coaccessable.

Example: Let $K = (P, \mathrm{S}, \mathfrak{A}, N, q)$ be a intuitionistic fuzzy non deterministic buchi automaton,

where,

$P = \{p_0(0.2,0.1), p_1(0.3,0.1), p_2(0.4,0.4)\}$

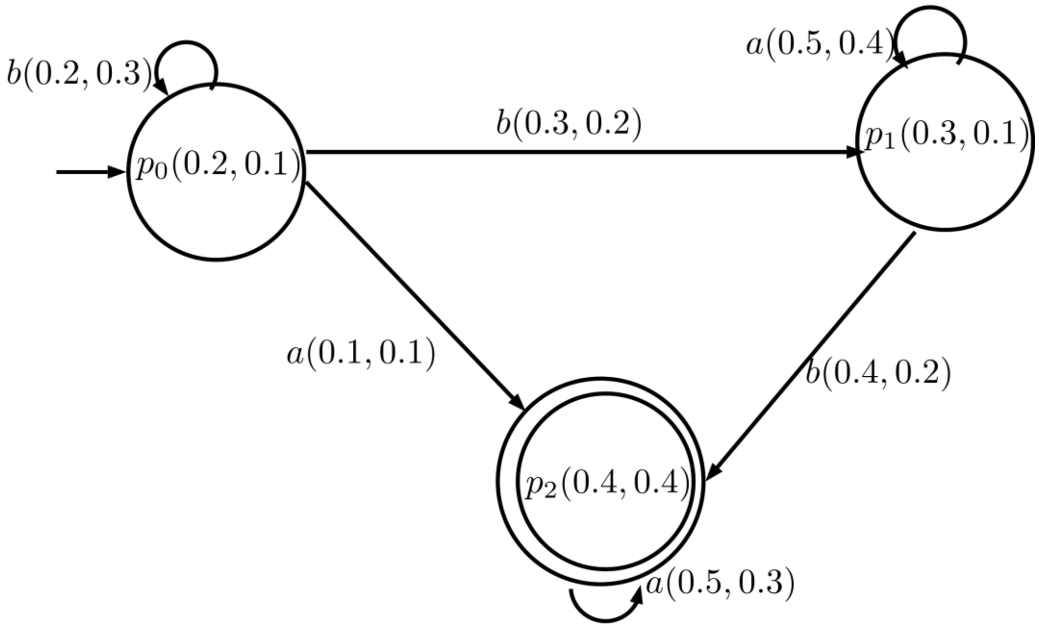
$S = p_0(0.2,0.1)$ is initial state,

$\mathfrak{A} = \{p_2(0.4,0.4)\}$,

$N = \{0,1\}$ and

$q(p_0, b, p_0) = (0.2,0.3)$, $q(p_0, b, p_1) = (0.3,0.2)$, $q(p_0, a, p_2) = (0.1,0.1)$ $q(p_1, a, p_1) = (0.5,0.4)$, $q(p_1, b, p_2) = (0.4,0.2)$, $q(p_2, a, p_2) = (0.5,0.3)$.

Then, $IL\ (K) = \{(b^* b a^* b a^*)(0.1,0.4)\}$.



$N^{w} = \{bbbbba\ldots,\ bbaaaa\ldots,\ baaa,\ldots .baaa\ldots,\}$

Run $= \{\mathrm{b} \xrightarrow{(p_0)} a \xrightarrow{(p_1)} a \xrightarrow{(p_1)} a,\ b^{(p)(p)(p)} \rightarrow^{2} b \rightarrow^{1} a \rightarrow^{1} a.\ \}$

$inf\ (\mathfrak{R}) = \{p_1\}$
$inf\ (\mathfrak{R}) = \{p_1\} \cap \{p_1\}$
$= \{p_1\}$

4 Construction of intuitionistic fuzzy buchi automata from intuitionistic fuzzy language.

Theorem 4.1. An intuitionistic fuzzy language recognized by a intuitionistic fuzzy Buchi automaton is also recognized by a trim intuitionistic fuzzy Buchi automaton and conversely.

Proof Let $K = (P, N, q, S, \mathfrak{A})$ be a intuitionistic fuzzy Buchi automaton and let $Z \subseteq P$ be the set of all accessible and coaccessible states of K. Construct a trim intuitionistic fuzzy Buchi automaton $K^{'} = (Z, N, q^{'}, S^{'}, \mathfrak{A}^{'})$ where $q^{'} : Z \times N \times Z \rightarrow (\delta_1, \delta_2)$ such that

$$\delta_1(p,n,z) = \begin{cases} q(p,n,z), & if\ q(p,n,z) \neq 0; \\ 1, & otherwise. \end{cases}$$

$$\delta_2(p,n,z) = \begin{cases} q(p,n,z), & if\ q(p,n,z) \neq 0; \\ 0, & otherwise. \end{cases}$$

$$S^{'}(\delta_1)(p) = \begin{cases} I(s), & if\ I(s) \neq 0; \\ 1, & otherwise. \end{cases}$$

$$S^{'}(\delta_2)(p) = \begin{cases} I(s), & if\ I(s) \neq 0; \\ 0, & otherwise. \end{cases}$$

$$\mathfrak{A}^{'}\delta_1(p) = \begin{cases} \mathfrak{A}(p), & if\ \mathfrak{A}(p) \neq 0; \\ 1, & otherwise. \end{cases}$$

$$\mathfrak{A}'\delta_2(p) = \begin{cases} \mathfrak{A}(p), & if\ \mathfrak{A}(p) \neq 0; \\ 0, & otherwise. \end{cases}$$

Every state in K' is a state of K and every intuitionistic fuzzy transition in K' is a fuzzy transition of K'. Clearly $IL\ (K') \subseteq IL(K)$.
Conversely, let $w = w_1w_2 \cdots \in IL(S)$ then there exists a run $R = p_1p_2p_3 \cdots$ of label w. These states $p_1, p_2, p_3, \cdots$ are clearly both accessible and coaccessible. The run R is in K' and the corresponding word w is in $IL\,(K')$. Therefore, $IL\ (K) \subseteq IL(K')$. Hence $IL\ (K) = IL(K')$ and the weight of the word also remains same in K and K'
The deterministic version of the above theorem is also true.

Theorem 4.2. An intuitionistic fuzzy language recognized by a intuitionistic fuzzy deterministic Buchi automaton is also recognized by a trim intuitionistic fuzzy Buchi automaton and conversely.
Proof Let $K = (P, N, q, S, \mathfrak{A})$ be a intuitionistic fuzzy deterministic Buchi automaton, then by constraints of above theorem 2.2.1$K' = (Z, N, q', S', \mathfrak{A}')$ is also deterministic.
Theorem 4.3. An intuitionistic fuzzy language recognized by a intuitionistic fuzzy Buchi automaton is also recognized by a complete intuitionistic fuzzy Buchi automaton.
Proof Let $K = (P, N, q, S, \mathfrak{A})$ be a intuitionistic fuzzy Buchi automaton. Construct a complete intuitionistic fuzzy Buchi automaton $K' = (P', N, q', S, \mathfrak{A})$ where $P' = P \cup \{e\}$, e is the new state and define $q' : P' \times N \times P' \to (\delta_1, \delta_2)$
where $0 < \delta_1 + \delta_2 \leq 1$ as follows:

$$q' = q_1 \cup q_2 \cup q_3$$

for all $n \in N$

$$q_1(e, n, e) = (1,0)$$

for all $p \in P$ and $n \in N$
$q_2(P, n, e) = (\delta_1, \delta_2)$

$$\delta_1(p, n, e) = \begin{cases} 1, if\ there\ is\ no\ z \in P\ such\ that\ q(p, n, z) \neq 0; \\ 0, otherwise. \end{cases}$$

$$\delta_1(p, n, e) = \begin{cases} 0, if\ there\ is\ no\ z \in P\ such\ that\ q(p, n, z) \neq 0; \\ 1, otherwise. \end{cases}$$

$$q_3(p, n, z) = q(p, n, z)$$

since all the state in K is a state in K', every transition in K is a transition in K' and the intuitionistic fuzzy initial and intuitionistic fuzzy final states are same as in K'.
Therefore, for every intuitionistic fuzzy Buchi automaton K there exists a complete intuitionistic fuzzy Buchi automaton K' such that K and K' recognize the same fuzzy w language.
Illustrative Example
Consider the intuitionistic fuzzy nondeterministic Buchi automata

$K = (P, N, q, S, F)$ given in Figure 4.1,

$P = \{s, t\}$

$N = \{a, b\}$

$q(s, a, s) = (O.2, 0.3),\ q(s, a, t) = (O.2, 0.2),\ q(t, b, t) = (O.1, O.4)$

$S = \{s(0.2, 0.1)\}$

$F = \{t(0.4, 0.4)\}$

The intuitionistic fuzzy ω language recognized by the intuitionistic fuzzy $\omega -$ automata is $IL\ (K) = \{w(\delta_1, \delta_2) \in a^+ b^{(j)}\}$

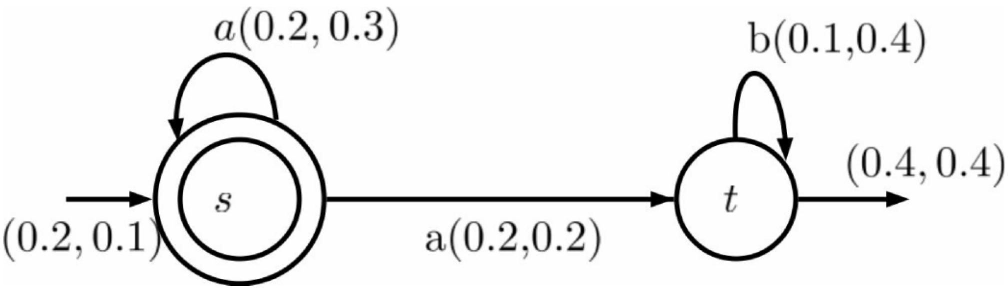


Figure 4.1: Intuitionistic Fuzzy nondeterministic Buchi automaton

and the weight of the intuitionistic language is

$$W(w)) = \begin{cases} 0.2, 0.3\ if\ w\ has\ atleast\ two\ a's \\ 0.2, 0.2, if\ w\ has\ exactly\ one\ a \end{cases}$$

Now we can construct a complete intuitionistic fuzzy Buchi automata $K' = (P', N, S, q, \mathfrak{A})$ by the construction procedure given in theorem 4.1

P′ is the finite set of states.

N is the finite set of input alphabets.

$S: P \rightarrow (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic fuzzy initial states.

$q: P \times N \times P \rightarrow (\delta_1, \delta_2),\ 0 < \delta_1 + \delta_2 \leq 1$ is the intuitionistic fuzzy transition function.

$\delta_1\left(q(p_i, w, p_j)\right) = \vee \{\delta_1(p_i, w_1, p_k) \wedge \delta_1(p_k, a, p_j)\}\ where, w = w_1 a_1$ for all $p_k \in P$

$\delta_2(q(p_i, w, p_j)) = \wedge \{\delta_2(p_i, w_1, p_k) \vee \delta_2(p_k, a, p_j)\}\ where\ , w = w_1 a_1$ for all $p_k \in P$

$\mathfrak{A}: P \rightarrow (\delta_1, \delta_{2)}$ where $0 < \delta_1 + \delta_2 \leq 1\ is\ the\ set\ of\ intuitionistic\ fuzzy\ final\ states.$

$P' = \{s, t, e\}$

$f(e, a, e) = (1,0)$

$f(e, b, e) = (1,0)$

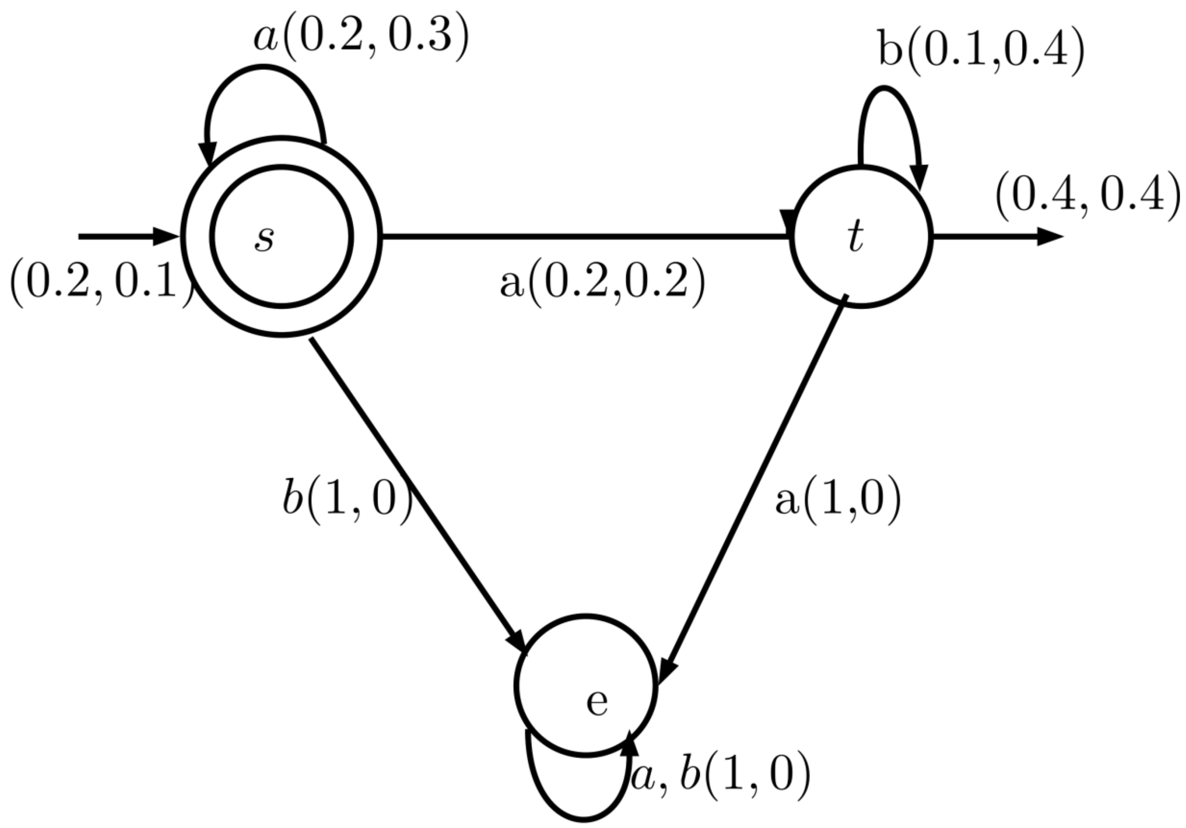
$f(s, a, s) = (0.2,0.3)$

$f(s, a, t) = (0.2,0.2)$

$f(s, b, e) = (1,0)$

$f(t, a, e) = (1,0)$

$f(t, b, t) = (0.1,0.4)$



The intuitionistic fuzzy transition diagram of P' is shown in Figure 4.2.

5 Conclusions

This paper generalize the classical regular grammar into intuitionistic fuzzy regular grammar and also proved that intuitionistic fuzzy regular grammar is equivalent to intuitionistic fuzzy automata.

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