



Simulation Design & Analysis functionality of Grid Connected (GC) Photo-Voltaic (PV) based on ANFIS MPPT

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Abstract— The weather has a big impact on how much electricity a photovoltaic (PV) array can generate. In this study, we demonstrated the grid-connected PV unit's continuous state functionality at various solar irradiances. The presented model was created in a MATLAB environment and includes a PV array controlled by an Adaptive Neural Fuzzy Interface System (ANFIS) based controller, the system here is well connected on the respect of DC-to-DC boost electronic system in advancement to this tracking system is holding accordance to the three phase multi-level (3) inverter. A transformer and low pass filter connected to the main utility grade for this solar unit. To determine the functioning mechanism of the provided model, daily meteorological conditions were used during lab creation. The suggested system's simulation findings demonstrate that it can deliver good power quality and grid performance. The number of cell modules employed in the proposed work was 90, the number of parallel strings was 6.0The system processed is giving 274 volts and 250 volts at the solar irradiance of 1000 wb/m²-and 600 wb/m² respectively.

Index Terms- Solar harvesting unit, Maximum solar unit, Boost converter, ANFIS

INTRODUCTION

Solar energy (SE) is a booming field of energy harvesting system in India. Given that India has about 305 apparent bright days per year. The SE source output over the course of a year exceeds the amounts of energy that can be obtained from India's priceless energy resource reserves. India, with its vast solar potential, has emerged as a global leader in solar energy production and deployment. Over the past decade, the country has made significant strides in harnessing solar power to meet its energy demands and combat climate change. This article explores the current status of solar energy in India, including its growth, government initiatives, challenges, and future prospects.

Solar Capacity Expansion: India has witnessed a remarkable growth in solar capacity installation, with cumulative installed capacity reaching over 40 GW as of 2021, compared to a mere 2.63 GW in 2014[1].

The Indian government has set ambitious targets to achieve 100 GW of solar capacity by 2022 under the National Solar Mission, which has provided a robust framework and policy support to drive solar energy adoption. The declining costs of solar photovoltaic (PV) modules, coupled with innovative financing models, have made solar energy increasingly competitive with conventional sources, attracting investments and fostering market growth.

The government has implemented various financial incentives, such as capital subsidies, tax benefits, and feed-in tariffs, to encourage solar power generation and attract investments in the sector. International Collaborations: India has collaborated with international organizations, including the International Solar Alliance (ISA), to

accelerate the adoption of solar energy globally and promote cooperation in research, technology, and capacity building. Solar Parks and Ultra Mega Solar Projects: The government has facilitated the development of solar parks and ultra-mega solar projects, providing land, infrastructure, and regulatory support to attract project developers and streamline the process [2].

The intermittent nature of solar power poses challenges for grid integration and stability, necessitating grid upgrades, energy storage solutions, and robust transmission infrastructure to balance power supply and demand. Acquiring large tracts of land for solar projects and developing the necessary infrastructure, such as transmission lines and substations, can be time-consuming and encounter regulatory hurdles.

While the cost of solar energy has decreased, accessing affordable financing remains a challenge for project developers, particularly small-scale enterprises, requiring innovative financing mechanisms and easier access to capital. The rapid expansion of solar energy demands a skilled workforce for installation, operation, and maintenance, necessitating the development of technical training programs and capacity building initiatives.

India is exploring the potential of floating solar projects on water bodies, which can address land scarcity issues and enhance energy generation efficiency, with several pilot projects already underway. India's vast untapped rooftop solar potential presents an opportunity for decentralized power generation, reducing transmission losses and improving energy access in urban and rural areas.

Advancements in energy storage technologies, such as batteries and pumped hydro storage, can address the intermittency of solar power and enhance grid stability, paving the way for increased solar energy integration.

Continued investments in research and development, along with technological advancements like perovskite solar cells and advanced PV technologies, hold the potential to further improve the efficiency and affordability of solar energy.

Although many RE programmers are large-scale, renewable energy systems are also ideal for remote and nonurban areas in developing nations where energy is crucial for human growth.

In comparison to standalone units, which primarily require electric batteries, global GC PV units contribute about 98.9% of the installed total capacity. A GC PV without electric batteries is less expensive and simply requires less regular maintenance. This helps to reduce the strain on other resources that supply power to the grid.

Out turns of an on-grid system is studied here with respect to grid while GC PV system is simulated upon.

OUT-TURN of ON-GRID SOLAR UNITS

PV frameworks may expose the lattice to a few unfavorable impacts if the PV entrance is high, such as:

1. Hindrance in detection of areas with islanding condition
2. Excessive voltage brought on by scattered feeders.
3. Difficulty controlling voltage.
4. Imbalance condition in current phasors.
5. An increase in reactive power
6. Power stream in reverse

The following three effects are thought about in the proposed work:

a. Problems/Harmonics in Power quality

Harmonics are electrical currents or voltages that have frequencies that are integer multiples of the fundamental frequency. In power systems, harmonics are often generated by nonlinear loads such as computers, variable frequency drives, and electronic devices. While harmonics themselves are not necessarily harmful, their presence can distort power quality and have various adverse effects. In the process of DC to AC conversion, some harmonics are generated which are transmitting toward the inverter model itself. Which is key reason of their own existence. Inverter is playing the role hold the total amount of the discharge of supply to the grid. Power loss from harmonic currents leads to the provided electricity becoming warped [3]. harmonics in power grids is the damage inflicted on electrical equipment. Non-linear loads, such as computers, variable speed drives, and electronic devices, produce harmonics that can overheat transformers, motors, and capacitors. The excessive heat generated by harmonics reduces the lifespan of equipment, increases maintenance costs, and can ultimately lead to premature failures. These equipment failures not only disrupt the power supply but

also result in significant financial losses for consumers and utilities.

voltage and current distortion, leading to an array of problems within power grids. Voltage distortion can affect sensitive equipment, leading to malfunctions, inaccurate measurements, and improper functioning of control systems. It can also result in flickering lights and excessive heating of appliances. Current distortion caused by harmonics affects power quality, increases system losses, and can result in overloading of conductors and transformers. Moreover, harmonic-induced current distortion can propagate across the power grid, affecting neighboring systems and causing a ripple effect of problems.

Detrimental impact on the efficiency of power grids. The presence of harmonic currents increases power losses in transformers, cables, and other components of the grid. These losses reduce the overall efficiency of the system and result in wastage of energy. Additionally, harmonics can cause a decrease in the power factor, leading to lower system efficiency and increased demand on the power generation capacity. Inefficiencies caused by harmonics not only have economic implications but also contribute to increased greenhouse gas emissions and environmental degradation.

Harmonics can induce resonance phenomena in power grids, which amplify the effects of harmonics and create power quality issues. Resonance occurs when the natural frequency of a system matches the harmonic frequencies present, resulting in excessive voltages and currents. Resonance can cause equipment malfunction, damage, and even complete system failures. Moreover, harmonics affect power quality by distorting waveforms, increasing voltage fluctuations, and reducing the overall system stability

b. Reactive Power Increment

In most cases, solar PV inverters have a power-factor of about one. A compensation-centered approach assesses the administrators of modest non-commercial PV units based on their kilowatt-hour production as opposed to their kilovolt-ampere-hour consumption. When adding solar units to the grid, the reactive power dynamics of the system can be affected. Reactive power is the portion of power that oscillates between the source and the load without being consumed. It is necessary for maintaining voltage levels and ensuring the efficient operation of the power system. Here's how the addition of solar units can impact reactive power. The photovoltaic model's ability to meet the expanding market need for reactive electrical power is severely limited. Due to this, The addition of solar units to the grid can affect voltage stability, particularly during periods of high solar generation. Excessive reactive power injection from PV systems can lead to overvoltage conditions, affecting the stability and performance of the system. Proper voltage control

mechanisms, such as voltage regulation devices and reactive power compensation equipment, may be necessary to maintain voltage stability within acceptable limits. [3].

c. Hindrance in detection of areas with islanding condition

Islanding refers to a situation in which a distributed generation source, such as a solar unit, continues to supply power to a portion of the grid despite a grid outage. Detecting islanding conditions is crucial for the safety of utility workers and to prevent damage to the equipment. However, the presence of solar units connected to the grid can introduce certain challenges in the detection of islanding conditions. Here are some hindrances in the detection of islanding conditions due to solar units [4].

All of these effects are mostly reliant on the PV unit's total size and placement. Large-scale models are evaluated over 500 KW. Simple arrangements are graded at 10 k-W or less. We evaluate a larger-scale, 4MW PV model created for simulation in this study.

Solar units, particularly those equipped with grid-tied inverters, often inject reactive power into the grid to meet power factor requirements or for voltage regulation purposes. This injection of reactive power can create the illusion of grid presence and make it more challenging to distinguish between islanded and grid-connected conditions. It complicates the detection process, as reactive power flow might falsely indicate grid connectivity.

The behavior and control mechanisms of inverters used in solar units can impact the detection of islanding conditions. Inverters may employ various techniques, such as frequency or voltage-based methods, to detect grid disturbances and initiate anti-islanding protection. However, the effectiveness and reliability of these methods can vary depending on the design, settings, and performance characteristics of the inverters.

The detection of islanding conditions is regulated and guided by interconnection standards and grid codes established by utility companies and regulatory bodies. These standards aim to ensure the safe and reliable operation of distributed generation systems. However, differing requirements and variations in interconnection standards across regions can introduce complexities in the implementation and performance of islanding detection methods for solar units.

In general, there are two methods for controlling 3-phase VSI: voltage control (VC) and current control (CC). In order to control the flow of electrical power, the VSI VC uses the phase angle between the inverter's output voltage and the grid voltage [6]. Two control loops are included in the control approach for the GC PV inverter. The first loop, which is an external holds the responsibility of out regulation. The second is internal, is accountable for

current calibration to achieve unit PF method [7]. The 3-phase inverter's suggested control strategy includes a method for managing the voltage in a DC connection, which might boost the power at the inverter's input and therefore enable the inverter to work efficiently.

Here are two common regulating strategies for 3-phase VSI, such as voltage and current controllers [6]. Two controlling loops are used in the controlling methodology while planning the PV inverter linked with the framework.

The PV device is generally connected to a low-voltage networking system using a power transformer that can verify the placement of power between the main grid and the PV unit. This method is often intended for use with community networking systems that typically have 3-phase lines with 20kV10% power.

To overcome these hindrances and enhance the detection of islanding conditions, advancements in anti-islanding technologies are being pursued. These include the development of advanced detection algorithms, improved communication and monitoring systems, and stricter regulatory standards. Ensuring the reliability and effectiveness of islanding detection for solar units is crucial to maintaining grid stability, protecting utility workers, and preventing potential equipment damage.

I. MODELING OF SYSTEM

The paper here is model according to solar GC considering a array which is detailed according to SPR-305 model of WHT company. WHT is a renowned name in the manufacturing of solar panels in India. A DC to DC boosting communication for raising the PV voltage to a level necessary for the inverter to produce the best output, a 3-phase VSI connected between DC-link capacitance (which in turn serves as a momentary power storing area to provide VSI with a constant flow of electricity), and finally an L-C filtration system. For a solar irradiation of 1,000 W/m², the standard PV voltage is 274 Vdc, which is converted into 500 Vdc using a DC-to-DC boost and then into AC voltage by a three-tier VSI near to 20 kV utilizing a step-up transformer as input to the UG.

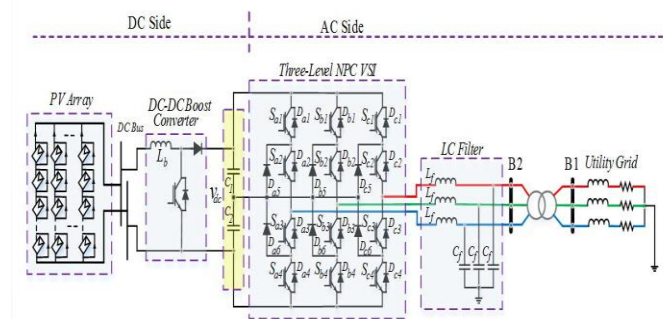


Fig. 1: Diagram of three-level PV grid associated inverter [8]

a. PV System Modeling

The electrical equivalent circuitry diagram for a PV cluster utilizing a revised two-diode modelling setup is shown in Fig. 2. Eq. 1 provides the PV array system's output current (I_{PV}). The PV array's output voltage is V_{PV} , and the respective PV cells' respective series and shunt resistances are R_s and R_{sh} . The numbers of series and parallel cells are N_s and N_p , respectively [10]. The two-diode system's parameters are set up as shown in Fig. 2: photo diode current (I_{ph}), flooding currents (I_{s1} , I_{s2}), series and shunt resistances (R_s , R_{sh}), and ideality two-diode factors (a_1 , a_2).

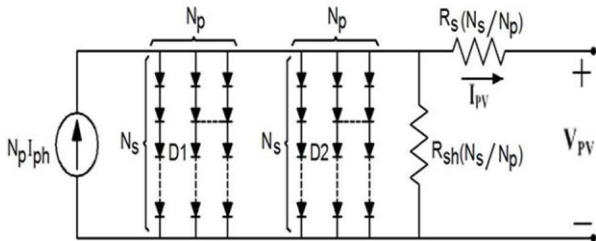


Fig. 2: Equal circuit model of single-diode for PV array [9]

b. Modeling for DC-DC Boost Converter

Equations (2:5) [11-18] can be used to compute the input and output associations of the DC-DC boost converter shown in Figure 3.

$$I_{PV} = N_p I_{PH} - N_p I_{S1} \left[\exp\left(\frac{1.0(V_{PV} + I_{PV} R_s)}{a_1 \left(\frac{V_{PV}}{N_s} + \frac{I_{PV} R_s}{N_p}\right)}\right) - 1 \right] - N_p I_{S2} \left[\exp\left(\frac{1.0(V_{PV} + I_{PV} R_s)}{a_2 \left(\frac{V_{PV}}{N_s} + \frac{I_{PV} R_s}{N_p}\right)}\right) - 1 \right] - \frac{1}{R_{sh}} \left(\frac{V_{PV}}{N_s} + \frac{I_{PV} R_s}{N_p}\right) \quad (1)$$

$$V_{DC} = \frac{V_{PV}}{1 - D} \quad (2)$$

$$L_{BOOST} = \frac{V_{PV} (V_{DC} - V_{PV})}{\Delta I_{LBOOST}} \quad (3)$$

$$C_B = \frac{\Delta I_L}{8 f_s \Delta V_{PV}} \quad (4)$$

$$C_{DC} = \frac{P_{PV}}{2.0 \omega_g V_{DC} \Delta V_{DC}} \quad (5)$$

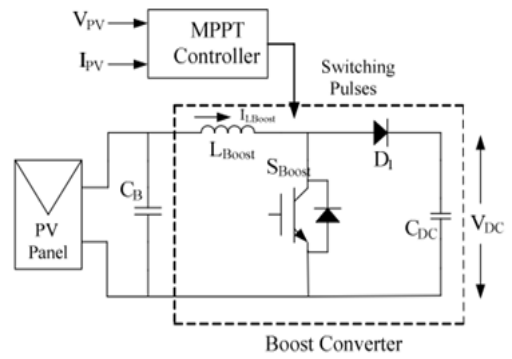


Fig:-3 DC-DC boost converter

c. Filter for Power

In order to lessen the harmonics produced by the inverter and the consequences of surges coming from one of the UG, filtration mechanisms are built into the UG user interface [12]. The 3-phase second order low-pass passive filter used as the electrical power filter system in this significant model is illustrated in Fig. 4. The filter is positioned next to the 3-phase VSI output.

Capacitance typically offers an alternative to both inductance and the reactive power produced by a capacitor at the primary frequency. Increased capacitor capacity can reduce inverter performance, whereas reduced LC filtering system capacitor value can increase inductor performance and voltage loss throughout the networking system. The reactive power related to capacitor value is typically advised to be less than 15% of the graded inverter power efficiency.

In this type of approach, 10% of the graded power connected to the inverter is used to select the reactive power [13-15].

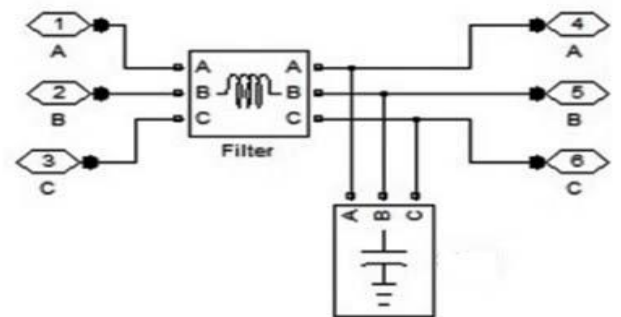


Fig. 4: LC low pass power filter used at the output terminals of PV inverter [12].

$$C_f = \frac{0.1 * P_{PV}}{3 * \omega_g V_g^2} \quad (6)$$

In order to avoid resonance with the grid network, the choice of the inductor in the filter, L_f , depends on the resonance frequency of the filter, which may be greater than or equivalent to the 1/10 of the grid frequency. As a result, the filter inductance in Equation 1:

$$L_f \leq \frac{1}{100 * w_g^2 C_f} \tag{7}$$

a. Adaptive Neuro-Fuzzy Inference System

The ANFIS is an advanced technique to calibrate the input system in preferred output this is performed with the processing on the ground of data learning [19].

This use two machine learning techniques (back propagation and least square error algorithms) into a single technique. To demonstrate the ANFIS architecture, two fuzzy IF-THEN rules based on a first-order Sugeno model are considered as follows [20]:

- RULE 1: If x is A_1 and y is B_1 , then $f_1 = p_1x + q_1y + r_1$
- RULE 2: If x is A_2 and y is B_2 , then $f_2 = p_2x + q_2y + r_2$

where x and y are the inputs, A_i and B_i are the fuzzy variables, f represents the outputs of fuzzy sets, and p_i, q_i , and r_1, r_2 are design parameters that the ANFIS system learns. Fig. 5 depicts the ANFIS architectural layout. In this diagram, a square denotes an adaptive node while a circle denotes a fixed node. The nodes in each layer of ANFIS' five-layer architecture perform similar tasks.

According to Fig. 6, an ANFIS-based MPPT controller consists of a DC-DC boost converter, FL power controller, and ANFIS reference model. This MPPT controller is based on the idea that the real-time MPP of a solar module can be precisely tracked by knowing the maximum power output of a PV module for a given set of solar irradiance and temperature.

The predicted value of the maximum power output from the PV modules at a particular temperature and irradiance is provided by the ANFIS reference model.

The actual power output from the PV module is measured and compared to the reference value from the ANFIS model at the same temperature and irradiance. The difference between the two power values is calculated to give an error, which is then fed to the FL power controller to generate a control signal. The signal generated by the FL power controller is given to the pulse width modulator (PWM). To control the duty cycle of the DC-DC power converter and force PV modules to operate at the MPP, the PWM generates a signal at a high level of frequency.

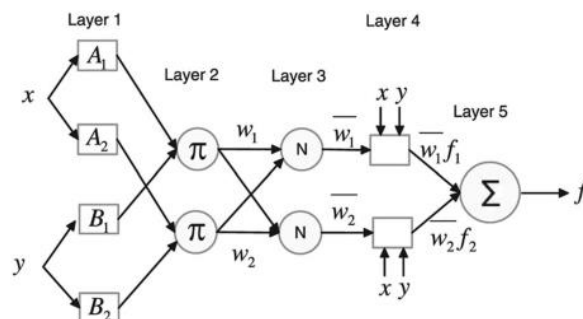
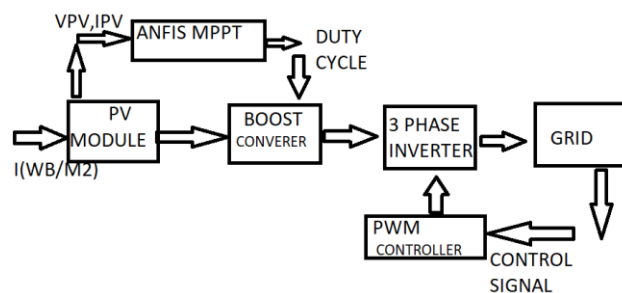


Fig.5: Adaptive neuro-fuzzy inference system architecture

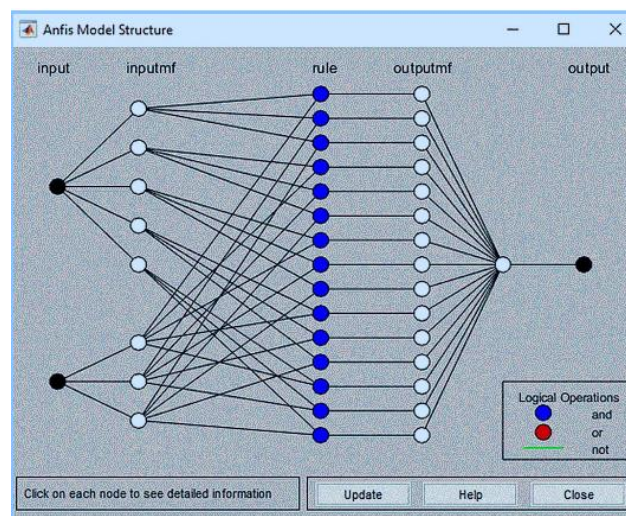


Fig.6: Architecture for the ANFIS reference model

b. Grid-Connected Control System

A VSI's regulating design is shown in Figure 7. PII place a vital role in supplying the required frequency and voltage it works in a simultaneous synchronous manner with inverter in accordance to grid. It adjust the voltage frequency with respect to that of the grid [18]. In order to advance its current regulator, the voltage regulator creates DC electric voltage (V_{dc})* and gives command current (I_d)*. The PI controllers for both the i_d and i_q currents make up the current regulator. A voltage regulator is used

to pull the command current (I_d), which is comparable to the grid current (I_d).

In order to reduce error and generate an addition signal with the built-up voltage computing signal (V_d), the evaluated signal (I_d) is examined by the PI controller. This allows for evaluation with L to develop (V_d)*. Additionally, in order to boost the inverter's PF close to unity, the command current (I_q)* approaches to zero. By using the PI controller to generate the addition signal with the produced voltage (V_q) as well as, L to generate (V_q)* instruction, the signal (I_q) is produced. Hysteresis band analysis is used to analyze the PI controller findings and correct for upper/lower limit errors. The $dq0$ is changed to accommodate 3-phase PWM.

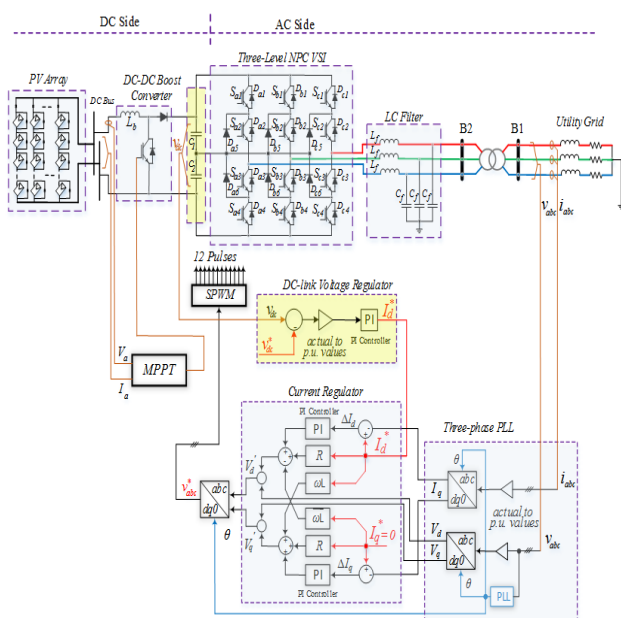


Fig. 7: System configuration of PV GC system

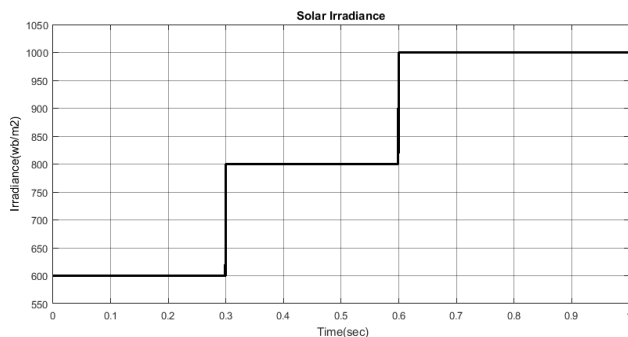


Figure 8 solar radiation

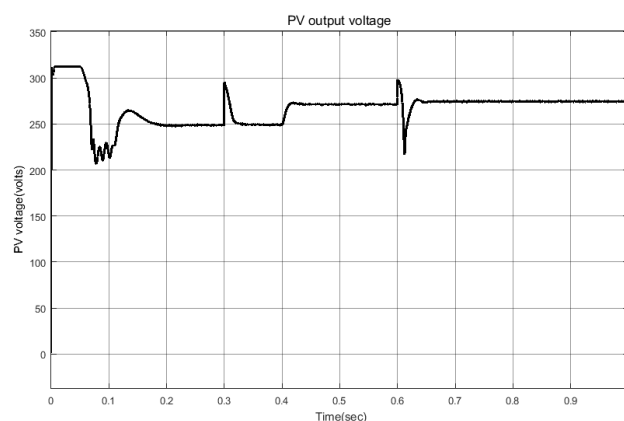


Figure 9 solar output voltage

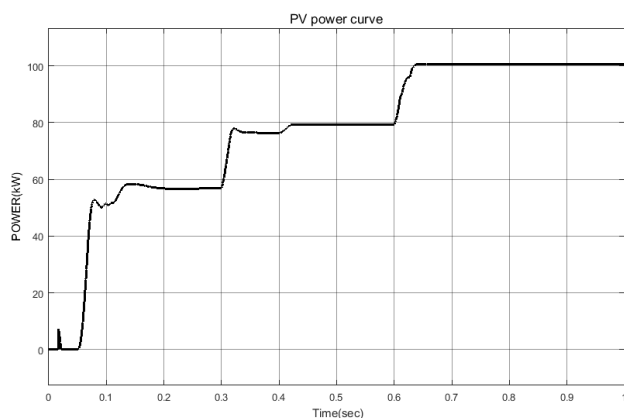


Figure 10 solar power generated

Figure 7 displays the proposed system's block diagram. Figure 8 depicts how the solar input irradiance varies over time. Figure 9 clearly illustrates how the PV generated volts vary in accordance with the variation in susceptibility. Figure 10 displays the solar-generated power, with curves illustrating how power values fluctuate as input irradiance changes.

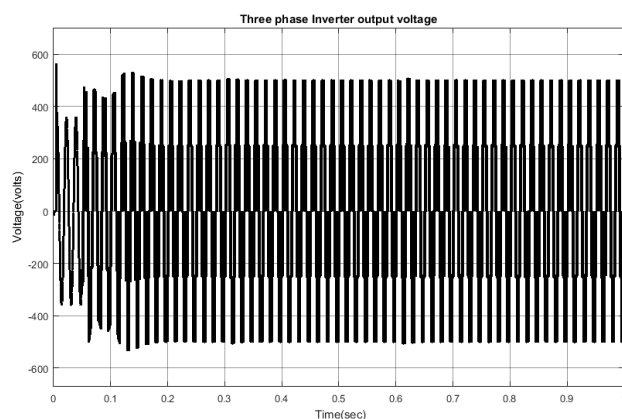


Figure 11 Inverter output voltage

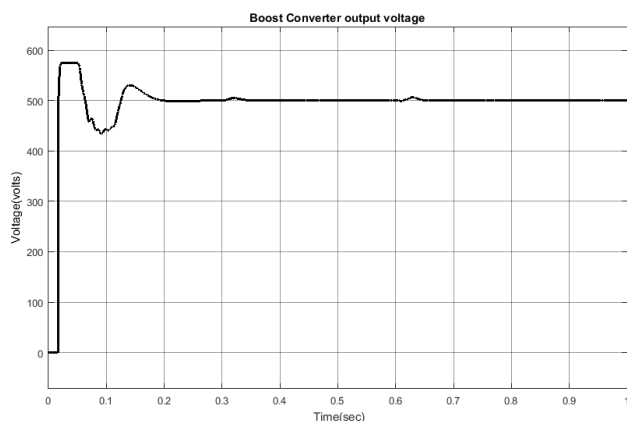


Figure 12 boost converter output voltage

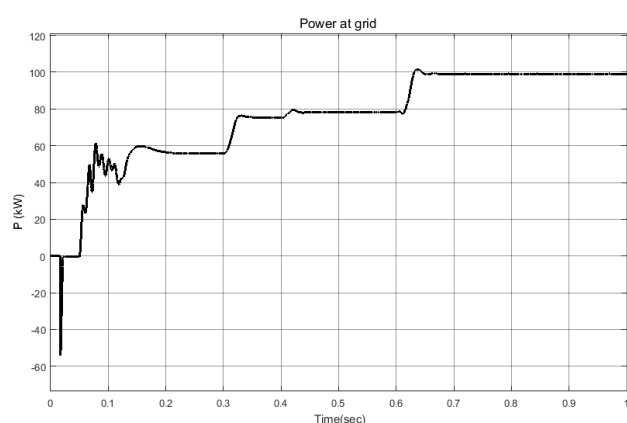


Figure 13 power at grid

Figure 11 demonstrates that the inverter output voltage remains constant after using ANFIS-based MPPT even when the resistance varies. Figure 12 depicts the boost converter's output, which is constant at 500 volts. This indicates that the intended MPPT and boost converter are accurately producing a constant output voltage. Figure 15 shows the variation in boost converter duty cycle to maintain constant voltage. Figure 13 displays the power transferred to the grid and also demonstrates variation. Figure 16 depicts the inverter's output following a harmonic filter.

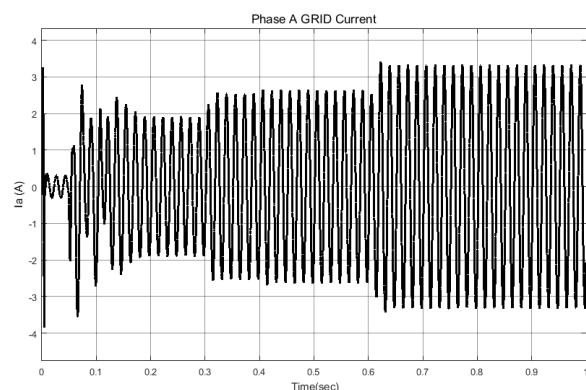


Figure 14 phase a grid current

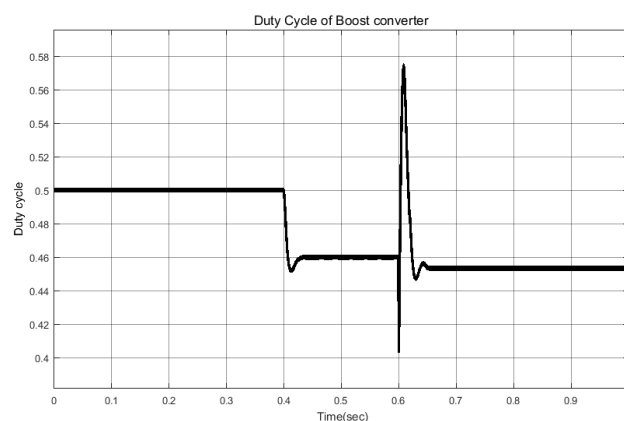


Figure 15 duty cycle of boost converter

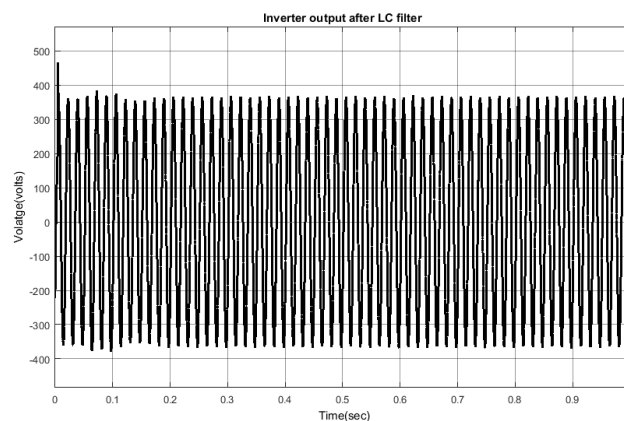


Figure 16 Inverter output voltage

II. CONCLUSION

This study examines the efficiency of a PV connected-grid unit made up of a PV array, an ANFIS-based MPPT tracking system, a three-level, three-phase inverter supplied with a filtration system made up of an inductor, capacitor, and step-up transformer. The grid code-compliant synchronous framework of the power grid,

along with higher inverter PF close to the unit, forms the foundation of the regulating structure. The inverter's enhanced power standard with LC filtration was designed to reduce harmonic distortion. The MPP tracking system employs irradiance levels designed to extract the maximum amount of power while maintaining DC voltage at a constant level of 500 VDC.

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