SOME FUNCTION IN INTUITIONISTIC TOPOLOGICAL SPACE

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ABSTRACT

In this paper is to introduce a new concept of Strongly $\mathrm{I}\alpha_{g}^{\wedge}$-homeomorphism and Slightly $\mathrm{I}\alpha_{g}^{\wedge}$-Continuous in intuitionistic topological spaces and investigate some of their basic properties and give characterizations for these spaces. We also study the relationship between the other intuitionistic sets are also discussed.

1 Introduction

The concept of intuitionistic sets in topological spaces was first introduced by Coker[1] in 1996. He also introduced the concept of intuitionistic points and investigated some fundamental properties of closed sets in intuitionistic topological spaces. [3] J.Arul Jesti and K.Heartlin introduced the concept of alpha^generalized closed sets in intuitionistic topological spaces and discuss some properties related to $\mathrm{I}\alpha_{g}^{\wedge}$-closed set in intuitionistic topological spaces. [5] J.Arul Jesti and K.Heartlin $\mathrm{I}\alpha_{g}^{\wedge}$-Closed Map and $\mathrm{I}\alpha_{g}^{\wedge}$-Homeomorphism in Intuitionistic Topological Spaces.The purpose of this paper is to develop Strongly $\mathrm{I}\alpha_{g}^{\wedge}$-homeomorphism and Slightly $\mathrm{I}\alpha_{g}^{\wedge}$-Continuous function in intuitionistic topological spaces and also study its relations with some of existing intuitionistic relations.

2. Preliminaries

Definition 2.1 [1]: Let $\mathcal{H}$ be a non-empty set. An intuitionistic set (IS for short) $\mathcal{A}$ is an object having the form $\mathcal{A} =<\mathcal{H}, \mathcal{A}_1, \mathcal{A}_2>$ Where $\mathcal{A}_1$ and $\mathcal{A}_2$ are subsets of $\mathcal{H}$ satisfying $\mathcal{A}_1 \cap \mathcal{A}_2 = \varphi$. The set $\mathcal{A}_1$ is called the set of members of $\mathcal{A}$, while $\mathcal{A}_2$ is called set of non members of $\mathcal{A}$.

Definition 2.2 [1]: Let $\mathcal{H}$ be a non-empty set and $\mathcal{A}$ and $\mathfrak{B}$ are intuitionistic set in the form $\mathcal{A} =< \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 >$, $\mathfrak{B} =< \mathcal{H}, \mathfrak{B}_1, \mathfrak{B}_2 >$ respectively. Then

a) $\mathcal{A} \subseteq \mathfrak{B}$ iff $\mathcal{A}_1 \subseteq \mathfrak{B}_1$ and $\mathcal{A}_2 \supseteq \mathfrak{B}_2$

b) $\mathcal{A} = \mathfrak{B}$ iff $\mathcal{A} \subseteq \mathfrak{B}$ and $\mathfrak{B} \subseteq \mathcal{A}$

c) $\mathcal{A}^C = < \mathcal{H}, \mathcal{A}_2, \mathcal{A}_1 >$

d) $\mathcal{A} - \mathfrak{B} = \mathcal{A} \cap \mathfrak{B}^C$

e) $\varphi = < \mathcal{H}, \varphi, \mathcal{H} >, \mathcal{H} =< \mathcal{H}, \mathcal{H}, \varphi >$

f) $\mathcal{A} \cup \mathfrak{B} = < \mathcal{H}, \mathcal{A}_1 \cup \mathfrak{B}_1, \mathcal{A}_2 \cap \mathfrak{B}_2 >$

g) $\mathcal{A} \cap \mathfrak{B} = < \mathcal{H}, \mathcal{A}_1 \cap \mathfrak{B}_1, \mathcal{A}_2 \cup \mathfrak{B}_2 >$.

Definition 2.3 [1]: Let $(\mathcal{H}, I\tau)$ be an ITS$(\mathcal{H})$. An intuitionistic set $\mathcal{A}$ of $\mathcal{H}$ is said to be

(i) Intuitionistic semi-open if $\mathcal{A} \subseteq I\mathrm{cl}(I\mathrm{int}\,(\mathcal{A}))$.

(ii) Intuitionistic pre-open if $\mathcal{A} \subseteq I\mathrm{int}(I\mathrm{cl}\,(\mathcal{A}))$.

(iii) Intuitionistic α-open if $\mathcal{A} \subseteq I\text{int}\big(I\text{cl}(I\text{int}(\mathcal{A}))\big)$.

(iv) Intuitionistic β-open if $\mathcal{A} \subseteq I\text{cl}(I\text{int}(I\text{cl}(\mathcal{A})))$.

(v) Intuitionistic regular-open if $\mathcal{A} = I\text{int}(I\text{cl}(\mathcal{A}))$.

Definition 2.4 [1]: Let $(\mathcal{H}, I\tau_\mu)$ and $(Y, I\tau_\theta)$ be two ITS'S and $f: \mathcal{H} \to Y$ be a function. Then f is said to be ***intuitionistic continuous*** iff the pre image of each Is in $I\tau_\theta$ is an Is in $I\tau_\mu$.

Definition 2.5 [3]: A subset $\mathcal{A}$ of $(\mathcal{H}, I\tau)$ is called an ***intuitionistic alpha ^ generalized* closed (**briefly ***$I\alpha_g^{\wedge}$-CS*)** if $I\text{gcl}(\mathcal{A}) \subseteq \mathcal{F}$, whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is $I\alpha$-O in $\mathcal{H}$. We denote the family of all $I\alpha_g^{\wedge}$-CS in space $\mathcal{H}$ by $I\alpha_g^{\wedge}$-$CS(\mathcal{H}, I\tau)$.

Definition 2.6 [5]:A function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ is a **$\mathrm{I}\alpha_g^{\wedge}$-*homeomorphism*** if

1. f is 1-1 and onto
2. f is $\mathrm{I}\alpha_g^{\wedge}$-continuous
3. f is $\mathrm{I}\alpha_g^{\wedge}$-open map

Definition 2.7 [4]: A function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ is said to be **$\mathrm{I}\alpha_g^{\wedge}$-*irresolute*** if $f^{-1}(O)$ is a $\mathrm{I}\alpha_g^{\wedge}$-closed in $(\mathcal{H}, I\tau_\mu)$ for every $\mathrm{I}\alpha_g^{\wedge}$- closed set O in $(\mathrm{Y}, I\tau_\theta)$.

Definition 2.8 [6]: A function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ is called a ***strongly $I\alpha_g^{\wedge}$-continuous function*** if the inverse image of every $\mathrm{I}\alpha_g^{\wedge}$-open set in $(\mathrm{Y}, I\tau_\theta)$ is I-open in $(\mathcal{H}, I\tau_\mu)$.

Definition 2.9 [6]: A map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ is said to be ***perfectly $I\alpha_g^{\wedge}$-continuous function*** if the inverse image of every $\mathrm{I}\alpha_g^{\wedge}$-open set in $(\mathrm{Y}, I\tau_\theta)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_\mu)$.

Definition 2.10 [4]: An Intuitionistic topological space $(\mathcal{H}, I\tau_\mu)$ is said to be a ***$I\alpha_g^{\wedge}$-$T_{1/2}$space*** if every $\mathrm{I}\alpha_g^{\wedge}$-closed set of $\mathcal{H}$ is I-closed in $\mathcal{H}$.

3. Strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism

Definition 3.1: A bijection $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ is said to be ***strongly* $\mathrm{I}\alpha_g^{\wedge}$- *homeomorphism*** if both f and f^{-1} are $\mathrm{I}\alpha_g^{\wedge}$-irresolute.

Example 3.2: Let $\mathcal{H} = \{\mathrm{a}, \mathrm{b}\} = \mathrm{Y}$ and family $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 = <\mathcal{H}, \{b\}, \{a\}>$, $\mathcal{A}_2 = <\mathcal{H}, \varphi, \{a\}>$,$I\tau_\theta$=$\{\mathrm{Y}, \phi, <Y, \{b\}, \varphi>, <Y, \varphi, \varphi>\}$. Then $\mathrm{I}\alpha_g^{\wedge}O(\mathcal{H}) = \{\mathcal{H}, \varphi, <\mathcal{H}, \varphi, \varphi>, <\mathcal{H}, \varphi, \{a\}>, <\mathcal{H}, \{b\}, \varphi>, <\mathcal{H}, \varphi, \{b\}>, <\mathcal{H}, \{b\}, \{a\}>\}$ and $\mathrm{I}\alpha_g^{\wedge}O(Y) = \{Y, \varphi, <Y, \varphi, \varphi>, <Y, \{a\}, \varphi>, <Y, \{b\}, \varphi>, <Y, \varphi, \{a\}>, <Y, \{b\}, \{a\}>\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ as $f(a) = b, f(b) = a$. Then clearly, f is strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism.

We denote the family of all strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism of ITS's $(\mathcal{H}, I\tau)$ into itself by $\mathrm{SI}\alpha_g^{\wedge}$-h($\mathcal{H}$).

Theorem 3.3: Every strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism is a $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism.

Proof: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ be a bijective map which is strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism. Then f and f^{-1} are $\mathrm{I}\alpha_g^{\wedge}$-irresolute. Since, every $\mathrm{I}\alpha_g^{\wedge}$-irresolute are $\mathrm{I}\alpha_g^{\wedge}$-continuous, f and $f^{-1}$are $\mathrm{I}\alpha_g^{\wedge}$-continuous. Since, $f^{-1}$is $\mathrm{I}\alpha_g^{\wedge}$-continuous, f is $\mathrm{I}\alpha_g^{\wedge}$-open map. Thus, f is both $\mathrm{I}\alpha_g^{\wedge}$-continuous and $\mathrm{I}\alpha_g^{\wedge}$-open. Therefore, f is $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism.

Remark 3.4: The converse of the above theorem need not be true.

Example 3.5: In example 3.2, Define Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (\mathrm{Y}, I\tau_\theta)$ as $f(a) = b, f(b) = a$. Then, f is $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism. Here, $f^{-1}(<Y, \{a\}, \varphi>) = <\mathcal{H}, \{a\}, \varphi>$ which is not $\mathrm{I}\alpha_g^{\wedge}$-open in $\mathcal{H}$. Hence, f is not strongly $\mathrm{I}\alpha_g^{\wedge}$-homeomorphism.

Theorem 3.6: If $f:(\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ and $g:(Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ are strongly $I\alpha_g^{\wedge}$-homeomorphism then $g \circ f : (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is also strongly $I\alpha_g^{\wedge}$-homeomorphism.

Proof: (i) $(g \circ f)$ is $I\alpha_g^{\wedge}$-irresolute.

Let P be a $I\alpha_g^{\wedge}$-open in Z. Now, $(g \circ f)^{-1}(P) = f^{-1}g^{-1}(P) = f^{-1}(Q)$ where $Q = g^{-1}(P)$. By hypothesis, $Q = g^{-1}(P)$ is $I\alpha_g^{\wedge}$-open in Y and again, by hypothesis $f^{-1}(Q)$ is $I\alpha_g^{\wedge}$-open in $\mathcal{H}$.

(ii) $(g \circ f)^{-1}$is $I\alpha_g^{\wedge}$-irresolute

Let G be a $I\alpha_g^{\wedge}$-open in $\mathcal{H}$. By hypothesis, $f(G)$ is $I\alpha_g^{\wedge}$-open in Y. Again, by hypothesis $(g \circ f)(G) = g(f(G))$ is $I\alpha_g^{\wedge}$-open in Z. Thus, $(g \circ f)^{-1}$is $I\alpha_g^{\wedge}$-irresolute.

From (i) and (ii), $g \circ f : (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is also strongly $I\alpha_g^{\wedge}$-homeomorphism.

Theorem 3.7: Every strongly $I\alpha_g^{\wedge}$-homeomorphism is $I\alpha_g^{\wedge}$-irresolute.

Proof: It is the consequence of the definition.

Remark 3.8: The converse of the above theorem need not be true.

Example 3.9: Let $\mathcal{H}$ = {a,b,c}=Y $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 = <\mathcal{H}, \{a\}, \varphi>$, $\mathcal{A}_2 = <\mathcal{H}, \{a\}, \{b\}>$, $\mathcal{A}_3 = <\mathcal{H}, \varphi, \varphi>$, $\mathcal{A}_4 = <\mathcal{H}, \{c\}, \varphi>$, $\mathcal{A}_5 = <\mathcal{H}, \varphi, \{b\}>$, $\mathcal{A}_6 = <\mathcal{H}, \{a, c\}, \varphi>$. and $I\tau_\theta = \{Y, \phi, <Y, \{a\}, \{c\}>, <Y, \{a, b\}, \varphi>, <Y, \varphi, \{a, c\}>\}$. Define $f:(\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = b, f(b) = c\ and\ f(c) = a$. Clearly, f is $I\alpha_g^{\wedge}$-irresolute. But $f(<Y, \varphi, \{c\}>) = <\mathcal{H}, \varphi, \{a\}>$ which is not $I\alpha_g^{\wedge}$-open in $\mathcal{H}$. Hence, f is not strongly $I\alpha_g^{\wedge}$-homeomorphism.

Theorem 3.10: The set S $I\alpha_g^{\wedge}$-h($\mathcal{H}$) is a group under the composition of maps.

Proof: Define a binary operation ‘ * ’ from S $I\alpha_g^{\wedge}$-h($\mathcal{H}$) ×S $I\alpha_g^{\wedge}$-h($\mathcal{H}$) → S $I\alpha_g^{\wedge}$-h($\mathcal{H}$) , by $f * g = f \circ g$ for all f and g in S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) and ∘ is the usual operation of composition of maps. Then by theorem 3.6, $f \circ g \in$ S$I\alpha_g^{\wedge}$-h($\mathcal{H}$).We know that the composition of maps are associative and the identity map $i : (\mathcal{H}, I\tau) \rightarrow (\mathcal{H}, \tau I)$ belonging to S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) serves as the identity element. If $f \in$ S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) then $f^{-1} \in$S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) such that $f \circ f^{-1} = f^{-1} \circ f = i$ and so inverse exists for each element of S$I\alpha_g^{\wedge}$-h($\mathcal{H}$). Therefore, S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) is a group under the composition of maps.

Theorem 3.11: Let$f:(\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a strongly $I\alpha_g^{\wedge}$-homeomorphism. Then f induces an isomorphism from the group S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) onto the group S $I\alpha_g^{\wedge}$-h(Y).

Proof: Using the map f, we define a map ψ_f: S$I\alpha_g^{\wedge}$-h($\mathcal{H}$) →S $I\alpha_g^{\wedge}$-h(Y) by ψ_f(h) = $f \circ h \circ f^{-1}$ for each $h \in$S$I\alpha_g^{\wedge}$-h($\mathcal{H}$). By theorem 3.6, ψ_f is well defined in general, because $f \circ h \circ f^{-1}$ is a strongly $I\alpha_g^{\wedge}$-homeomorphism for every strongly $I\alpha_g^{\wedge}$-homeomorphism $f : (\mathcal{H}, I\tau_\mu) \rightarrow (Y, I\tau_\theta)$. Clearly, ψ_f is bijective. Further for all h_1, $h_2 \in$ S$I\alpha_g^{\wedge}$-h($\mathcal{H}$), $\psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \psi_f(h_1) \circ \psi_f(h_2)$. Therefore, ψ_f is a homeomorphism and hence it induces an isomorphism induced by f.

Theorem 3.12: Strongly $I\alpha_g^{\wedge}$-homeomorphism is an equivalence relation on the collection of all ITS’s.

Proof: Reflexivity and symmetry are immediate and transitivity follows from theorem 3.6.

4 Slightly $I\alpha_g^{\wedge}$-Continuous

Definition 4.1: A function $f:(\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is called ***Slightly $I\alpha_g^{\wedge}$-Continuous*** at a point x∈ $\mathcal{H}$ if for each I-clopen subset V of Y containing f(x), there exists an $I\alpha_g^{\wedge}$-open subset U in $\mathcal{H}$ containing x such that f(U) ⊑V. The function f is said to be slightly $I\alpha_g^{\wedge}$-continuous if f is slightly $I\alpha_g^{\wedge}$-continuous at each of its points.

Definition 4.2: A function $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ is said to be ***Slightly*** $\boldsymbol{I\alpha_g^{\wedge}}$***-Continuous*** if the inverse image of every I-clopen set in $(\mathrm{Y}, I\tau_{\theta})$ is I$\alpha_g^{\wedge}$-open in $(\mathcal{H}, I\tau_{\mu})$.

Example 4.3: Let $\mathcal{H} = \{\mathrm{a,b}\} = \mathrm{Y}$ and family $I\tau_{\mu} = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 = <\mathcal{H}, \{b\}, \varphi>$, $\mathcal{A}_2 = <\mathcal{H}, \varphi, \varphi>$ and $I\tau_{\mu}$={Y, ϕ, $<Y, \{a\}, \varphi>, <Y, \varphi, \{a\}>, <Y, \varphi, \varphi>$}. Define $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ as $f(a) = a, f(b) = b$. Then clearly, f is slightly I$\alpha_g^{\wedge}$-continuous.

Proposition 4.4: The definition 4.1 and 4.2 are equivalent.

Proof: Suppose the definition 4.1 holds. Let O be a I-clopen set in Y and x∈$f^{-1}(O)$. Then f(x)∈O and thus there exists an I$\alpha_g^{\wedge}$-open set $\mathrm{U_x}$ such that x $\in \mathrm{U_x} \subseteq f^{-1}(O)$ and $f^{-1}(O) = \cup_{\mathrm{x} \in f^{-1}(O)} \mathrm{U_x}$. Since, arbitrary union of I$\alpha_g^{\wedge}$-open set is I$\alpha_g^{\wedge}$-open. $f^{-1}(O)$ is I$\alpha_g^{\wedge}$-open in $\mathcal{H}$ and therefore, f is slightly I$\alpha_g^{\wedge}$-continuous . Suppose, the definition 4.2 holds. Let f (x) ∈O where, O is a I-clopen set in Y. Since, f is slightly I$\alpha_g^{\wedge}$-continuous, x ∈$f^{-1}(O)$ where $f^{-1}(O)$ is I$\alpha_g^{\wedge}$-open in $\mathcal{H}$. Let U = $f^{-1}(O)$. Then U is I$\alpha_g^{\wedge}$- open in $\mathcal{H}$, x ∈ $\mathcal{H}$ and f(U) ⊆ O.

Theorem 4.5: For a function $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$, the following statements are equivalent.

(i) f is slightly I$\alpha_g^{\wedge}$-continuous.

(ii) The inverse image of every I-clopen set O of Y is I$\alpha_g^{\wedge}$-open in $\mathcal{H}$.

(iii) The inverse image of every I-clopen set O of Y is I$\alpha_g^{\wedge}$-closed in $\mathcal{H}$.

(iv) The inverse image of every I-clopen set O of Y is I$\alpha_g^{\wedge}$-clopen in $\mathcal{H}$.

Proof:

(i) ⇒(ii): Follows from the proposition 4.4

(ii)⇒(iii): Let O be a I-clopen set in Y which implies O^c is I-clopen in Y. By (ii), f^{-1} (O^c) =(f^{-1} $(O))^c$ is I$\alpha_g^{\wedge}$-open in $\mathcal{H}$. Therefore, $f^{-1}(O)$ is I$\alpha_g^{\wedge}$- closed in $\mathcal{H}$.

(iii) ⇒ (iv): By (ii) and (iii), $f^{-1}(O)$is I$\alpha_g^{\wedge}$-clopen in $\mathcal{H}$.

(iv) ⇒ (i): Let O be a I-clopen set in Y containing f (x), by (iv) $f^{-1}(O)$is I$\alpha_g^{\wedge}$-clopen in $\mathcal{H}$. Take U = $f^{-1}(O)$, then f(U) ⊆ O. Hence, f is slightly I$\alpha_g^{\wedge}$-continuous .

Theorem 4.6: Every slightly I-continuous function is slightly I$\alpha_g^{\wedge}$-continuous.

Proof: Let $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ be a slightly I-continuous function. Let O be a I-clopen set in Y. Then, $f^{-1}(O)$ is I-open in $\mathcal{H}$. Since, every I-open set is I$\alpha_g^{\wedge}$-open. Hence, f is slightly I$\alpha_g^{\wedge}$-continuous .

Remark 4.7: The converse of the above theorem need not be true.

Example 4.8: Let $\mathcal{H} = \{\mathrm{a,b}\} = \mathrm{Y}$ and $I\tau_{\mu} = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 = <\mathcal{H}, \{b\}, \{a\}>$, $\mathcal{A}_2 = <\mathcal{H}, \varphi, \{a\}>$, $I\tau_{\theta}$={Y, ϕ, $<Y, \{b\}, \varphi>, <Y, \varphi, \varphi>$}. Define $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ as $f(a) = b, f(b) = a$. Clearly, f is slightly I$\alpha_g^{\wedge}$-continuous but not slightly I-continuous. Since, $f^{-1}(<Y, \varphi, \varphi>) = <\mathcal{H}, \varphi, \varphi>$ where $<Y, \varphi, \varphi>$ is I-clopen in Y but $<\mathcal{H}, \varphi, \varphi>$ is not I-open in $\mathcal{H}$.

Theorem 4.9: Every I$\alpha_g^{\wedge}$-continuous function is slightly I$\alpha_g^{\wedge}$-continuous.

Proof: $f:(\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ be a I$\alpha_g^{\wedge}$-continuous function. Let O be a I-clopen set in Y. Then, $f^{-1}(O)$ is I$\alpha_g^{\wedge}$-open in $\mathcal{H}$ and I$\alpha_g^{\wedge}$-closed in $\mathcal{H}$. Hence, f is slightly I$\alpha_g^{\wedge}$-continuous.

Remark 4.10: The converse of the above theorem need not be true.

Example 4.11: In example 4.8, The function f is slightly I$\alpha_g^{\wedge}$-continuous but not I$\alpha_g^{\wedge}$-continuous, since, f^{-1} $(<Y, \{b\}, \varphi>) = <\mathcal{H}, \{a\}, \varphi>$ is not I$\alpha_g^{\wedge}$-open in $\mathcal{H}$.

Theorem 4.12: Every contra I$\alpha_g^{\wedge}$-continuous function is slightly I$\alpha_g^{\wedge}$-continuous.

Example 4.3: Let $\mathcal{H} = \{a, b\} = Y$ and family $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{b\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \varphi, \varphi >$ and $I\tau_\mu$={Y, ϕ, $< Y, \{a\}, \varphi >, < Y, \varphi, \{a\} >, < Y, \varphi, \varphi >$}. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. Then clearly, f is slightly $I\alpha^{\wedge}_{g}$-continuous.

Proof: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a contra $I\alpha^{\wedge}_{g}$-continuous function. Let O be I-clopen set in Y. Then, $f^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, f is slightly $I\alpha^{\wedge}_{g}$-continuous.

Remark 4.13: The converse of the above theorem need not be true.

Example 4.14: In example 4.8, The function f is slightly $I\alpha^{\wedge}_{g}$-continuous but not contra $I\alpha^{\wedge}_{g}$-continuous, since, $f^{-1}(< \mathcal{H}, \{b\}, \varphi >) =< Y, \{a\}, \varphi >$ is not $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$.

Remark 4.15: Composition of two slightly $I\alpha^{\wedge}_{g}$-continuous need not be slightly $I\alpha^{\wedge}_{g}$-continuous as it can be seen from the following example.

Example 4.16: Let $\mathcal{H}$=Y=Z={a,b} and $I\tau_\mu = \{X, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{b\}, \{a\} >$, $\mathcal{A}_2 =< \mathcal{H}, \varphi, \{a\} >$, $I\tau_\theta$={Y, ϕ, $< Y, \{b\}, \varphi >, < Y, \varphi, \varphi >$} and $I\tau_\rho$ = {Z, ϕ, $< Z, \{a\}, \varphi >, < Z, \varphi, \{a\} >, < Z, \varphi, \varphi >$}. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ by f(a) = a , f(b) = b. Clearly, f is slightly $I\alpha^{\wedge}_{g}$-continuous. Define g: $(Y, I\tau_\theta) \longrightarrow (Z, I\tau_\rho)$ by g(a) = a, g(b) = b. Clearly, g is slightly $I\alpha^{\wedge}_{g}$-continuous. But $(g \circ f)$: $(\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is not slightly $I\alpha^{\wedge}_{g}$-continuous, since $(g \circ f)^{-1}$ $(< Z, \{a\}, \varphi >) = f^{-1}(g^{-1}(< Z, \{a\}, \varphi >)$ $= f^{-1}$ $(< Y, \{a\}, \varphi >) = < \mathcal{H}, \{a\}, \varphi >$ is not a $I\alpha^{\wedge}_{g}$-open in $(\mathcal{H}, I\tau_\mu)$

Theorem 4.17: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ and g: $(Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ be functions. Then the following properties hold:

(i) If f is $I\alpha^{\wedge}_{g}$-irresolute and g is slightly $I\alpha^{\wedge}_{g}$-continuous then $(g \circ f)$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

(ii) If f is $I\alpha^{\wedge}_{g}$-irresolute and g is $I\alpha^{\wedge}_{g}$-continuous then $(g \circ f)$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

(iii) If f is $I\alpha^{\wedge}_{g}$- irresolute and g is slightly I-continuous then $(g \circ f)$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

(iv) If f is $I\alpha^{\wedge}_{g}$-continuous and g is slightly I-continuous then $(g \circ f)$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

(v) If f is strongly $I\alpha^{\wedge}_{g}$-continuous and g is slightly $I\alpha^{\wedge}_{g}$-continuous then $(g \circ f)$ is slightly I-continuous.

(vi) If f is slightly $I\alpha^{\wedge}_{g}$-continuous and g is perfectly $I\alpha^{\wedge}_{g}$-continuous then $(g \circ f)$ is $I\alpha^{\wedge}_{g}$-irresolute.

Proof:

(i) Let O be an I-clopen set in Z. Since, g is slightly $I\alpha^{\wedge}_{g}$-continuous, $g^{-1}(O)$is $I\alpha^{\wedge}_{g}$-open in Y. Since, f is $I\alpha^{\wedge}_{g}$-irresolute, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Since, $(g \circ f)^{-1}(O)= f^{-1}(g^{-1}(O))$, $g \circ f$ is slightly $I\alpha^{\wedge}_{g}$-continuous .

(ii) Let O be a I-clopen set in Z. Since, g is $I\alpha^{\wedge}_{g}$-continuous, $g^{-1}(O)$is $I\alpha^{\wedge}_{g}$-open in Y. Since, f is $I\alpha^{\wedge}_{g}$-irresolute, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, $g \circ$ f is slightly $I\alpha^{\wedge}_{g}$-continuous.

(iii) Let O be a I-clopen set in Z. Since, g is slightly I-continuous, $g^{-1}(O)$ is I-open in Y . Since, f is $I\alpha^{\wedge}_{g}$-irresolute, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, $g\circ$ f is slightly $I\alpha^{\wedge}_{g}$-continuous.

(iv) Let O be a I-clopen set in Z. Since, g is slightly I-continuous, $g^{-1}(O)$ is I-open in Y . Since, f is $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, $g \circ f$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

(v) Let O be an I-clopen set in Z. Since, g is slightly $I\alpha^{\wedge}_{g}$-continuous, $g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in Y. Since, f is strongly $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(g^{-1}(O))$ is I-open in $\mathcal{H}$. Therefore, $g \circ f$ is slightly I- continuous.

(vi) Let O be a $I\alpha^{\wedge}_{g}$-open in Z. Since, g is perfectly $I\alpha^{\wedge}_{g}$-continuous, $g^{-1}(O)$ is I-open and I-closed in Y. Since, f is slightly $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, $g \circ f$ is $I\alpha^{\wedge}_{g}$- irresolute.

Theorem 4.18: If the function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is slightly $I\alpha^{\wedge}_{g}$-continuous and $(\mathcal{H}, I\tau_\mu)$ is $I\alpha^{\wedge}_{g}$-$T_{1/2}$ space, then f is slightly I-continuous.

Proof: Let O be a I-clopen set in Y. Since, g is slightly $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Since, $\mathcal{H}$ is $I\alpha^{\wedge}_{g}$-$T_{1/2}$ space, $f^{-1}(O)$ is I-open in $\mathcal{H}$. Hence, f is slightly I-continuous.

Theorem 4.19: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ and $g: (Y, I\tau_\theta) \longrightarrow (Z, I\tau_\rho)$ be functions. If f is surjective and pre $I\alpha^{\wedge}_{g}$-open and $(g \circ f): (\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is slightly $I\alpha^{\wedge}_{g}$-continuous, then g is slightly $I\alpha^{\wedge}_{g}$-continuous.

Proof: Let O be a I-clopen set in $(Z, I\tau_\rho)$. Since, $(g \circ f): (\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is slightly $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(g^{-1}(O))$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Since, f is surjective and pre $I\alpha^{\wedge}_{g}$-open $f(f^{-1}(g^{-1}(O))= g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in Y. Hence, g is slightly $I\alpha^{\wedge}_{g}$-continuous.

Theorem 4.20: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ and $g: (Y, I\tau_\theta) \longrightarrow (Z, I\tau_\rho)$ be functions. If f is surjective , pre $I\alpha^{\wedge}_{g}$-open and $I\alpha^{\wedge}_{g}$-irresolute, then $(g \circ f): (\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is slightly $I\alpha^{\wedge}_{g}$-continuous if and only if g is slightly $I\alpha^{\wedge}_{g}$-continuous.

Proof: Let O be a I-clopen set in $(Z, I\tau_\rho)$. Since, $(g \circ f): (\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is slightly $I\alpha^{\wedge}_{g}$-continuous, $f^{-1}(g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Since, f is surjective and pre $I\alpha^{\wedge}_{g}$-open $f(f^{-1}\left(g^{-1}(O)\right) = g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in Y. Hence, g is slightly $I\alpha^{\wedge}_{g}$-continuous. Conversely, let g is slightly $I\alpha^{\wedge}_{g}$-continuous. Let O be a I-clopen set in $(Z, I\tau_\rho)$, then $g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in Y. Since, f is $I\alpha^{\wedge}_{g}$- irresolute, $f^{-1}(g^{-1}(O)$ is $I\alpha^{\wedge}_{g}$-open in $\mathcal{H}$. Hence, $(g \circ f): (\mathcal{H}, I\tau_\mu) \longrightarrow (Z, I\tau_\rho)$ is slightly $I\alpha^{\wedge}_{g}$-continuous.

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