Decision making using Gaussian Double Refined Indeterminate Neutrosophic Number

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Abstract: Single Valued Neutrosophic Numbers in numerous forms are useful tool for dealing with indeterminate and inconsistent information encountered in real world. At times more emphasis have to be given to the indeterminate information and in such case these concepts may not be sufficient to deal with it. To overcome this, in this paper we have proposed the definition of Gaussian Double Refined Indeterminate Neutrosophic Number (GDRINN).Also we developed a ranking method based on the α $-$cut and is applied to the decision making algorithm. The method is exhibited with a hypothetical case study.

Keywords: Gaussian fuzzy number, Gaussian Single valued Neutrosophic Number, Gaussian Double Refined Indeterminate Neutrosophic Number. Decision making

1. Introduction

Fuzzy numbers are widely used to quantity the associated uncertainty in several real life problems. From the introduction of Fuzzy set theory by Zadeh in 1965, many developments have been done by different researchers on fuzzy set theory. Different types of fuzzy numbers also have been introduced by different researchers. Out of different fuzzy numbers, Gaussian fuzzy number is very important in different real life problems.

Dutta and Ali [3] used Gaussian fuzzy number to represent epistemic type uncertainty, fused with triangular fuzzy numbers and carried out risk assessment under fuzzy environment. Garg and Singh [5] suggested the numerical solution for fuzzy system of equations by using the Gaussian membership function to the fuzzy numbers considering in its parametric form. Sen et.al [21] proposed a new similarity measure of Gaussian fuzzy numbers based on on the exponent distance about mean, standard deviation and height of the Gaussian fuzzy numbers. Bharatraj, J [1] defined IVIFSs using Gaussian membership functions (GMFs), and new measures of the distance, the overlap, and the angle between two sets and used it to determine the similarities between test subjects in genetic brain profiling. Ranjan kumar et.al. [20] exhibited a novel method for finding the neutrosophic shortest path problem (NSSPP) considering Gaussian valued Neutrosophic number.

Multi-attribute decision making (MADM) which is an important part of decision science is to find an optimal alternative, which are characterized in terms of multiple attributes, from alternative sets. In many practical problems, the decision makers may be not able to evaluate exactly the values of the MADM problems due to uncertain and asymmetric information between decision makers and hence the values of the MADM problems are not measured by accurate numbers. Though we can use some sets such as; a fuzzy set, intuitionistic set and neutrosophic set to represent an uncertainty of values of the MADM problems ,in recent years neutrosophic sets (NSs) have become a subject of great interest for researchers [15],[16],[17],[18],[19] and have been widely applied to multi-criteria group decision-making (MCGDM) problems. Mondal and Pramanik[8-10] applied neutrosophic sets in multi criteria decision making for diverse concepts.

Peng et al. [14] introduced the concept of multi-valued neutrosophic power aggregation operators and applied to multi-criteria group decision-making problems. Further as an advancement Karaaslan [6] introduced Gaussian single-valued neutrosophic number and used it to solve multicriteria decision making problem. In order to provide more emphasis for the indeterminacy, we have introduced the definition of Gaussian Double Refined Indeterminate Neutrosophic number and applied it to solve the multicriteria decision making problem.

2.Preliminaries

2.1 Gaussian Fuzzy Number: A fuzzy number is said to be a Gaussian Fuzzy Number $G_{FN}(\mu\ ,\ \sigma)$ whose membership function is given by

$$g(x) = exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), -\infty < x < \infty$$

where μ denotes the mean and σ denotes the standard deviation of the distribution.

2.2 $\alpha - \mathbf{cut}$ of Gaussian Fuzzy Number: If the Gaussian membership function is given by

$g(x) = exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$, then the $\alpha - cut$ is given by $A_{FN_\alpha} = \left[\mu - \sigma\sqrt{-2\ log\alpha}\ ,\mu + \sigma\sqrt{-2\ log\alpha}\right]$

2.3 Gaussian Single Valued Neutrosophic Number:[6] A single Valued Neutrosophic number is said to be Gaussian Single Valued Neutrosophic Number $G_{SVNN}\left((\mu_t\ ,\sigma_t),(\mu_i\ ,\sigma_i)(\mu_f\ ,\sigma_f)\right)$ whose truth membership function, indeterminacy membership function and falsity membership function are defined as

$$\varphi\ (x_t)\ =\ exp\left(-\frac{1}{2}\left(\frac{x_t-\ \mu_t}{\sigma_t}\right)^2\right),$$

$$\varphi\ (x_i)\ = 1-\left(exp\left(-\frac{1}{2}\left(\frac{x_i-\ \mu_i}{\sigma_i}\right)^2\right)\right),$$

$$\varphi\ \left(x_f\right)\ = 1-\left(exp\left(-\frac{1}{2}\left(\frac{x_f-\ \mu_f}{\sigma_f}\right)^2\right)\right),$$

where $\mu_t\ ,\mu_i\ ,\mu_f$ denotes the mean of the truth membership, indeterminacy membership and falsity membership distribution functions , $\sigma_t\ ,\sigma_i\ ,\sigma_f$ denotes the standard deviation of the truth membership, indeterminacy membership and falsity membership distribution functions.

3. Gaussian Double Refined Indeterminate Neutrosophic Number(GDRINN):

In this section we have proposed the definition of GDRINN,$\alpha\ -$cut of GDRINN and its arithmetic operations using $\alpha\ -$cut

3.1 Definition: A GDRINN $\bar{\bar{\mathcal{A}}}_{GN}$=$\left((\overline{\mu_t},\sigma_t),(\overline{\mu_{\iota t}},\sigma_{it}),\left(\overline{\mu_{\iota f}},\sigma_{if}\right),\left(\overline{\mu_f},\sigma_f\right)\right)$ whose truth membership function, indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership function are given as follows.

$$\psi\ (x_t) = \exp\left(-\frac{1}{2}\left(\frac{x_t-\overline{\mu_t}}{\sigma_t}\right)^2\right)$$

$$\psi(x_{it}) = \exp\left(-\frac{1}{2}\left(\frac{x_{it}-\overline{\mu_{\iota t}}}{\sigma_{it}}\right)^2\right)$$

$$\psi\left(x_{if}\right) = 1-\exp\left(-\frac{1}{2}\left(\frac{x_{if}-\overline{\mu_{\iota f}}}{\sigma_{if}}\right)^2\right)$$

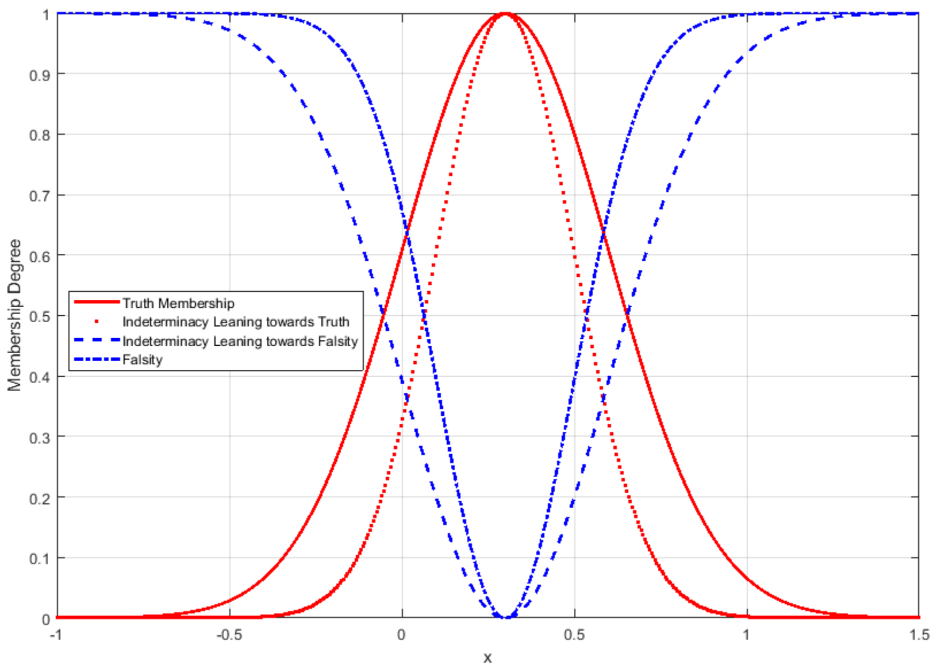
$$\psi\ \left(x_f\right) = 1-\exp\left(-\frac{1}{2}\left(\frac{x_f-\overline{\mu_f}}{\sigma_f}\right)^2\right)$$

respectively.

Here $\overline{\mu_t},\overline{\mu_{\iota t}},\overline{\mu_{\iota f}},\overline{\mu_f}$ denotes the mean of truth membership function, indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership function value. $\sigma_t\ ,\sigma_{it},\sigma_{if},\sigma_f$ denotes the standard deviation of distribution of truth membership function, indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership value.

3.2 Graphical Representation of GDRINN:

The GDRINN $\bar{\bar{\mathcal{A}}}_{GN} = \big((0.3, 0.3), (0.3,\ 0.2), (0.3, 0.3), (0.3, 0.2)\big)$ is depicted in Fig 1.



3.3 $\alpha - cut$ of Gaussian Double Refined Indeterminate Neutrosophic Number:

The $\alpha - cut$ of GDRINN are as follows.

$$\bar{\bar{\mathcal{A}}}_{GN\,t\alpha} = \left[\bar{\mu}_t - \sigma_t\sqrt{-2\log\alpha}\ ,\qquad \bar{\mu}_t + \sigma_t\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,it\alpha} = \left[\overline{\mu_{it}} - \sigma_{it}\sqrt{-2\log\alpha}\ ,\qquad \overline{\mu_{it}} + \sigma_{it}\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,if\alpha} = \left[\overline{\mu_{if}} - \sigma_{if}\sqrt{-2\log(1-\alpha)}\ ,\qquad \overline{\mu_{if}} + \sigma_{if}\sqrt{-2\log(1-\alpha)}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,f\alpha} = \left[\overline{\mu_f} - \sigma_f\sqrt{-2\log(1-\alpha)}\ ,\qquad \overline{\mu_f} + \sigma_f\sqrt{-2\log(1-\alpha)}\right]$$

3.4 Arithmetic Operations of Gaussian Double Refined Indeterminate Neutrosophic Number based on $\alpha - cut$:

Let $\bar{\bar{\mathcal{A}}}_{GN} = \left(\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t}, \sigma_{\bar{\bar{\mathcal{A}}}_t}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}}, \sigma_{\bar{\bar{\mathcal{A}}}_{it}}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}}, \sigma_{\bar{\bar{\mathcal{A}}}_{if}}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f}, \sigma_{\bar{\bar{\mathcal{A}}}_f}\right)\right)$ and

$\bar{\bar{\mathcal{B}}}_{GN} = \left(\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}, \sigma_{\bar{\bar{\mathcal{B}}}_t}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}, \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}, \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\right), \left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}, \sigma_{\bar{\bar{\mathcal{B}}}_f}\right)\right)$

be two Gaussian Double Refined Indeterminate Neutrosophic Numbers. Then their $\alpha - $cuts of these are as follows:

$$\bar{\bar{\mathcal{A}}}_{GN\,t\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} - \sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,it\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} - \sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,if\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} - \sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}\right]$$

$$\bar{\bar{\mathcal{A}}}_{GN\,f\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} - \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}\right]$$

and

$$\bar{\bar{\mathcal{B}}}_{GN\,t\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t} - \sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{B}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{B}}}_{GN\,it\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}} - \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}\right]$$

$$\bar{\bar{\mathcal{B}}}_{GN\,if\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}} - \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}\right]$$

$$\bar{\bar{\mathcal{B}}}_{GN\,f\alpha} = \left[\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} - \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}\ ,\quad \bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}\right]$$

Based on $\alpha -$ cuts, the arithmetic operations between $\bar{\bar{\mathcal{A}}}_{GN}$ and $\bar{\bar{\mathcal{B}}}_{GN}$ are defined as follows.

1. Addition:

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN\,t\alpha} + \bar{\bar{\mathcal{B}}}_{GN\,t\alpha} &= \Big[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\right)\sqrt{-2\log\alpha}\ ,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right) \\ &\quad + \left(\sigma_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\right)\sqrt{-2\log\alpha}\Big]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN\,it\alpha} + \bar{\bar{\mathcal{B}}}_{GN\,it\alpha} &= \Big[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\right)\sqrt{-2\log\alpha}\ ,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right) \\ &\quad + \left(\sigma_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\right)\sqrt{-2\log\alpha}\Big]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN\,if\alpha} + \bar{\bar{\mathcal{B}}}_{GN\,if\alpha} &= \Big[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\right)\sqrt{-2\log(1-\alpha)}\ ,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right) \\ &\quad + \left(\sigma_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\right)\sqrt{-2\log(1-\alpha)}\Big]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN\,f\alpha} + \bar{\bar{\mathcal{B}}}_{GN\,f\alpha} &= \Big[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\right)\sqrt{-2\log(1-\alpha)}\ ,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} + \bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right) \\ &\quad + \left(\sigma_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\right)\sqrt{-2\log(1-\alpha)}\Big]\end{aligned}$$

Truth membership function, indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership function of addition of Gaussian Double Refined Indeterminate Neutrosophic Numbers $\bar{\bar{\mathcal{A}}}_{GN}$ and $\bar{\bar{\mathcal{B}}}_{GN}$ are as follows.

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}+\bar{\bar{\mathcal{B}}}_{GN})}(x_t) = \exp\left(-\frac{1}{2}\left(\frac{x_t-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t}+\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_t}+\sigma_{\bar{\bar{\mathcal{B}}}_t}}\right)^2\right)$$

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}+\bar{\bar{\mathcal{B}}}_{GN})}(x_{it}) = \exp\left(-\frac{1}{2}\left(\frac{x_{it}-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}}+\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_{it}}+\sigma_{\bar{\bar{\mathcal{B}}}_{it}}}\right)^2\right)$$

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}+\bar{\bar{\mathcal{B}}}_{GN})}(x_{if}) = 1 - \exp\left(-\frac{1}{2}\left(\frac{x_{if}-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}}+\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_{if}}+\sigma_{\bar{\bar{\mathcal{B}}}_{if}}}\right)^2\right)$$

and

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}+\bar{\bar{\mathcal{B}}}_{GN})}(x_f) = 1 - \exp\left(-\frac{1}{2}\left(\frac{x_f-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f}+\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_f}+\sigma_{\bar{\bar{\mathcal{B}}}_f}}\right)^2\right)$$

respectively.

2. Subtraction :

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{t\alpha}} - \bar{\bar{\mathcal{B}}}_{GN_{t\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\right)\sqrt{-2\log\alpha}\,,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right)\right.\\ &\quad \left. + \left(\sigma_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\right)\sqrt{-2\log\alpha}\right]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{it\alpha}} - \bar{\bar{\mathcal{B}}}_{GN_{it\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\right)\sqrt{-2\log\alpha}\,,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right)\right.\\ &\quad \left. + \left(\sigma_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\right)\sqrt{-2\log\alpha}\right]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{if\alpha}} - \bar{\bar{\mathcal{B}}}_{GN_{if\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\right)\sqrt{-2\log(1-\alpha)}\,,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right)\right.\\ &\quad \left. + \left(\sigma_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\right)\sqrt{-2\log(1-\alpha)}\right]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{f\alpha}} - \bar{\bar{\mathcal{B}}}_{GN_{f\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right) - \left(\sigma_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\right)\sqrt{-2\log(1-\alpha)}\,,\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} - \bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right)\right.\\ &\quad \left. + \left(\sigma_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\right)\sqrt{-2\log(1-\alpha)}\right]\end{aligned}$$

Truth membership function, indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership function of subtraction of Gaussian Double Refined Indeterminate Neutrosophic Numbers $\bar{\bar{\mathcal{A}}}_{GN}$ and $\bar{\bar{\mathcal{B}}}_{GN}$ are as follows.

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}-\bar{\bar{\mathcal{B}}}_{GN})}(x_t) = \exp\left(-\frac{1}{2}\left(\frac{x_t-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t}-\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_t}+\sigma_{\bar{\bar{\mathcal{B}}}_t}}\right)^2\right)$$

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}-\bar{\bar{\mathcal{B}}}_{GN})}(x_{it}) = \exp\left(-\frac{1}{2}\left(\frac{x_{it}-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}}-\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_{it}}+\sigma_{\bar{\bar{\mathcal{B}}}_{it}}}\right)^2\right)$$

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}-\bar{\bar{\mathcal{B}}}_{GN})}(x_{if}) = 1-\exp\left(-\frac{1}{2}\left(\frac{x_{if}-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}}-\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_{if}}+\sigma_{\bar{\bar{\mathcal{B}}}_{if}}}\right)^2\right)$$

and

$$\psi_{(\bar{\bar{\mathcal{A}}}_{GN}-\bar{\bar{\mathcal{B}}}_{GN})}(x_f) = 1-\exp\left(-\frac{1}{2}\left(\frac{x_f-\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f}-\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f}\right)}{\sigma_{\bar{\bar{\mathcal{A}}}_f}+\sigma_{\bar{\bar{\mathcal{B}}}_f}}\right)^2\right)$$

3.Multiplication:

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{t\alpha}}\cdot\bar{\bar{\mathcal{B}}}_{GN_{t\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t}-\sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}\right)\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}-\sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}\right),\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t}+\sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}\right)\right.\\ &\quad\left.\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t}+\sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}\right)\right]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{it\alpha}}\cdot\bar{\bar{\mathcal{B}}}_{GN_{it\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}}-\sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}\right)\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}-\sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}\right),\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}}+\sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}\right)\right.\\ &\quad\left.\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}}+\sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}\right)\right]\end{aligned}$$

$$\begin{aligned}\bar{\bar{\mathcal{A}}}_{GN_{if\alpha}}\cdot\bar{\bar{\mathcal{B}}}_{GN_{if\alpha}} &= \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}}-\sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}\right)\right.\\ &\quad\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}-\sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}\right),\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}}+\sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}\right)\\ &\quad\left.\cdot\left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}}+\sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}\right)\right]\end{aligned}$$

$$\bar{\bar{\mathcal{A}}}_{GN_{f\alpha}} \cdot \bar{\bar{\mathcal{B}}}_{GN_{f\alpha}} = \left[\left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} - \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}\right) \cdot \left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} - \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}\right),\ \left(\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}\right) \cdot \left(\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}\right)\right]$$

4. Division:

$$\frac{\bar{\bar{\mathcal{A}}}_{GN_{t\alpha}}}{\bar{\bar{\mathcal{B}}}_{GN_{t\alpha}}} = \left[\frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} - \sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t} + \sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}}, \frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_t} + \sigma_{\bar{\bar{\mathcal{A}}}_t}\sqrt{-2\log\alpha}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_t} - \sigma_{\bar{\bar{\mathcal{B}}}_t}\sqrt{-2\log\alpha}}\right]$$

$$\frac{\bar{\bar{\mathcal{A}}}_{GN_{it\alpha}}}{\bar{\bar{\mathcal{B}}}_{GN_{it\alpha}}} = \left[\frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} - \sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}} + \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}}, \frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{it}} + \sigma_{\bar{\bar{\mathcal{A}}}_{it}}\sqrt{-2\log\alpha}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{it}} - \sigma_{\bar{\bar{\mathcal{B}}}_{it}}\sqrt{-2\log\alpha}}\right]$$

$$\frac{\bar{\bar{\mathcal{A}}}_{GN_{if\alpha}}}{\bar{\bar{\mathcal{B}}}_{GN_{if\alpha}}} = \left[\frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} - \sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}} + \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}}, \frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_{if}} + \sigma_{\bar{\bar{\mathcal{A}}}_{if}}\sqrt{-2\log(1-\alpha)}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_{if}} - \sigma_{\bar{\bar{\mathcal{B}}}_{if}}\sqrt{-2\log(1-\alpha)}}\right]$$

$$\frac{\bar{\bar{\mathcal{A}}}_{GN_{f\alpha}}}{\bar{\bar{\mathcal{B}}}_{GN_{f\alpha}}} = \left[\frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} - \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} - \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}}, \frac{\bar{\mu}_{\bar{\bar{\mathcal{A}}}_f} + \sigma_{\bar{\bar{\mathcal{A}}}_f}\sqrt{-2\log(1-\alpha)}}{\bar{\mu}_{\bar{\bar{\mathcal{B}}}_f} + \sigma_{\bar{\bar{\mathcal{B}}}_f}\sqrt{-2\log(1-\alpha)}}\right]$$

4. Decision making using Gaussian Double Refined Indeterminate Neutrosophic Number

We have considered the decision making problem adapted from [4] and we have extended the algorithm of Karaaslan [6] with proposed definition and by applying the newly defined score function using α $-$cut .

Let us denote the set of Patients by **P** = $\{p_1, p_2, p_3, \dots p_p\}$,set of symptoms by**S**= $\{s_1, s_2, s_3, \dots s_s\}$and set of diseases by **D** = $\{d_1, d_2, d_3, \dots d_d\}$.

Table:1 Double Refined Indeterminate Neutrosophic number for Linguistic terms

Linguistic terms	**Linguistic values for DRINN**
Absolutely Low	$\langle 0.05,0.10,0.95,0.95\rangle$
Low	$\langle 0.20,0.25,0.75,0.80\rangle$
Fairly Low	$\langle 0.35,0.40,0.60,0.65\rangle$
Medium	$\langle 0.50,0.50,0.50,0.50\rangle$

Fairly High	$\langle 0.65,0.60,0.40,0.35\rangle$
High	$\langle 0.80,0.75,0.25,0.20\rangle$
Absolutely High	$\langle 0.95,0.95,0.10,0.05\rangle$

Patients p_i (i = 1, 2, ..., p) are evaluated by experts using Table 1 for each symptom s_j (j = 1, 2, ..., s), and patient-symptom (**PS**) matrix is given as follows:

$$\mathbf{PS} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ \vdots & \vdots & \vdots & \vdots \\ p_{p1} & p_{p2} & \ldots & p_{ps} \end{pmatrix}$$

$p_{ij} = \langle p_{t_{ij}}, p_{it_{ij}}, p_{if_{ij}}, p_{f_{ij}}\rangle$ denotes the DRINN value of patient $p_i$related to symptom s_j. Next the Symptoms s_j (j = 1, 2, ..., s) are evaluated by Gaussian Double Refined Indeterminate Neutrosphic numbers for each disease $d_k (k = 1,2, \ldots d)$ and the symptom- disease (**SD)** matrix is given as follows:

$$\mathbf{SD} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1d} \\ q_{21} & q_{22} & \cdots & q_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ q_{s1} & q_{s2} & \ldots & q_{sd} \end{pmatrix}$$

$q_{jk} = GDRINN \langle \left(\bar{\mu}_{q_{t_{jk}}}, \sigma_{q_{t_{jk}}}\right), \left(\bar{\mu}_{q_{it_{jk}}}, \sigma_{q_{it_{jk}}}\right), \left(\bar{\mu}_{q_{if_{jk}}}, \sigma_{q_{if_{jk}}}\right), \left(\bar{\mu}_{q_{f_{jk}}}, \sigma_{q_{f_{jk}}}\right)\rangle$ denotes the Gaussian DRINN of symptom s_j related to disease d_k .

Decision matrix of Patient-Disease (**PD)** is obtained by the composition of patient-symptom (**PS**) and symptom- disease (**SD)** as follows

$$\mathbf{PD} = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1d} \\ r_{21} & r_{22} & \cdots & r_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ r_{p1} & r_{p2} & \ldots & r_{pd} \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ \vdots & \vdots & \vdots & \vdots \\ p_{p1} & p_{p2} & \ldots & p_{ps} \end{pmatrix} \circ \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1d} \\ q_{21} & q_{22} & \cdots & q_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ q_{s1} & q_{s2} & \ldots & q_{sd} \end{pmatrix}$$

Where

$r_{ik} = \langle p_{t_{ij}}, p_{it_{ij}}, p_{if_{ij}}, p_{f_{ij}}\rangle \, GDRINN \langle \left(\bar{\mu}_{q_{t_{jk}}}, \sigma_{q_{t_{jk}}}\right), \left(\bar{\mu}_{q_{it_{jk}}}, \sigma_{q_{it_{jk}}}\right), \left(\bar{\mu}_{q_{if_{jk}}}, \sigma_{q_{if_{jk}}}\right), \left(\bar{\mu}_{q_{f_{jk}}}, \sigma_{q_{f_{jk}}}\right)\rangle$

$$= \left\langle \begin{array}{c} \left(p_{t_{ij}}\overline{\mu_t} - p_{t_{ij}}\sigma_t\sqrt{-2\log\alpha}\ , p_{t_{ij}}\overline{\mu_t} + p_{t_{ij}}\sigma_t\sqrt{-2\log\alpha}\right), \\ \left(p_{it_{ij}}\overline{\mu_{\iota t}} - p_{it_{ij}}\sigma_{it}\sqrt{-2\log\alpha}\ ,\ p_{it_{ij}}\overline{\mu_{\iota t}} + p_{it_{ij}}\sigma_{it}\sqrt{-2\log\alpha}\right), \\ \left(p_{if_{ij}}\overline{\mu_{\iota f}} - p_{if_{ij}}\sigma_{if}\sqrt{-2\log(1-\alpha)}\ ,\ p_{if_{ij}}\overline{\mu_{\iota f}} + p_{if_{ij}}\sigma_{if}\sqrt{-2\log(1-\alpha)}\right), \\ \left(p_{f_{ij}}\overline{\mu_f} - p_{f_{ij}}\sigma_f\sqrt{-2\log(1-\alpha)}\ ,\ p_{f_{ij}}\overline{\mu_f} + p_{f_{ij}}\sigma_f\sqrt{-2\log(1-\alpha)}\right) \end{array} \right\rangle$$

For the above Gaussian DRINN the score function is defined as

$$\begin{aligned} S_{r_{ik}} = \Big[4 + \Big(& p_{t_{ij}}\overline{\mu_t} - p_{t_{ij}}\sigma_t\sqrt{-2\log\alpha} + p_{it_{ij}}\overline{\mu_{\iota t}} - p_{it_{ij}}\sigma_{it}\sqrt{-2\log\alpha} - p_{if_{ij}}\overline{\mu_{\iota f}} \\ & - p_{if_{ij}}\sigma_{if}\sqrt{-2\log(1-\alpha)} - p_{f_{ij}}\overline{\mu_f} - p_{f_{ij}}\sigma_f\sqrt{-2\log(1-\alpha)}\Big) \\ & + \Big(p_{t_{ij}}\overline{\mu_t} + p_{t_{ij}}\sigma_t\sqrt{-2\log\alpha} + p_{it_{ij}}\overline{\mu_{\iota t}} + p_{it_{ij}}\sigma_{it}\sqrt{-2\log\alpha} - p_{if_{ij}}\overline{\mu_{\iota f}} \\ & + p_{if_{ij}}\sigma_{if}\sqrt{-2\log(1-\alpha)} - p_{f_{ij}}\overline{\mu_f} + p_{f_{ij}}\sigma_f\sqrt{-2\log(1-\alpha)}\Big)\Big] / 8 \end{aligned}$$

If max $\{S_{r_{ik}}\} = S_{r_{it}}$ for $1 \le t \le k$, then it is said that patient p_i suffers from disease d_t. In case max $S_{r_{ik}}$ occurs for more than one value, for $1 \le t \le k$, then symptoms can be re-examined.

Algorithm:

The patient-symptom matrix obtained according to opinion of expert is considered as the input, analysis is performed and the disease for each patient is diagnosed and given as output.

Step1: Construct Patient Symptom matrix **PS** according to opinions of experts by using Table 1.

Step 2: Construct Symptom -Disease matrix **SD** by using GDRINNs.

Step 3.Calculate decision matrix **PD** as the composition of Patient Symptom matrix **PS** and Symptom -Disease matrix **SD** .

Step 4. Compute score values of each elements of decision matrix **PD**.

Step 5. Find t for which max $\{S_{r_{ik}}\} = S_{r_{it}}$ for $1 \le t \le k$

Numerical Illustration:

We have considered the hypothetical case study in Double Refined Indeterminate Neutrosophic Number to illustrate the proposed method.

Let us assume that there are five patients p_1, p_2, p_3, p_4 and p_5 who is considered to be suffering from viral fever (d_1), tuberculosis (d_2), typhoid (d_3), throat disease (d_4) or malaria (d_5). In these diseases, common symptoms are temperature (s_1), cough (s_2), throat pain (s_3), headache (s_4), body pain (s_5).

Step 1: As per the Opinion and observation made by an expert, suppose that matrix **PS** is as follows:

$$
\mathbf{P\ S} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\
\begin{pmatrix}
\langle 0.95,0.95,0.10,0.05\rangle & \langle 0.65,0.60,0.40,0.35\rangle & \langle 0.20,0.25,0.75,0.80\rangle & \langle 0.20,0.25,0.75,0.80\rangle & \langle 0.80,0.75,0.25,0.20\rangle \\
\langle 0.35,0.40,0.60,0.65\rangle & \langle 0.65,0.60,0.40,0.35\rangle & \langle 0.50,0.50,0.50,0.50\rangle & \langle 0.80,0.75,0.25,0.20\rangle & \langle 0.95,0.95,0.10,0.05\rangle \\
\langle 0.65,0.60,0.40,0.35\rangle & \langle 0.50,0.50,0.50,0.50\rangle & \langle 0.80,0.75,0.25,0.20\rangle & \langle 0.20,0.25,0.75,0.80\rangle & \langle 0.80,0.75,0.25,0.20\rangle \\
\langle 0.20,0.25,0.75,0.80\rangle & \langle 0.80,0.75,0.25,0.20\rangle & \langle 0.95,0.95,0.10,0.05\rangle & \langle 0.35,0.40,0.60,0.65\rangle & \langle 0.20,0.25,0.75,0.80\rangle \\
\langle 0.80,0.75,0.25,0.20\rangle & \langle 0.35,0.40,0.60,0.65\rangle & \langle 0.05,0.10,0.95,0.95\rangle & \langle 0.80,0.75,0.25,0.20\rangle & \langle 0.20,0.25,0.75,0.80\rangle
\end{pmatrix}
\end{array}
$$

Step 2: Suppose that the matrix **SD** is as follows:

$$
\mathbf{SD} = \begin{pmatrix}
\begin{matrix} <(0.3,0.1). \\ (0.25,0.05), \\ (0.25,0.05), \\ (0.2,0.03)> \end{matrix} &
\begin{matrix} <(0.25,0.03), \\ (0.35,0.01), \\ (0.35,0.01), \\ (0.5,0.01)> \end{matrix} &
\begin{matrix} <(0.25,0.15\}, \\ (0.125,0.0075), \\ (0.125,0.0075), \\ (0.5,0.2)> \end{matrix} &
\begin{matrix} <(0.6,0.03), \\ (0.2,0.09), \\ (0.2,0.09), \\ (0.2,0.1)> \end{matrix} &
\begin{matrix} <(0.71,0.04), \\ (0.26,0.025), \\ (0.26,0.025), \\ (0.45,0.02)> \end{matrix} \\
\begin{matrix} <(0.65,0.12), \\ (0.24,0.01), \\ (0.24,0.01), \\ (0.25,0.04)> \end{matrix} &
\begin{matrix} <(0.25,0.03), \\ (0.16,0.01), \\ (0.16,0.01), \\ (0.6,0.01)> \end{matrix} &
\begin{matrix} <(0.6,0.1), \\ (0.32,0.03), \\ (0.32,0.03), \\ (0.18,0.018)> \end{matrix} &
\begin{matrix} <(0.6,0.09), \\ (0.4,0.0005), \\ (0.4,0.0005), \\ (0.4,0.13)> \end{matrix} &
\begin{matrix} <(0.6,0.05), \\ (0.15,0.025), \\ (0.15,0.025), \\ (0.2,0.02) \end{matrix} \\
\begin{matrix} <(0.4,0.1), \\ (0.115,0.04), \\ (0.115,0.04), \\ (0.5,0.01)> \end{matrix} &
\begin{matrix} <(0.5,0.06), \\ (0.175,0.01), \\ (0.175,0.01), \\ (0.32,0.03)> \end{matrix} &
\begin{matrix} <(0.45,0.12), \\ (0.2,0.075), \\ (0.2,0.075), \\ (0.56,0.03)> \end{matrix} &
\begin{matrix} <(0.45,0.1), \\ (0.45,0.0075), \\ (0.45,0.0075), \\ (0.5,0.18)> \end{matrix} &
\begin{matrix} <(0.88,0.02), \\ (0.3,0.35), \\ (0.3,0.35), \\ (0.4,0.02)> \end{matrix} \\
\begin{matrix} <(0.9,0.35), \\ (0.215,0.025), \\ (0.215,0.025), \\ (0.8,0.07)> \end{matrix} &
\begin{matrix} <(0.25,0.01), \\ (0.06,0.045), \\ (0.06,0.045) \\ (0.44,0.04)> \end{matrix} &
\begin{matrix} <(0.6,0.22), \\ (0.15,0.095), \\ (0.15,0.095), \\ (0.13,0.022)> \end{matrix} &
\begin{matrix} <(0.65,0.14), \\ (0.115,0.006), \\ (0.115,0.006), \\ (0.14,0.2)> \end{matrix} &
\begin{matrix} <(0.9,0.06), \\ (0.15,0.1), \\ (0.15,0.1), \\ (0.65,0.01)> \end{matrix} \\
\begin{matrix} <(0.5,0.09), \\ (0.16,0.0105), \\ (0.16,0.0105), \\ (0.44,0.06)> \end{matrix} &
\begin{matrix} <(0.33,0.02), \\ (0.35,0.04), \\ (0.35,0.04), \\ (0.6,0.07)> \end{matrix} &
\begin{matrix} <(0.75,0.03), \\ (0.25,0.055), \\ (0.25,0.055), \\ (0.25,0.02)> \end{matrix} &
\begin{matrix} <(0.48,0.12), \\ (0.215,0.01), \\ (0.215,0.01), \\ (0.41,0.04)> \end{matrix} &
\begin{matrix} <(0.28,0.02), \\ (0.315,0.1), \\ (0.315,0.1), \\ (0.5,0.08)> \end{matrix}
\end{pmatrix}
$$

Step 3: The decision matrix **PD** = **P S** ∘ **SD** is obtained .

For ease , we denote $\sqrt{-2\log\alpha} = c_1$ and $\sqrt{-2\log(1-\alpha)} = c_2$

$$
\begin{aligned}
r_{11} = {} & (0.285 - 0.095c_1, 0.285 + 0.095c_1), (0.238 - 0.048\,c_1, 0.238 + 0.048\,c_1), \\
& (0.025 - 0.005\,c_2, 0.025 + 0.005\,c_2), (0.01 - 0.002\,c_2, 0.01 + 0.002\,c_2) \\
& + (0.423 - 0.078\,c_1, 0.423 + 0.078\,c_1), (0.144 - 0.006\,c_1, 0.144 + 0.006\,c_1), \\
& (0.096 - 0.004\,c_2, 0.096 + 0.004\,c_2), (0.088 - 0.014\,c_2, 0.088 + 0.014\,c_2) \\
& + (0.08 - 0.02\,c_1, 0.08 + 0.02\,c_1), (0.029 - 0.01\,c_1, 0.029 + 0.01\,c_1), \\
& (0.086 - 0.03\,c_2, 0.086 + 0.03\,c_2), (0.4 - 0.008c_2, 0.4 + 0.008c_2)
\end{aligned}
$$

$+ (0.18 - 0.07c_1, 0.18 + 0.07c_1), (0.054 - 0.006c_1, 0.054 + 0.006c_1),$

$(0.161 - 0.019c_2, 0.161 + 0.019c_2), (0.64 - 0.056c_2, 0.64 + 0.056c_2)$

$+ (0.4 - 0.072c_1, 0.4 + 0.072c_1), \quad (0.12 - 0.008c_1, 0.12 + 0.008c_1),$

$(0.04 - 0.003c_2, 0.04 + 0.003c_2), (0.088 - 0.012c_2, 0.088 + 0.012c_2)$

$r_{11} = \big((1.368 - 0.335c_1, 1.368 + 0.335c_1), (0.585 - 0.078c_1, 0.585 + 0.078c_1), (0.408 - 0.061c_2, 0.408 + 0.061c_2), (1.226 - 0.092c_2, 1.226 + 0.092c_2)\big)$

Similarly the other elements are also calculated and we have

$r_{12} = \big((0.815 - 0.079c_1, 0.815 + 0.079c_1), (0.751 - 0.06c_1, 0.751 + 0.06c_1), (0.363 - 0.057c_2, 0.363 + 0.057c_2), (0.963 - 0.075c_2, 0.963 + 0.075c_2)\big)$

$r_{13} = \big((1.438 - 0.3c_1, 1.438 + 0.3c_1), (0.587 - 0.109c_1, 0.587 + 0.109c_1), (0.467 - 0.154c_2, 0.467 + 0.154c_2), (0.69 - 0.062c_2, 0.69 + 0.062c_2)\big)$

$r_{14} = \big((1.564 - 0.232c_1, 1.564 + 0.232c_1), (0.733 - 0.098c_1, 0.733 + 0.098c_1), (0.658 - 0.023c_2, 0.658 + 0.023c_2), (0.96 - 0.363c_2, 0.96 + 0.363c_2)\big)$

$r_{15} = \big((1.645 - 0.103c_1, 1.645 + 0.103c_1), (0.686 - 0.227c_1, 0.686 + 0.227c_1), (0.503 - 0.376c_2, 0.503 + 0.376c_2), (1.033 - 0.048c_2, 1.033 + 0.048c_2)\big)$

$r_{21} = \big((1.923 - 0.529c_1, 1.923 + 0.529c_1), (0.615 - 0.075c_1, 0.615 + 0.075c_1), (0.374 - 0.061c_2, 0.374 + 0.061c_2), (0.6 - 0.056c_2, 0.6 + 0.056c_2)\big)$

$r_{22} = \big((1.015 - 0.088c_1, 1.015 + 0.088c_1), (0.702 - 0.087c_1, 0.702 + 0.087c_1), (0.412 - 0.03c_2, 0.412 + 0.03c_2), (0.813 - 0.038c_2, 0.813 + 0.038c_2)\big)$

$r_{23} = \big((1.896 - 0.383c_1, 1.896 + 0.383c_1), (0.693 - 0.182c_1, 0.693 + 0.182c_1), (0.366 - 0.085c_2, 0.366 + 0.085c_2), (0.710 - 0.156c_2, 0.710 + 0.156c_2)\big)$

$r_{24} = \big((1.801 - 0.346c_1, 1.801 + 0.346c_1), (0.835 - 0.055c_1, 0.835 + 0.055c_1), (0.556 - 0.061c_2, 0.556 + 0.061c_2), (0.623 - 0.243c_2, 0.623 + 0.243c_2)\big)$

$r_{25} = \big((2.065 - 0.124c_1, 2.065 + 0.124c_1), (0.700 - 0.370c_1, 0.700 + 0.370c_1), (0.500 - 0.235c_2, 0.500 + 0.235c_2), (0.718 - 0.036c_2, 0.718 + 0.036c_2)\big)$

$r_{31} = \big((1.42 - 0.347c_1, 1.42 + 0.347c_1), (0.53 - 0.079c_1, 0.53 + 0.079c_1), (0.45 - 0.057c_2, 0.45 + 0.057c_2), (1.023 - 0.101c_2, 1.023 + 0.101c_2)\big)$

$r_{32} = \big((1.002 - 0.101c_1, 1.002 + 0.101c_1), (0.699 - 0.06c_1, 0.699 + 0.06c_1), (0.397 - 0.056c_2, 0.397 + 0.056c_2), (1.011 - 0.061c_2, 1.011 + 0.061c_2)\big)$

r_{33}= $\big((1.543\ -\ 0.312\, c_1\, , 1.543\ +\ 0.312\, c_1)\, , (0.611\ - 0.141\, c_1\, ,\ 0.611\ + 0.141\, c_1)\, ,$
$(0.436\ -\ 0.122\, c_2\, , 0.436\ +\ 0.122\, c_2)\, , (0.531\ -\ 0.107\, c_2\, ,\ 0.531\ +\ 0.107\, c_2)\big)$

r_{34}= $\big((1.564\ -\ 0.269\, c_1\, , 1.564\ +\ 0.269\, c_1)\, , (0.848\ - 0.070\, c_1\, ,\ 0.848\ + 0.070\, c_1)\, ,$
$(0.533\ -\ 0.046\, c_2\, , 0.533\ +\ 0.046\, c_2)\, , (0.780\ -\ 0.304\, c_2\, ,\ 0.780\ +\ 0.304\, c_2)\big)$

r_{35}= $\big((1.870\ -\ 0.095\, c_1\, , 1.870\ +\ 0.095\, c_1)\, , (0.700\ - 0.391\, c_1\, , 0.700\ + 0.391\, c_1)\, ,$
$(0.446\ -\ 0.211\, c_2\, , 0.446\ +\ 0.211\, c_2)\, , (0.958\ -\ 0.045\, c_2\, ,\ 0.958\ +\ 0.045\, c_2)\big)$

r_{41}= $\big((1.375\ -\ 0.352\, c_1\, , 1.375\ +\ 0.352\, c_1)\, , (0.478\ - 0.0072\, c_1\, , 0.478\ + 0.0072\, c_1)\, ,$
$(0.509\ -\ 0.068\, c_2\, , 0.509\ +\ 0.068\, c_2)\, , (1.107\ -\ 0.127\, c_2\, ,\ 1.107\ +\ 0.127\, c_2)\big)$

r_{42}= $\big((0.879\ -\ 0.095\, c_1\, , 0.879\ +\ 0.095\, c_1)\, , (0.486\ - 0.049\, c_1\, , 0.486\ + 0.049\, c_1)\, ,$
$(0.620\ -\ 0.069\, c_2\, , 0.620\ +\ 0.069\, c_2)\, , (1.302\ -\ 0.094\, c_2\, ,\ 1.302\ +\ 0.094\, c_2)\big)$

r_{43}= $\big((1.318\ -\ 0.307\, c_1\, , 1.318\ +\ 0.307\, c_1)\, , (0.584\ - 0.148\, c_1\, , 0.584\ + 0.148\, c_1)\, ,$
$(0.472\ -\ 0.120\, c_2\, , 0.472\ +\ 0.120\, c_2)\, , (0.749\ -\ 0.196\, c_2\, ,\ 0.749\ +\ 0.196\, c_2)\big)$

r_{44}= $\big((1.352\ -\ 0.246\, c_1\, , 1.352\ +\ 0.246\, c_1)\, , (0.878\ -\ 0.035\, c_1\, , 0.878\ + 0.035\, c_1)\, ,$
$(0.525\ -\ 0.081\, c_2\, , 0.525\ +\ 0.081\, c_2)\, , (0.860\ -\ 0.277\, c_2\, ,\ 0.860\ +\ 0.277\, c_2)\big)$

r_{45}= $\big((1.829\ -\ 0.092\, c_1\, , 1.829\ +\ 0.092\, c_1)\, , (0.602\ -\ 0.423\, c_1\, , 0.602\ +\ 0.423\, c_1)\, ,$
$(0.589\ -\ 0.195\, c_2\, , 0.589\ +\ 0.195\, c_2)\, , (1.243\ -\ 0.092\, c_2\, ,\ 1.243\ +\ 0.092\, c_2)\big)$

r_{51}= $\big((1.308\ -\ 0.425\, c_1\, , 1.308\ +\ 0.425\, c_1)\, , (0.497\ -\ 0.068\, c_1\, , 0.497\ +\ 0.068\, c_1)\, ,$
$(0.490\ -\ 0.071\, c_2\, , 0.490\ -\ 0.071\, c_2)\, , (1.190\ -\ 0.104\, c_2\, ,\ 1.190\ +\ 0.104\, c_2)\big)$

r_{52}= $\big((0.579\ -\ 0.05\, c_1\, , 0.579\ +\ 0.05\, c_1)\, , (0.478\ -\ 0.057\, c_1\, , 0.478\ +\ 0.057\, c_1)\, ,$
$(0.628\ -\ 0.060\, c_2\, , 0.628\ +\ 0.060\, c_2)\, , (1.362\ -\ 0.102\, c_2\, ,\ 1.362\ +\ 0.102\, c_2)\big)$

r_{53}= $\big((1.063\ -\ 0.343\, c_1\, , 1.063\ +\ 0.343\, c_1)\, , (0.418\ -\ 0.111\, c_1\, , 0.418\ +\ 0.111\, c_1)\, ,$
$(0.639\ -\ 0.156\, c_2\, , 0.639\ +\ 0.156\, c_2)\, , (0.975\ -\ 0.101\, c_2\, ,\ 0.975\ +\ 0.101\, c_2)\big)$

r_{54}= $\big((1.329\ -\ 0.197\, c_1\, , 1.329\ +\ 0.197\, c_1)\, , (0.495\ -\ 0.077\, c_1\, , 0.495\ +\ 0.077\, c_1)\, ,$
$(0.908\ -\ 0.040\, c_2\, , 0.908\ +\ 0.040\, c_2)\, , (1.185\ -\ 0.348\, c_2\, , 1.185\ +\ 0.348\, c_2)\big)$

r_{55}= $\big((1.598\ -\ 0.103\, c_1\, , 1.598\ +\ 0.103\, c_1)\, , (0.477\ -\ 0.164\, c_1\, , 0.477\ +\ 0.164\, c_1)\, ,$
$(0.714\ -\ 0.454\, c_2\, , 0.714\ +\ 0.454\, c_2)\, , (1.130\ -\ 0.102\, c_2\, , 1.130\ +\ 0.102\, c_2)\big)$

Step 4: The score value of each element of the decision matrix **P D** and the score matric is given by

$$S_M = \begin{pmatrix} 0.5798 & 0.56 & \mathbf{0.717} & 0.6698 & 0.6988 \\ \mathbf{0.891} & 0.623 & 0.8783 & 0.8643 & 0.8868 \\ 0.6193 & 0.5733 & \mathbf{0.7968} & 0.7748 & 0.7915 \\ 0.5593 & 0.3608 & 0.6703 & \mathbf{0.7113} & 0.6498 \\ 0.5313 & 0.2668 & 0.4668 & 0.4328 & \mathbf{0.5578} \end{pmatrix}$$

Step 5: From the score matrix S_M, we say that patient 1 suffer from typhoid, patient 2 suffer from viral fever, patient 3 suffer from typhoid, patient 4 suffer from threat disease and patient 5 suffer from malaria.

Conclusion: Double Refined Indeterminate Neutrosophic Numbers will provide us with a thorough study of indeterminacy. With this outlook we have defined GDRINN and proposed an approach for solving decision making problems with GDRINN. The illustrative example shows that the final result produced by the method proposed in this paper is precise with the result produced in [6]. Future scope of this research work could be to investigate the application of GDRINN in various optimization techniques.

REFERENCES

[1] Bharatraj J. Interval Valued Intuitionistic Fuzzy Gaussian Membership Function: A Novel Extension. Intelligent and Fuzzy Techniques: Smart and Innovative Solutions. 2020 Jun 10;1197:372–80. doi: 10.1007/978-3-030-51156-2_44. PMCID: PMC7351572.

[2]P.Chi and P. Liu, An Extended TOPSIS Method for the Multiple Attribute Decision Making Problems Based on Interval Neutrosophic Set, Neutrosophic Sets and Systems, 1 (2013), 63-70.

[3] P. Dutta and T. Ali, Uncertainty modelling in risk analysis: A fuzzy set approach, International Journal of Computer Applications, 43(17) (2012), 35-3.

[4] Y. C¸ elik and S. Yamak, Fuzzy soft set theory applied to medical diagnosis using fuzzy arithmetic operations, Journal of Inequalities and Applications, 82 (2013), 1-9.

[5] A. Garg and S.R Singh, Solving fuzzy system of equations using Gaussian membership function, International Journal of Computational Cognition, 7 (2009), 25-32.

[6] Karaaslan, Faruk. "Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making." Neutrosophic Sets and Systems, vol. 22, Nov. 2018, pp. 101-117.

[7] P. Liu, Y. Chu, Y. Li, and Y. Chen, Some Generalized Neutrosophic Number Hamacher Aggregation Operators and Their Application to Group Decision Making, International Journal of Fuzzy Systems, 16(2) (2014), 242-255.

[8] K. Mondal and S. Pramanik. Multi-criteria group decision making approach for teacher

recruitment in higher education under simplified Neutrosophic environment, Neutrosophic Sets and Systems, 6 (2014), 28-34.

[9] K. Mondal and S. Pramanik, Neutrosophic tangent similarity measure and its application to multiple attribute decision making, Neutrosophic Sets and Systems, 9 (2015), 80-87.

[10] K. Mondal and S. Pramanik, Neutrosophic decision making model of school choice, Neutrosophic Sets and Systems, 7 (2015), 62-68.

[11] K. Mondal, S. Pramanik, and B. C. Giri. Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy, Neutrosophic Sets and Systems, 20 (2018), 3-11. http://doi.org/10.5281/zenodo.1235383.

[12] K. Mondal, S. Pramanik, and B. C. Giri, Hybrid binary logarithm similarity measure for MAGDM problems under SVNS assessments. Neutrosophic Sets and Systems, 20 (2018), 12-25. http://doi.org/10.5281/zenodo.1235365.

[13] K. Mondal, S. Pramanik, and B. C. Giri, Interval neutrosophic tangent similarity measure based MADM strategy and its application to MADM problems, Neutrosophic Sets and Systems, 19 (2018), 47-56. http://doi.org/10.5281/zenodo.1235201.

[14] Peng JJ, Wang JQ, Wu XH, Wang J, Chen XH (2014) Multivalued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. Int J Comput Intell Syst 8(2):345–363

[15] S. Pramanik, P. Biswas, and B. C. Giri, Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment, Neural Computing and Applications, 28 (5) (2017), 1163-1176. DOI 10.1007/s00521-015-2125-3.

[16] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache, and T.K. Roy, IN-cross entropy based MAGDM strategy under interval neutrosophic set environment, Neutrosophic Sets and Systems, 18 (2017), 43-57. http://doi.org/10.5281/zenodo.1175162

[17] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache, and T. K. Roy NS-cross entropy based MAGDM under single valued neutrosophic set environment. Information, 9(2) (2018), 37. doi:10.3390/info9020037.

[18] S. Pramanik, R. Mallick, and A. Dasgupta, Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment, Neutrosophic Sets and Systems, 20 (2018), 108-131. http://doi.org/10.5281/zenodo.1284870.

[19] S. Pramanik, S. Dalapati, and T. K. Roy, (2018), Neutrosophic multi-attribute group decision making strategy for logistic center location selection. In F. Smarandache, M. A. Basset and V. Chang (Eds.), Neutrosophic Operational Research, Vol. III. Pons Asbl, Brussels 13-32.

[20] Ranjan Kumar, S A Edalatpanah, Sripati Jha , Ramayan Singh , A Novel Approach to Solve

Gaussian Valued Neutrosophic Shortest Path Problems , International Journal of Engineering and Advanced Technology (IJEAT) ISSN: 2249 – 8958, Volume-8 Issue-3, February 2019

[21] Sanjib Sen , Kartik Patra and Shyamal Kumar Mondal , Similarity Measure of Gaussian Fuzzy Numbers and Its Application , Int. J. Appl. Comput. Math (2021) 7:96.