Establishing an inventory management finite planning horizon model, comprising Linear Demand, Exponential Deterioration, and Trade Credit Policy

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Abstract: In modern-day business transactions, trade credit is having a greater importance in the payment process. In this study, we focused on a supply chain model that evaluates, over a finite planning horizon, the trade credit policy between the two important players:the supplier and the retailer. We specifically created a mathematical model for commodities that depreciate exponentially and used Mathematica version 12.0 to find a solution in later part.We give a numerical instance to illustrate how to establish the optimum replenishment schedule for our finite planning horizon model. It is observed that the total profit function is convex and its optimality is depicted.Our analysis presents managerial guidelines.

Keywords: Demand credit period sensitive, Exponential deterioration, Trade credit, Finite planning horizon.

1. Introduction

Trade credit refers to an agreement between the suppliers and retailers for the purchase of goods or services without paying the money upfront. The terms of trade credit may vary based on the conditions between the parties, including factors such as the amount of credit extended, the interest rate (if any), the repayment period, and any late payment penalties. When a supplier offers a more extended period of waiting before the payment of an invoice, it can incentivize buyers to purchase more products or services from that supplier. Conversely, if the credit period is shorter, it may discourage buyers from making a purchase, as they may not have sufficient cash flow within the given timeframe. Thus, a longer credit period equates to a larger investment in receivables.

The supplier has implemented a credit period that works progressively as follows: In case the retailer clears the unpaid bill within M-time units, no interest is charged. However, if the payment is made between M and N (where N is greater than M), any outstanding balance incurs interest at a rate of Ic_1 as charged by the supplier. In case the retailer clears the debt after N, they will be subject to an interest rate of Ic_2 (where Ic_2 is higher than Ic_1). Several studies have developed mathematical models to address this type of credit policy. A new model for inventory management was put forward by Soni and Shah in 2005that has a steady rate of deterioration in inventory units and the supplier providing two successive credit periods. Similarly, Soni et al. (2005) developed a mathematical framework for the random units received by the retailer, and the supplier's provision of two credit periods in a progressive manner. Huang (2006) expanded on the inventory model proposed by Huang (2003) by integrating two tiers of trade credit, while also accounting for the limited storage capacity of the retailer. The optimal cycle time is determined using straightforward analytic theorems.

The responsiveness of demand to credit periods can vary based on factors such as the industry, product or service type, and the financial condition of the buyer. For instance, industries that involve high material and equipment expenses, like construction or manufacturing, may be more sensitive to credit periods. Conversely, industries that offer low-cost products or services may be less sensitive to changes in credit periods.

Various studies have explored assimilating trade credit into inventory modelling, with Baumol's (1952) model being the earliestthat aimed to regulate the payment timings minimizing the total inventory expenses for the buyers. This model accounted for holding costs, ordering costs, and trade credit costs, and showed that trade credit policies can significantly affect the optimal payment time. The findings emphasized the importance of considering trade credit in inventory management.

Numerous scholars have further investigated the influence of trade credit on inventory modeling after Baumol's pioneering work, such as Chiu et al. (2019) and Tang et al. (2020). These studies have demonstrated that integrating trade credit policies into inventory management models can result in noteworthy enhancements in profitability and cost reduction.
The present study has a specific emphasis on a critical facet of inventory management, that is, the construction of an inventory management model with linear demand, exponential deterioration, and trade credit policy within a finite planning horizon. The model is centred around a single product with a finite shelf life, characterized by linear demand and exponential deterioration. Furthermore, the model encompasses a trade credit policy where the supplier offers the buyer credit, enabling them to postpone payment.

The structure of the research paper is outlined as follows: Section 2 offers an outline of the literature review, while Section 3 explains the notations and assumptions used in the study. Section 4 is dedicated to the mathematical model, including the derivation of the non-linear differential equation. Numerical examples and solution analysis are provided in Section 5. In Section 6 and section 7, the paper offers managerial insights, concluding remarks, and a list of potential avenues for future research.

2. Literature Review

One of the early attempts to incorporate trade credit policies into inventory modelling was made by Chapman and his colleagues in 1984. They developed an economic order quantity model that takes in account the credit periods offered by suppliers. Being highly vulnerable to the length of the credit period allowed, the model also focuses attention to the correlation between the credit period and inventory levels.The effects of ordering costs are less significant, and it's added reliance on the estimation of inventory item demand makes it different from the traditional economic order quantity model. They provided a numerical example to demonstrate how assimilating a credit period can lead to substantial reductions in the amount required to maintain the goods that are not yet sold to the customer (inventory costs).

In 1985, Goyal was the first to develop a mathematical model that considers a supplier's repayment terms in resolving an account; if the account is cleared within the permissible hold time, no interest is charged on the outstanding amount. The supplier will impose a higher interest rate if the account is uncleared beyond the specified delay period. The system benefits monetarily from this arrangement since the retailer can make interest payments on the revenue earned during the period of delay. Shah et al. (1988) expanded the model to account for shortages, while Mandal and Phaujdar (1989a, b) included interest earned from the sales revenue on the shares that remained after the settlement period. A finite replenishment rate was proposed to Goyal’s model by Chung and Huang in 2003.

The study conducted by Abad and Jaggi (2003) focused on analysing the seller-buyer channel, where the price and trade credit offered by the supplier can influence the final demand for the product. In this model, the seller’s unit price and the extent of the buyer’s credit period are considered as policy for the seller as they have an impact on the product’s ultimate demand.

P. Singh et al. (2017), Sarker et al. (2001), Ouyang et al. (2005a) and S. Saxenaet al. (2022) investigated the optimal payment time when an inventory is susceptible to deterioration and payment is subject to a permissible delay. Meanwhile, Jamal et al. (2000) analysed a situation where a retailer can pay the supplier either at the conclusion of the credit period or later, incurring interest charges on the unpaid balance for the overdue time. They developed a model to minimize costs and determine the optimal cycle and payment time for the retailer under various system parameters. The wholesaler permitted a certain credit period for payment without penalties and the study accounted for a constant rate of inventory depreciation. The conclusion drawn from the study suggests that based on variables including interest rates, unit purchase, selling prices and the pace of unit deterioration, the retailer may decide to make a payment after the allowable credit period.

Chung and Liao (2004) enhanced the model initially introduced by Hwang and Shinn (1997) and Khouja and Mehrez (1996) by incorporating exponential deterioration of inventory items, while accounting for the possibility of permissible delay in payments. They worked with the assumption that the delay in payment is determined by the quantity of goods ordered.

This study primarily focuses on the management of inventory over a finite planning horizon. While prior research by scholars such as P. Singh et al. (2017), V. Singh et al. (2018 and 2019), S. Saxena et al. (2020), Nitin Kumar Mishra &Ranu (2022), and Ranu& Nitin Kumar Mishra (2023) has explored this topic, none of them have examined the interaction between linear time-dependent demand, exponentially deteriorating items, and trade credit under a finite planning horizon.

Research Gap: There is a research gap in the literature regarding the inclusion of trade credit policies and exponential deterioration within the context of inventory management models under a finite planning horizon. While some studies have explored this area, they have primarily focused on general attributes of the supply chain. Furthermore, only a few studies have specifically examined trade credit policies within a finite planning horizon, and none have considered trade credit with time-dependent linear demand and exponential deterioration for unequal cycle lengths.

Problem Identified: The aim of this study is to develop a supply chain inventory approachfor declining goods that considers time depends on demand and a trade-credit scheme within a finite planning horizon. The primary objective is to minimize the overall costs for both the supplier and the retailerwhile achieving an optimal ordering scheme, including product order costs, deteriorating costs, set-up costs, and holding costs. To find a solution, a non-linear programming approach will be employed. The model's validity is demonstrated through a solution of a numerical problem with the numerical iterative method using Mathematica software.

3. Assumptions and Notations

3.1. Assumptions

1. In this model, it is assumed that there will be no stockouts or shortages.
2. In this model, it is assumed that orders will be fulfilled immediately when they are placed. Therefore, lead time considered zero.
3. It is presumed that the planning horizon is finite.
4. The terms of credit policy are "α/M1 net M", which indicates that if payments are made within M1 days of invoice date, a α percent reduction of sale price is provided. Otherwise, they are required to pay the full sale price within M days of the invoice date.
5. In this model, the costof holding inventory in the supply chain is assumed to be I per unit per unit time.
6. A single supplier-single retailer supply chain channel is viewed as consisting of a single item.
7. This model assumes that there is only single product with single supplier and single retailer in the supply chain channel.
8. The demand for the goods is linear time dependent over the planning horizon, which means that the demand increases with time.

3.2. Notations

1. a - The initial demand rate at the start of the inventory management period per annum.
2. b - The demand rate increases over time, on a year.
3. $I_{i+1}(t)$- The inventory level between t_i and t_{i+1} during the $(i+1)^{th}$ cycle, where $t_i$is the start time of the cycle and t is any time between $t_i$and t_{i+1}.
4. Q_{nt}, which represents the quantity ordered during the $(i)^{th}$ cycle at time t where $t_i \leq t \leq t_{i+1}$.
5. S_s – The cost includes both the setup cost and the transportation cost and is measured in dollars per order.
6. θ- The rate at which the product deteriorates over time.
7. r- The opportunity cost, expressed as a discount rate, per unit of time.
8. A_0- The ordering cost per order at the start of the inventory management period
9. C_0- Unit cost of the item
10. h- The rate of inflation expressed per unit of time.
11. $C -$ Represents the cost incurred for buying a single unit, expressed in dollars per unit.
12. The variable "n_1" denotes the total count of replenishment cycles.

4. Mathematical structure

Demand function can be written as

$$D(t) = a + bt$$

where a > 0, b ≥ 0 and t is within a positive time frame.

The variation in the inventory level $I_{i+1}(t)$ with respect to time can be expressed using the following differential equation, which applies to the (i+1)th cycle:

$$\frac{\partial(\boldsymbol{I_{i+1}(t)})}{\partial t} + \theta I_{i+1}(t) = -D(t)\,, \quad t_i \leq t \leq t_{i+1} (1)$$

Solving (2) we get

$$I_{i+1}(t) = (a + bt_{i+1})\frac{e^{\theta t\, i+1}}{\theta\, e^{\theta t}} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\left[1 - \frac{e^{\theta t_{i+1}}}{e^{\theta t}}\right] (2)$$

$$Q_{i+1} = I_{i+1}(t_i) = e^{\theta t_{i+1}}\frac{(a+bt_{i+1})}{\theta\, e^{t_i}} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\left[1 - \frac{e^{\theta t_{i+1}}}{e^{\theta t\, i}}\right] \quad (3)$$

Decentralized approach:

Indecentralized approach, retailers and suppliers operate independently, with each member making decisions autonomously. While this structure allows for individual decision-making, it can lead to a lack of coordinationor alignment with the overall objectives of the supply chain.

The total cost of retailer for the$(i + 1)^{\text{th}}$cyclein decentralized scenario is:

$$TC_r^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots\ldots, t_{n1}) = (n_1 * A_0) + \sum_{i=1}^{n_1}\left(\int_{t_i}^{t_{i+1}} Q_{i+1}(t_i)(1-\alpha)e^{-h*M_1} + I * C_0(1-\alpha)e^{-h*M_1}\int_{t_i}^{t_{i+1}} I_{i+1}(t)e^{-rt}\,dt\right)$$

$$TC_r^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots\ldots, t_{n1}) = (n_1 * A_0) + \sum_{i=1}^{n_1}\left(\int_{t_i}^{t_{i+1}} C_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)}\int_{t_i}^{t_{i+1}}(a+bt)\,dt + I * C_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)}\int_{t_i}^{t_{i+1}} I_{i+1}(t)e^{-rt}\,dt\right)$$

$$TC_r^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots\ldots, t_{n1}) = (n_1 * \mathrm{A}_0) + \sum_{i=1}^{n_1}\left(\int_{t_i}^{t_{i+1}} \mathrm{C}_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}}(a + bt)\, dt + \mathrm{I} * \mathrm{C}_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}} (a + bt_{i+1})\frac{e^{\theta t_{i+1}}}{\theta e^{\theta t}} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\left[1 - \frac{e^{\theta t_{i+1}}}{e^{\theta t}}\right] e^{-rt}\, dt\right) \quad (4)$$

$$TC_r^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots\ldots, t_{n1}) = (n_1 * \mathrm{A}_0) + \sum_{i=1}^{n_1}\left(\int_{t_i}^{t_{i+1}} \mathrm{C}_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}}(a + bt)\, dt + \mathrm{I} * \mathrm{C}_0(1-\alpha)e^{-\frac{h}{2}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}} (a + bt_{i+1})\frac{e^{\theta t_{i+1}}}{\theta e^{\theta t}} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\left[1 - \frac{e^{\theta t_{i+1}}}{e^{\theta t}}\right] e^{-rt}\, dt\right) \quad (5)$$

The total cost of supplier for decentralized case:

$$TC_s^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots\ldots, t_{n1}) = (n_1 * S_s) + \sum_{i=0}^{{n_1}^*-1} C * \left(\frac{((a+b*t_{i+1})*e^{\theta * t_{i+1}})}{\theta * e^{\theta * t_i}} - \frac{(a+b*t_i)}{\theta} + \frac{b}{\theta^2} - \frac{(b*e^{\theta * t_{i+1}})}{\theta^2 * e^{\theta * t_i}}\right) \quad (6)$$

Total quantity represented as:

$$\sum_{i=0}^{n_1-1} Q_{i+1}(t_i) = \sum_{i=0}^{n_1-1} e^{\theta t_{i+1}}\frac{(a+bt_{i+1})}{\theta e^{t_i}} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\left[1 - \frac{e^{\theta t_{i+1}}}{e^{\theta t_i}}\right] \quad (7)$$

One way to determine the root value of ti is to take a partial derivative of Eqn (4) with respect to t_i and then solve for t_i by taking $\frac{\partial TC_r^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots, t_{n1})}{\partial t_i} = 0$. Then, calculation of the minimum total cost of the retailer and supplier, along with the optimal order quantity, is possible once the optimal and distinct values of t_i have been identified.

$$\frac{\partial TC_s^{Dc}(n_1, t_0, t_1, \ldots\ldots\ldots, t_{n1})}{\partial t_i} =$$

$$C_0 * (1-\alpha) * \left(\left(e^{\frac{-h}{2}*(t_{i+1}-t_i)} * \left(-a + b * t_i + \frac{h}{2} * \frac{((t_{i+1}-t_i)*(2*a+b*t_i+b*t_{1+i}))}{2}\right)\right) + e^{\frac{-h*(t_i-t_{i-1})}{2}} * \left(a + b * t_i - \frac{h}{2} * \frac{1}{2}((t_i - t_{i-1}) * (2 * a + b * t_{i-1} + b * t_i))\right)\right) + I * \left(e^{\frac{-h*(t_{i+1}-t_i)}{2}}\left(\frac{(a+b*t_i)*e^{-r*t_i}}{\theta} - \frac{(a+b*t_{i+1})*e^{\theta * t_{i+1}}}{\theta * e^{t_i *(r+\theta)}} + \frac{1}{(\theta)^3} * b * e^{-(1+r)*t_i} * (e^{t_{i+1}*(\theta)} - e^{t_i} * \theta) + \frac{h}{2} * \left(\frac{1}{r^2 * (\theta)^2 * (r+\theta) * e^2} * (e^{-(r)*t_{i+1}} * \theta *\right.\right.\right.$$

$(b*(r+\theta)+(-b*r+a*r*\theta)*e+b*r*\theta*e*t_{1+i})-e^{-(r+\theta)*t_i}*(e^{t_i*(\theta)}*(r+\theta)*(b*\theta+r*e*(-b+a*\theta+b*\theta*t_i))+e^{t_{i+1}*(\theta)}*r^2*e*(b-\theta*(a+b*t_{1+i}))))) +$
$e^{\frac{-h}{2}*(t_i-t_{i-1})}*(\frac{(a+b*t_i)}{r+\theta}*(\frac{e^{t_j}}{e^{t_{j-1}*(r+\theta)}}-\frac{1}{e^{t_i*(r+\theta-1)}})-\frac{h}{2}*(\frac{1}{(\theta)^2}*(\frac{b*(e^{-t_i*(r)}-e^{-t_{i-1}*(r)})*\theta}{r^2*(e)^2}+\frac{1}{r*(r+\theta)*e}*(a*$
$e^{-(t_i+t_{i-1})*(r)}*(-e^{(t_i)*(r)}+e^{(t_{i-1})*(r)})*\theta*(r+\theta)+b*(-e^{t_i*\theta-(r+\theta)*t_{i-1}}*r+e^{-(t_{i-1})*(r)}*$
$(r+\theta)+e^{-(t_i)*(r)}*\theta*(-1+t_i*(r+\theta)))) +$
$\frac{\left((a+b*t_i)*\left(e^{t_i*\theta}\right)^{-(r+\theta)*(t_i+t_{i-1})}*\left(\left(e^{t_i*\theta}\right)^{t_i*(r+\theta)}-\left(e^{t_i*\theta}\right)^{(r+\theta)*t_{i-1}}\right)*\theta\right)}{((r+\theta)*e*t_i*\theta)}-\frac{b*e^{-r*t_{i-1}*\theta*t_{i-1}}}{r*e}))))));(8)$

Centralized approach:

A centralized supply chain is supervised by a centralized entity or decision-making body that oversees and coordinates the activities of both the retailer and supplier. This approach ensures that the decision-making aligns with the overall objectives of the supply chain, resulting in favourable outcomes. The centralized approach facilitates superior coordination and decision-making, which ultimately enhances the efficiency, sustainability, effectiveness, and better utilization of resources in achieving the objectives of the supply chain.

When a retailer and supplier collaborate in a centralized scenario, they can enjoy the benefits of shorter replenishment cycles, resulting in reduced total costs for both parties compared to a decentralized approach. They can achieve these benefits by sharing their savings between the two involved parties.

$$OP_c=\sum_{i=0}^{n_1-1}(T_r^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})*d\%*(t_{i+1}-0.1-t_i))(9)$$

Retailer's cost for in centralized case:

$$T_r^{Ce}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})=\left\{T_r^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})-\frac{OP_c}{2}\right\}(10)$$

Supplier's cost for in the centralized case:

$$T_s^{Ce}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})=\left\{T_s^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})-\frac{OP_c}{2}\right\}(11)$$

Now percentage Improved cost of retailer:

$$=\left\{\frac{T_r^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})-T_r^{Ce}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})}{T_r^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})}\right\}\times 100\ \ (12)$$

Now percentage Improved cost of supplier:

$$=\left\{\frac{T_s^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})-T_s^{Ce}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})}{T_s^{Dc}(n_1,t_0,t_1,\dots\dots\dots,t_{n1})}\right\}\times 100\ \ (13)$$

5. Arithmetical exhibit

Let us consider the parametric values$\alpha = 0.2$, $C_0 = 0.3$, θ = 2.4, a=132, b=5, I = 2, h= 0.3, $A_0 = 20$, $S_s = 85$, r = 0.0, $C = 0.001$ and d = 0.01. Solve this numerical problem with the help of mathematics iterative method by Mathematica software.

Table 1, Table 2, Table 3, Figure 1, Figure 2and Figure 3 provide a detailed analysis of the retailer's replenishment time and optimal overall cost for the values of " $\boldsymbol{a = 132}$ ". For the decentralized case, Optimal total cost values for a=132 is $791.66, respectively, and are achieved at the 6th optimal replenishment cycle for all subsequent cycles after reaching the minimum at n=6, for a=132, respectively, the overall cost gradually increases. In the centralized case, Table 1, Table 2, Table 3 Figure 1, Figure 2and Figure 3, show an analysis of replenishment time and optimal total cost values for a =132, which is $238.41, respectively, and are again achieved at the 5th optimal replenishment cycle.

5.1.The solution to the numerical problem is presented in both tabular and graphical formats.

Decentralized and Centralizes system:

Table 1. The overall expense incurred by the retailer in the decentralized scenario.

↓ a	→n_1		1	2	3	4	5	6	7
132		Decentralized	861.52	1417.80	1641.25	1525.18	1111.	**791.66**	6039.94
		Centralized	588.64	993.04	1058.42	790.66	**238.**4	248.80	3987.56

Table 2 presents the determined values for the most favourable retailer cost, supplier cost, number of replenishment cycles, and replenishment quantity in the decentralized scenario.

↓ a	→ t_i	t_0	t_1	t_2	t_3	t_4	t_5	t_6	Q_{i+1}	T_r^{Dc}	T_s^{Dc}
132		0	1.524	2.2626	2.5299	2.7490	3.0088	**4**	419.814	**791.66**	260.90

Table 3 displays the determined values for the most favourable retailer cost, supplier cost, number of replenishment cycles, and replenishment quantity in both the decentralized and centralized scenarios.

		Decentralized case	Centralized case	
Parameters	n_1			st
132	6			

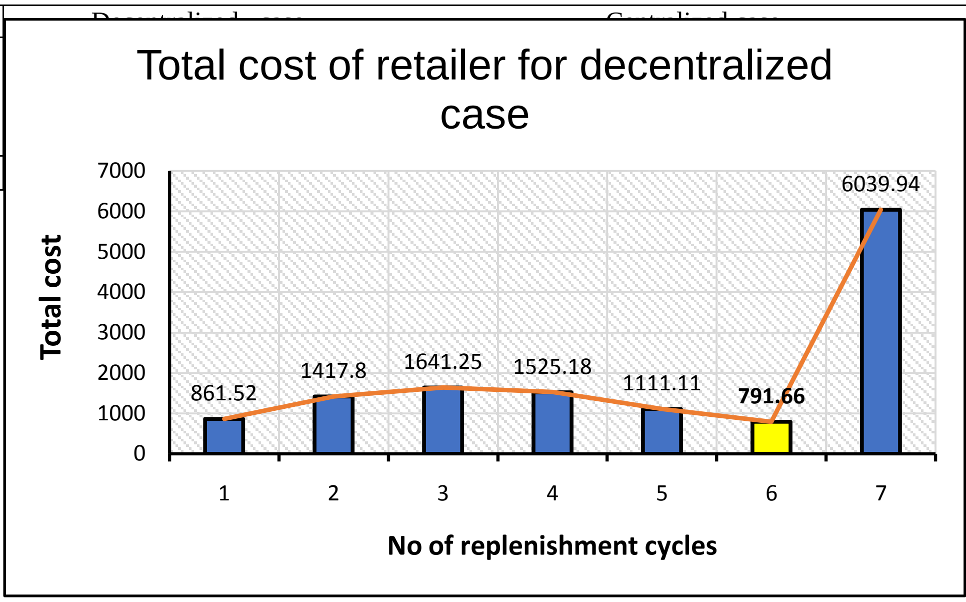

Figure 1. Increased order of replenished time to place an order.

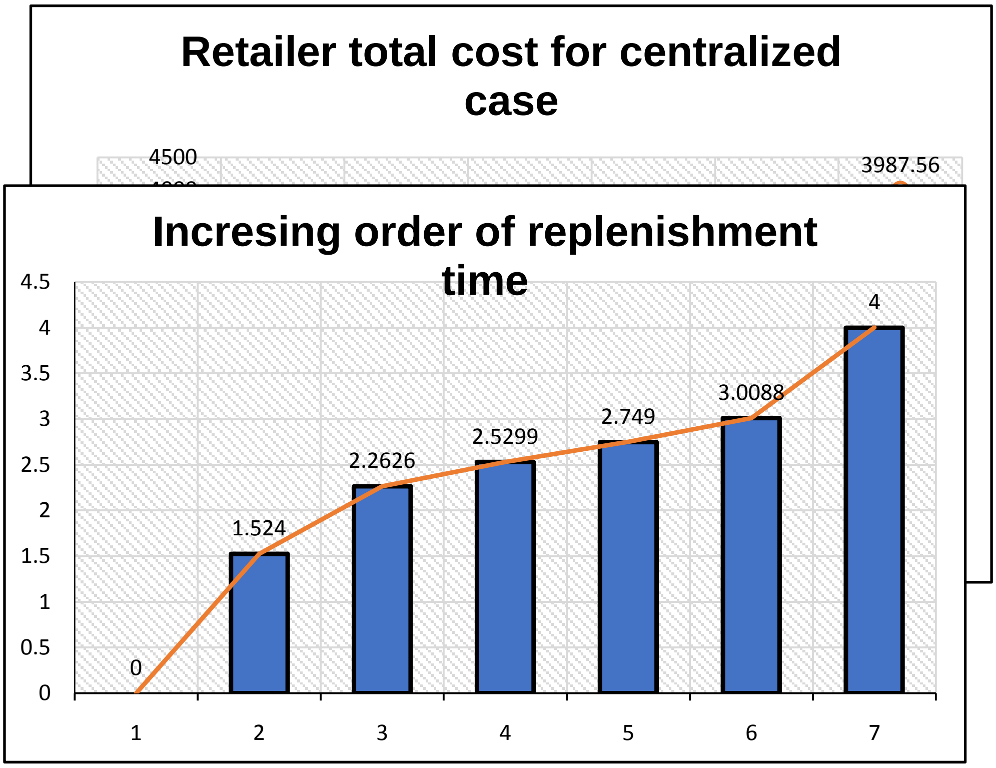

Figure 2. An optimal level of retailer cost in the case of decentralized.

Figure 3. The centralized case shows that the retailer's cost has reached its minimum cost.

6. Executive perception

To make sure that the most effective replenishment plant is being used, it is imperative to routinely check inventory levels. The ideal replenishment pattern is not a one-time fix and should be examined and updated on a regular basis to adjust for any changes in demand, issues with the supply chain, or other important variables. To assure that the optimal replenishment schedule stays current and effective, consider setting in place a continuous improvement approach.

7. Recapitulation

The optimum replenishment schedule was created by the authors of the research using a finite planning horizon model. They were adept at demonstrating the schedule to be optimal by using a convex total cost function and a mathematical analysis of a numerical example. The results of the research could be used in supply chain management to save costs and regulate inventories, among several other.

The proposed work could be improved in the future in a variety of forms. One approach is to include the ideas and research on fuzzy logic, inflation, carbon emissions or sustainability from S.Mishra et al. (2020), V. Singh et al. (2017), and S. Saxena et al. (2020). Another choice is to consider extra factors like credit and demand that are influenced by pricing and inventory. Moreover, Ranu and Nitin Kumar Mishra (2023) might provide useful insight on these subjects.

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