SOFT FIXED POINT THEOREM FOR CONTRACTION CONDITIONS IN SOFT S-METRIC SPACE

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ABSTRACT

In the present paper, we define soft S-metric space and establish existence and uniqueness of soft fixed point theorem satisfying contraction conditions in soft S-metric space, finally, in order to demonstrate the relevance of our ideas and to bolster our findings. Our outcomes sum up a few late outcomes in the setting of S-metric space. These established results improve and modify some existing results in the literature. An illustrative example is provided.

KEYWORDS

Soft set; fixed point; Contraction mapping: Soft metric space; Soft S-metric space

AMS subject Classifications: 47H10, 54H25

INTRODUCTION

Molodtsov [10] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties and has shown several applications of this theory in solving many practical problems in various disciplines like as economics, engineering, etc. Maji *et al.* [8, 9] studied soft set theory in detail and presented an application of soft sets in decision making problems. Chen *et al.* [1] worked on a new definition of reduction and addition of parameters of soft sets, Shabir and Naz [11] studied about soft topological spaces and explained the concept of soft point by various techniques. Das and Samanta introduced a notion of soft real set and number [5], soft complex set and number [6], soft metric space [6, 7]. Sushma et al. [12] proved fixed soft point theorem with soft contractive condition for self-mapping by using implicit relation on complete soft S −metric space.

In the present paper, we have proved the unique soft fixed point theorem of a contractions mapping in the context of soft S −metric space. Before starting to prove main result, some basic definitions are required.

2 Definitions and Preliminaries

Definition 2.1. ([10]) A pair (F, E) is called a soft set over X, where F is a function given by $F: E \rightarrow P(X)$ and E is a set of parameters. In other words, a soft set over X is a parameterized family of subsets of the universe X. For any parameter $x \in E, F(x)$ may be considered as the set of x −approximate elements of the soft set (F, E).

Definition 2.2. [10] Let (F, E) and (G, D) be two soft sets over X. We say that (F, E) is a sub soft set of (G, D) and denote it by $(F, E) \subset (G, D)$ if:

1) $E \subseteq D$, and
2) $F(e) \subseteq G(e), \forall\, e \in E$.

(F, E) Is said to be a super soft set for (G, D), if (G, D) is a sub soft set of (F, E) we denote it by $(F, E) \supseteq (G, D)$

Definition 2.3. [10] Let (F, E) be a soft set over X. then

1) (F, E) is said to be a null soft denoted by $\tilde{\phi}$ if for every $e \in E,\ F(e) = \phi$.
2) (F, E) is said to be an absolute soft set denoted by $\tilde{X}$, if for every $e \in E, F(e) = X$

Definition 2.4. [9] Let $A \subseteq E$ be a set of parameters. The ordered pair (a, r), where $r \in R$ and $a \in A$, is called a soft parametric scalar. The parametric scalar (a, r) is called nonnegative if $r \geq 0$. Let (a, r) and (b, r') be two soft parametric scalar, then (a, r) is called no less than (b, r') denoted by $(a, r) \geq (b, r')$ if $r \geq r'$.

3 Soft S −metric spaces

In order to get our main results, we introduce some definitions and give one example to support our results.

Definition 3.1 [3] A soft S-metric on $\tilde{X}$ is a mapping S:$S: SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \rightarrow \mathbb{R}(E)$ which satisfies the following conditions:

$(\widetilde{S_1})$ $S(\widetilde{u_a}, \widetilde{v_b}, \widetilde{w_c}) \geq \tilde{0}$;

$(\widetilde{S_2})$ $S(\widetilde{u_a}, \widetilde{v_b}, \widetilde{w_c}) = \tilde{0}$; *if and only if* $\widetilde{u_a} = \widetilde{v_b} = \widetilde{w_c}$

$(\widetilde{S_3})$ $S(\widetilde{u_a}, \widetilde{v_b}, \widetilde{w_c}) \leq S(\widetilde{u_a}, \widetilde{u_a}, \widetilde{t_d}) + S(\widetilde{v_b}, \widetilde{v_b}, \widetilde{t_d}) + S(\widetilde{w_c}, \widetilde{w_c}, \widetilde{t_d})$

For all $\widetilde{u_a}, \widetilde{v_b}, \widetilde{w_c}, \widetilde{t_d} \in SP(X)$, then the soft set $\tilde{X}$ with a soft S-metric S is called soft S-metric space and denoted by $(\tilde{X}, \tilde{S}, E)$.

Definition 3.2 [3] A soft sequence $\{\widetilde{u_n}, \widetilde{v_n}\}_{n=1} \infty$ in $(\tilde{X}, \tilde{S}, E)$ is called convergent to $\tilde{u}$ if $\lim_{n \to +\infty} d(\widetilde{u_n}, \tilde{u}) = d(\tilde{u}, \tilde{u})$.

Definition 3.3. [3] A soft sequence $\{\widetilde{u_n}, \widetilde{v_n}\}_{n=1} \infty$ in $(\tilde{X}, \tilde{S}, E)$ is called cauchy if $\lim_{n,m \to +\infty} d(\widetilde{u_n}, \widetilde{u_m}) = 0$.

Definition 3.4. [2] Let $(\tilde{X}, \tilde{S}, E)$ is said to be complete soft metric space over $\tilde{U}$. A Cauchy soft sequence $\{\widetilde{u_n}, \widetilde{v_n}\}_{n=1} \infty$ in $(\tilde{U}, \tilde{V})$, there exists an $\tilde{u} \in \tilde{U}$ such that $\lim_{n,m \to +\infty} d(\widetilde{u_n}, \widetilde{u_m}) =$

$\lim_{n\to+\infty} d(\widetilde{u_n},\tilde{u}) = d(\tilde{u},\tilde{u})$.

Corollary 3.5. [2] Let $(\tilde{X},\tilde{S},E)$ be a complete soft metric space with $s \geq 1$ and the two soft mappings $\tilde{f}$ *and* $\tilde{g}$ have a unique point of coincidence in $\tilde{X}$. Moreover, if the two soft maps $\tilde{f}$ *and* $\tilde{g}$ are weakly compatible, and then $\tilde{f}$ *and* $\tilde{g}$ have a unique common fixed point.

Example 3.6. Let $\tilde{X} = [0,+\infty)$ and a soft S-metric space $\tilde{S}: \tilde{X} \times \tilde{X} \to [0,+\infty)$ defined by

$$S(\tilde{U},\tilde{\eta}) = (\tilde{U}+\tilde{\eta})^2$$

Then $(\tilde{X},\tilde{S},E)$ is a complete soft S-metric space with $s = 2$ a constant. Define $\tilde{F}\tilde{U} = \frac{\tilde{U}}{4}$ and $\tilde{\tilde{G}}U = \left(1+\frac{\tilde{U}}{8}\right)$ are soft mappings $\tilde{f}$ *and* $\tilde{g}$ on $\tilde{X}$. since $\tilde{k} \geq (1+\tilde{k})$ for each $\tilde{k} \in [0,+\infty), \forall\ \tilde{U},\tilde{\eta} \in \tilde{X}$, we have

$$\tilde{S}(\tilde{F}\tilde{U},\tilde{F}\tilde{\eta}) = \left(\frac{\tilde{U}}{4}+\frac{\tilde{\eta}}{4}\right)^2 = \left(2\frac{\tilde{U}}{8}+2\frac{\tilde{\eta}}{8}\right)^2$$

$$= 4\left(\frac{\tilde{U}}{2}+\frac{\tilde{\eta}}{2}\right)^2$$

$$\geq 4\left(\left(1+\frac{\tilde{U}}{8}\right)+\left(1+\frac{\tilde{\eta}}{8}\right)\right)^2 = 4S(\tilde{G}\tilde{U},\tilde{G}\tilde{\eta}),$$

Which means that $\tilde{S}(\tilde{F}\tilde{U},\tilde{F}\tilde{\eta}) \geq \tilde{\alpha}\tilde{S}(\tilde{G}\tilde{U},\tilde{G}\tilde{\eta})$, where $\tilde{\alpha} = 4$. Hence all the conditions of corollary 3.5 are satisfied, hence the mappings $\tilde{F}$ *and* $\tilde{G}$ have a unique point of coincidence actually 0 is the unique point of coincidence. Further by $\widetilde{FG}0 = \widetilde{GF0}$, we observe that 0 is unique fixed point of $\tilde{F}$ *and* $\tilde{G}$

Lemma 3.7. [3] Let $(\tilde{X},\tilde{S},E)$ be a complete soft metric space with $s \geq 1$. Then

1) If $S(\tilde{u},\tilde{v}) = 0$, then $d(\tilde{u},\tilde{u}) = d(\tilde{v},\tilde{v}) = 0$
2) If $\widetilde{\{u_n\}}$ is a sequence such that $\lim_{n\to+\infty} S(\widetilde{u_n},\widetilde{u_{n+1}}) = 0$, then we have $\lim_{n\to\infty} S(\widetilde{u_n},\widetilde{u_n}) = \lim_{n\to\infty} S(\widetilde{u_{n+1}},\widetilde{u_{n+1}}) = 0$
3) If $\widetilde{u_n} \neq \widetilde{v_n}$, then $S(\tilde{u},\tilde{v}) > 0$

Definition 3.8. [3] The soft S −metric space $(\tilde{X},\tilde{S}\ E)$ is called complete, if every cauchy sequence in $\tilde{X}$ convergence to some point of $\tilde{X}$.

Definition 3.9. [3] Let $(\tilde{X},\tilde{S},E)$ be a soft S −metric space. A function $(f,\varphi): (\tilde{X},\tilde{S},E) \to (\tilde{X},\tilde{S},E)$ is called a soft contraction mapping if there is a soft real number $\alpha \in R, 0 \leq \alpha < 1$ such that for every point $\widetilde{x_\lambda},\widetilde{y_\mu} \in SP(X)$, we have

$$S\left((f,\varphi)(\widetilde{x_\lambda}),(f,\varphi)(\widetilde{y_\mu})\right) \leq \alpha S(\widetilde{x_\lambda},\widetilde{y_\mu}),$$

Definition 3.10. [3] Let $(f,\varphi): (\tilde{X},\tilde{S},E) \to (\tilde{Y},\tilde{S'},E')$ be a soft mapping from soft S −metric space $(\tilde{X},\tilde{S},E)$ to a soft S −metric space $(\tilde{Y},\tilde{S'},E')$. Then (f,φ) is a soft continuous at a soft point $\widetilde{u_a} \in SP(\tilde{X})$ if and only if $(f,\varphi)(\{\widetilde{u_a}\}) \to (f,\varphi)(\widetilde{u_a})$.

4. MAIN RESULTS

Theorem 4.1. Let $(\tilde{X},\tilde{S},E)$ be a complete soft S −metric space and let $T: \tilde{X} \to \tilde{X}$ be a mapping satisfying the following condition.

$$\tilde{S}(T\tilde{x},T\tilde{y}) \leqq \alpha\tilde{S}(\tilde{x},\tilde{y}) + \beta\frac{\tilde{S}(\tilde{x},T\tilde{x})\tilde{S}(\tilde{y},T\tilde{y})}{\tilde{S}(\tilde{x},\tilde{y})}$$

$$+\gamma[\tilde{S}(\tilde{x},T\tilde{x})+\tilde{S}(\tilde{y},T\tilde{y})] + \delta[\tilde{S}(\tilde{x},T\tilde{y})+\tilde{S}(\tilde{y},T\tilde{x})]$$

$$+\eta\frac{\tilde{S}(\tilde{x},\tilde{y})\left[1+\sqrt{\tilde{S}(\tilde{x},\tilde{y})\tilde{S}(\tilde{x},T\tilde{x})}\right]^2}{\left[1+\tilde{S}(\tilde{x},\tilde{y})\right]^2}$$

for all $\tilde{x},\tilde{y} \in \tilde{X}$; $\alpha,\beta,\gamma,\eta \geq 0$; $\alpha+\beta+\gamma+\eta > 1$.

Then T has a unique fixed point.

Proof: Define a sequence $\{\tilde{x}_n\}$as follows

Let $\tilde{x}_0 \in \tilde{X}$ we define a sequence $\{\tilde{x}_n\}$ in X by

$$\tilde{x}_{n+1} = T\tilde{x}_n \; for\; all\; n = 0,1,2,3, \dots, \dots, \dots, \dots.$$

Where $\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) \neq 0$, it follows from (4.1) and

$$\tilde{S}(\tilde{x}_n, \tilde{x}_n) \leqq \tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})$$

Consider $\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1}) = \tilde{S}(T\tilde{x}_{n-1}, T\tilde{x}_n)$

$$\widetilde{\leqq}\; \alpha\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \beta\frac{\tilde{S}(\tilde{x}_{n-1}, T\tilde{x}_{n-1})\tilde{S}(\tilde{x}_n, T\tilde{x}_n)}{\tilde{d}(\tilde{x}_{n-1}, \tilde{x}_n)}$$

$$+ \gamma[\tilde{S}(\tilde{x}_{n-1}, T\tilde{x}_{n-1}) + \tilde{S}(\tilde{x}_n, T\tilde{x}_n)]$$

$$+ \delta[\tilde{S}(\tilde{x}_{n-1}, T\tilde{x}_n) + \tilde{S}(\tilde{x}_n, T\tilde{x}_{n-1})]$$

$$+ \eta\frac{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\left[1 + \sqrt{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\tilde{S}(\tilde{x}_{n-1}, T\tilde{x}_{n-1})}\right]^2}{\left[1 + \tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\right]^2}$$

$$= \alpha\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \beta\frac{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})}{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)}$$

$$+ \gamma[\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})]$$

$$+ \delta[\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_{n+1}) + \tilde{S}(\tilde{x}_n, \tilde{x}_n)]$$

$$+ \eta\frac{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\left[1 + \sqrt{\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)}\right]^2}{\left[1 + \tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)\right]^2}$$

$$\widetilde{\leqq}\; \alpha\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \beta\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})$$

$$+ \gamma\left[\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})\right]$$

$$+ \delta\left[\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_{n+1}) + \tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})\right]$$

$$+ \eta\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)$$

$$\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1}) \;\widetilde{\leqq}\; (\alpha + \gamma + 2\delta + \eta)\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + (\beta + \gamma + 2\delta)\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1})$$

Hence we have

$$\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1}) \;\widetilde{\leqq}\; \frac{\alpha + \gamma + 2\delta + \eta}{1 - (\beta + \gamma + 2\delta)}\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)$$

$$\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1}) \;\widetilde{\leqq}\; L\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n)$$

Where $L = \frac{\alpha+\gamma+2\delta+\eta}{1-(\beta+\gamma+2\delta)}$, $0 \;\widetilde{\leqq}\; L < 1$

Similarly, we have

$$\tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) \;\widetilde{\leqq}\; L\tilde{S}(\tilde{x}_{n-2}, \tilde{x}_{n-1}).$$

Continuing this process, we conclude that

$$\tilde{S}(\tilde{x}_n, \tilde{x}_{n+1}) \;\widetilde{\leqq}\; L^n\tilde{S}(\tilde{x}_0, \tilde{x}_1)$$

Now for $n > m$, using triangular inequality we have

$$\tilde{S}(\tilde{x}_n, \tilde{x}_m) \;\widetilde{\leqq}\; \tilde{S}(\tilde{x}_{n-1}, \tilde{x}_n) + \tilde{S}(\tilde{x}_{n-2}, \tilde{x}_{n-1}) + \dots\dots\dots\dots\dots + \tilde{S}(\tilde{x}_m, \tilde{x}_{m+1})$$

$$\widetilde{\leqq}\; [L^{n-1} + L^{n-2} + L^{n-3} + L^{n-4} + \dots\dots\dots\dots + L^m]\tilde{S}(\tilde{x}_0, \tilde{x}_1)$$

$$\widetilde{\leqq}\; \frac{L^m}{1 - L}\tilde{S}(\tilde{x}_0, \tilde{x}_1)$$

For a natural number N_1 let $c < 0$ such that $\frac{L^m}{1-L}\tilde{S}(\tilde{x}_0, \tilde{x}_1) < c, \forall\, m \geq N_1$.

Thus $\tilde{S}(\tilde{x}_n, \tilde{x}_m) \;\widetilde{\leqq}\; \frac{L^m}{1-L}\tilde{S}(\tilde{x}_0, \tilde{x}_1) < c$ for $n > m$.

Therefore $\{\tilde{x}_n\}$ is a Cauchy Sequence in a complete soft S-metric space $(\tilde{X}, \tilde{S})$, $\exists\, \tilde{z}^* \in X$ such that $\tilde{x}_n \to \tilde{z}^*$ as $n \to \infty$. As T is continuous, so $T\lim_{n\to\infty} \tilde{x}_n = T\tilde{z}^*$ implies $\lim_{n\to\infty} T\tilde{x}_n = T\tilde{z}^*$ implies $\lim_{n\to\infty} \tilde{x}_{n-1} = T\tilde{z}^*$ implies $T\tilde{z}^* = \tilde{z}^*$. Hence $\tilde{z}^*$ is a fixed point T.

For uniqueness of fixed point $\tilde{z}^*$, let $\tilde{z}^{**}(\tilde{z}^* \neq \tilde{z}^{**})$ be another fixed point of T. We have to prove that $\tilde{S}(\tilde{z}^*, \tilde{z}^*) = \tilde{S}(\tilde{z}^{**}, \tilde{z}^{**}) = 0$

Consider $\tilde{S}(\tilde{z}^*,\tilde{z}^*)=\tilde{S}(T\tilde{z}^*,T\tilde{z}^*)$, then by (4.1) we have

$$\tilde{S}(\tilde{z}^*,\tilde{z}^*)\leq\alpha\tilde{S}(\tilde{z}^*,\tilde{z}^*)+\beta\frac{\tilde{S}(\tilde{z}^*,T\tilde{z}^*)\tilde{S}(\tilde{z}^*,T\tilde{z}^*)}{\tilde{S}(\tilde{z}^*,\tilde{z}^*)}+\gamma\left[\tilde{S}(\tilde{z}^*,T\tilde{z}^*)+\tilde{S}(\tilde{z}^*,T\tilde{z}^*)\right]+\delta\left[\tilde{S}(\tilde{z}^*,T\tilde{z}^*)+\tilde{S}(\tilde{z}^*,T\tilde{z}^*)\right]$$

$$+\eta\frac{\tilde{S}(\tilde{z}^*,\tilde{z}^*)\left[1+\sqrt{\tilde{S}(\tilde{z}^*,\tilde{z}^*)\tilde{S}(\tilde{z}^*,T\tilde{z}^*)}\right]^2}{\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^*)\right]^2}\quad(4.2)$$

$$\tilde{S}(\tilde{z}^*,\tilde{z}^*)\lesseqqgtr(\alpha+\beta+\gamma+\delta+\eta)\tilde{S}(\tilde{z}^*,\tilde{z}^*)$$

Which is a contradiction due to $0\lesseqqgtr\alpha+\beta+2\gamma+4\delta+\eta<1$. , Hence $\tilde{S}(\tilde{z}^*,\tilde{z}^*)=0$.

Similarly, we can show that $\tilde{S}(\tilde{z}^{**},\tilde{z}^{**})=0$.

Now consider $\tilde{S}(\tilde{z}^*,\tilde{z}^{**})=\tilde{S}(T\tilde{z}^*,T\tilde{z}^{**})$

$$\lesseqqgtr\alpha\tilde{S}(\tilde{z}^*,\tilde{z}^{**})+\beta\frac{\tilde{S}(\tilde{z}^*,T\tilde{z}^*)\tilde{S}(\tilde{z}^{**},T\tilde{z}^{**})}{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})}+\gamma\left[\tilde{S}(\tilde{z}^*,T\tilde{z}^*)+\tilde{S}(\tilde{z}^{**},T\tilde{z}^{**})\right]+\delta\left[\tilde{S}(\tilde{z}^*,T\tilde{z}^{**})+\tilde{S}(\tilde{z}^{**},T\tilde{z}^*)\right]$$

$$+\eta\frac{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\left[1+\sqrt{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\tilde{S}(\tilde{z}^*,T\tilde{z}^*)}\right]^2}{\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\right]^2}$$

$$\lesseqqgtr\alpha\tilde{S}(\tilde{z}^*,\tilde{z}^{**})+\beta\frac{\tilde{S}(\tilde{z}^*,\tilde{z}^*)\tilde{S}(\tilde{z}^{**},\tilde{z}^{**})}{\tilde{S}(z^*,\tilde{z}^{**})}+\gamma\left[\tilde{S}(\tilde{z}^*,\tilde{z}^*)+\tilde{S}(\tilde{z}^{**},\tilde{z}^{**})\right]$$

$$+\delta\left[\tilde{S}(\tilde{z}^*,\tilde{z}^{**})+\tilde{S}(\tilde{z}^{**},\tilde{z}^*)\right]$$

$$+\eta\frac{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\left[1+\sqrt{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\tilde{S}(\tilde{z}^*,\tilde{z}^*)}\right]^2}{\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\right]^2}\quad(4.3)$$

Since $1\lesseqqgtr 1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})$,

So $1\lesseqqgtr\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\right]^2$

$$\Rightarrow\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\lesseqqgtr\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\right]^2\tilde{S}(\tilde{z}^*,\tilde{z}^{**})$$

$$\Rightarrow\frac{\tilde{S}(\tilde{z}^*,\tilde{z}^{**})}{\left[1+\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\right]^2}\lesseqqgtr\tilde{S}(\tilde{z}^*,\tilde{z}^{**})$$

Thus (4.3) becomes

$$\tilde{S}(\tilde{z}^*,\tilde{z}^{**})\lesseqqgtr(\alpha+2\delta+\eta)\tilde{S}(\tilde{z}^*,\tilde{z}^{**})$$

This is contradiction thus $\tilde{S}(\tilde{z}^*,\tilde{z}^{**})=0$

Similarly we can show that $\tilde{S}(\tilde{z}^{**},\tilde{z}^*)=0$ which implies $\tilde{z}^*=\tilde{z}^{**}$ is the unique soft fixed point of T.

Corollary 4.2. Let $(\tilde{X},\tilde{S},E)$ be a complete soft S −metric space and let $T:\tilde{X}\to\tilde{X}$ be a mapping satisfying the following condition.

$$\tilde{S}(T\tilde{x},T\tilde{y})\lesseqqgtr\alpha\tilde{S}(\tilde{x},\tilde{y})+\beta\frac{\tilde{S}(\tilde{x},T\tilde{x})\tilde{S}(\tilde{y},T\tilde{y})}{\tilde{S}(\tilde{x},\tilde{y})}+\gamma\left[\tilde{S}(\tilde{x},T\tilde{x})+\tilde{S}(\tilde{y},T\tilde{y})\right]+\delta\left[\tilde{S}(\tilde{x},T\tilde{y})+\tilde{S}(\tilde{y},T\tilde{x})\right]+\eta\frac{\tilde{S}(\tilde{x},\tilde{y})\left[1+\sqrt{\tilde{S}(\tilde{x},\tilde{y})\tilde{S}(\tilde{x},T\tilde{x})}\right]^2}{\left[1+\tilde{S}(\tilde{x},\tilde{y})\right]^2}+\zeta\tilde{S}(\tilde{x},\tilde{y})^2$$

for all $\tilde{x},\tilde{y}\in\tilde{X}$; $\alpha,\beta,\gamma,\eta,\zeta\geq 0$; $\alpha+\beta+\gamma+\eta+\zeta>1$. Then T has a unique fixed point.

Example 4.3. Let $\tilde{x}=[0,1]$. Define a complete soft S −metric by $\tilde{S}(\tilde{x},\tilde{y})^2=|y|,\forall\,\tilde{x},\tilde{y}\in\tilde{X}$ and define a continuous self-mapping T by $T\tilde{y}=\frac{y}{4}\forall\,\tilde{y}\in\tilde{X}$.

Set $\alpha=\frac{1}{8},\beta=\frac{1}{12},\gamma=\frac{1}{20},\delta=\frac{1}{24}$ and $\eta=\frac{1}{30}$. Then T satisfies all assumptions of Theorem 4.1 and $\eta=0$ is the unique fixed point of T in $\tilde{X}$.

Conclusion

In this paper we have proved a fixed point theorem for contraction mapping in soft S −metric space. Established result improves and modify results due Cigdem *et., al* [3] and Sushma *et., al* [12].

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