On Cover Edge Pebbling Number of Helm Graph, Crown Graph and Pan Graph

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ABSTRACT

Let G be a connected graph. An edge pebbling move on G is the process of removing two pebbles from one edge and placing one pebble on the adjacent edge. The cover edge pebbling number of G, denoted by $CP_E(G)$ is the minimum number of pebbles required to place a pebble on all the edges of G, however might be the initial configuration is. In this paper, we compute the cover edge pebbling number for Helm graph, Crown graph and Pan graph.

KEYWORDS:

Cover edge pebbling number, Helm graph, Crown graph, Pan graph

1 INTRODUCTION

Lagarias and Saks first suggested the game of pebbling. Later by Chung [1], it was introduced into the literature. Removal of two pebbles from one vertex and placement of one pebble on the adjacent vertex is called pebbling move. Given a connected graph G. The pebbling number P(G) of G is the least number of pebbles needed in a graph G so that we can move a pebble to any arbitrary target vertex by a sequence of pebbling move whatever might be the initial configuration is.

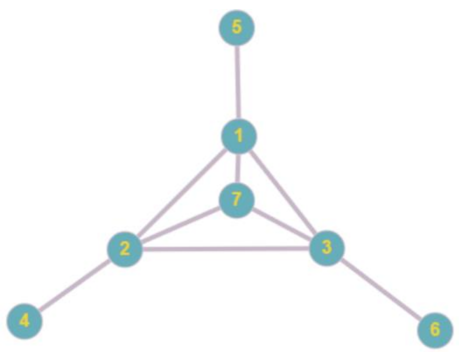
The concept of cover pebbling was first introduced by Crull [2]. The cover pebbling number CP(G) is the least number of pebbles needed in a graph G so that we can move one pebble to all the vertices of the graph G.

In [6] a new concept namely edge pebbling number and cover edge pebbling number has been introduced and cover edge pebbling number for certain standard graphs namely path, complete graph, friendship graph and star graph have been determined.

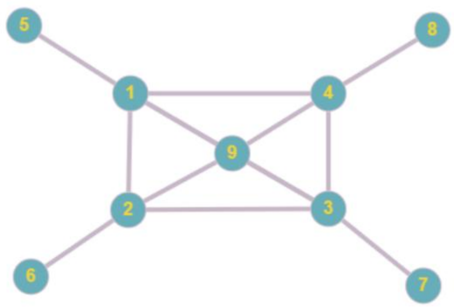
In edge pebbling, pebbles will be distributed on the edges of the graph instead of the vertices. An edge pebbling move is the process of removing two pebbles from one edge and placing one pebble on the adjacent edge. Edge pebbling number $P_E(G)$ is the minimum number of pebbles needed in a graph G to reach any arbitrary target edge by a sequence of edge pebbling move regardless of initial configuration of pebbles. The cover edge pebbling number $CP_E(G)$ is the least number of pebbles needed in a graph G so that we can move one pebble to all the edges of the graph G. In this paper we establish the cover edge pebbling number for Helm graph, Crown graph and Pan graph and the theorems are proved by applying the recommendations given in [3].

DEFINITION 1.1: The Helm graph H_n is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

Example:



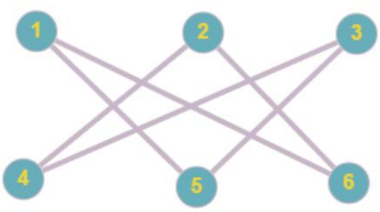
H_3



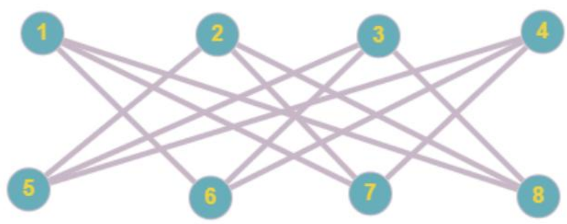
H_4

DEFINITION 1.2: The crown graph $H_{n,n}$ is the graph obtained from an complete bipartitite graph $K_{n,n}$ with the removal of the edges, in the perfect matching of $K_{n,n}$.

Example:



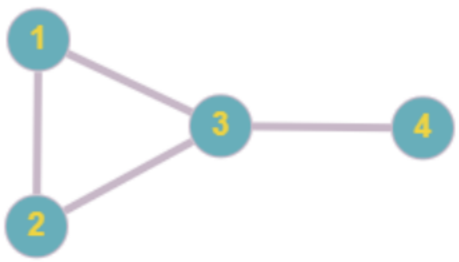
$H_{3,\,3}$



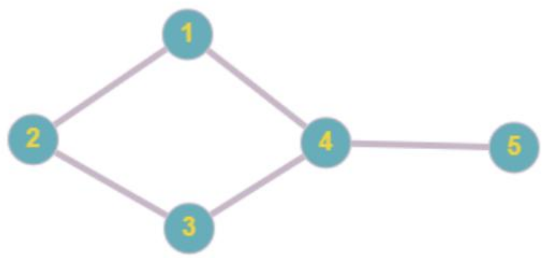
$H_{4,4}$

DEFINITION 1.3: The n-pan graph $P_{n,1}$ is the graph obtained by joining a cycle C_n to a singleton graph K_1 with a bridge.

Example:



$P_{3,1}$



$P_{4,1}$

2 COVER EDGE PEBBLING NUMBER

DEFINITION 2.1 [6] A cover edge pebbling number $CP_E(G)$ of a graph G is defined as, however the pebbles are initially placed in the edges, the minimum number of pebbles required to place a pebble in all the edges.

DEFINITION 2.2 [6] The distance between two edges x and y is defined as, $d(x,y)=d(v_i, v_j)-2$ where $x=v_iv_{i+1}$, $y=v_{j-1}v_j$ and $d(v_i, v_j)$ is the length of the shortest path between v_i and v_j

DEFINITION 2.3 [6]The distance d(x) of an edge x in a graph G is the sum of the distances from x to each other edge of E(G), where E(G) is the edge set of G.

(i.e.) $d(x)=\sum_{x\in E(G)} d(y,x) \forall y\in E(G), x\neq y$

DEFINITION 2.4[6]Let $x\in E(G)$, then x is called a key edge if d(x) is a maximum.

THEOREM 2.5:

The cover edge pebbling number of the Helm graph H_3 is 27.

i.e.,$CP_E(H_3) = 27$.

Proof:

In this graph pendant edges are key edges.

Choose any one of the key edges from H_3.

Let it be e^*. Each of the key edges are adjacent with 3 edges.

To place one pebble on each of those 3 edges, 3.2=6 pebbles are to be placed on e^*. To pebble the five remaining edges,on finding the shortest path exactly one edge is to be crossed.

∴ Minimum number of pebbles to be placed one^* to place one pebble on those 5 edges is $2^2.5$ pebbles.

Altogether 6+ $2^2.5$ pebbles are required for the above considered cases.

After covering all the edges with one pebble, a pebble has to be placed on e^* to cover e^*.

∴ The cover edge pebbling of H_3 =6+2^2.5 + 1 = 27

THEOREM 2.6:

The cover edge pebbling number of the Helm graphH_n is 20n – 37, n>3.

i.e., $\boldsymbol{CP_E(H_n)} = 20n - 37$, n > 3.

Proof

In this graph pendant edges are key edges.Choose any one of the key edges from H_n.

Let it be e^*. e^* is exactly adjacent with 3 edges. Among those edges, two edges are on the cycle C_n and the remaining one the edge incident with the universal vertex.

To place one pebble on each of those 3 edges, 3.2=6 pebbles are needed on e^*.

Now, the n – 2 edges on the cycleC_n, n – 1 pendant edges and n – 1 edges that are incident with the universal vertex are to be pebbled.

Among those n – 2 edges from the cycleC_n, 2 edges can be reached by crossing exactly one edge and remaining n – 4 edges can be reached by crossing 2 edges.

∴ $2.2^2 + (n-4)\, 2^3$ pebbles are to be placed on e*.

Among those n – 1 pendant edges, two edges can be reached by crossing exactly one edge and the remaining n – 3 edges can be reached by crossing 2 edges.

∴ $2.2^2 + (n-3)\, 2^3$pebbles are to be placed on e*.

All n – 1 edges which are incident with the universal vertex can be reached by crossing exactly one edge.

∴ 2^2. (n – 1) pebbles are needed on e^*.

Altogether $6 + 2.2^2 + (n-4)\, 2^3 + 2.2^2 + (n-3)\, 2^3 + 2^2.\, (n-1)$ pebbles are needed for the above considered cases.

After covering all the edges with one pebble, a pebble has to be placed on $e^*$to cover e^*.

∴ The cover edge pebbling $\boldsymbol{CP_E(H_n)} = 6 + 2.2^2 + (n-4)\, 2^3 + 2.2^2 + (n-3)\, 2^3 + 2^2.\, (n-1)$

$$= 6 + 2(4) + (n-4)\, 8 + 2(4) + (n-3)\, 8 + 4\, (n-1) + 1$$

= 6 + 8 + 8n – 32 + 8 + 8n – 24 + 4n – 4 + 1

= 23– 60 + 20n

= 20n – 37

∴The cover edge pebbling $CP_E(H_n)$= 20n – 37, n > 3

THEOREM2.7: Thecoveredgepebblingnumberforcrown graph $H_{3,3}$ is 21. i.e., $CP_E(H_{3,3}) = 21$.

Proof:

In crown graph $H_{3,3}$, all the edges are key edges.

Choose any one of the edgesfrom$H_{3,3}$. Let it be e^*.

e^* is exactly adjacent with 2 edges. To place one pebble on each of these edges, 2. 2 = 4 pebbles are needed on e^*.

And exactly one edge say e' is at the distance two from e*. To reach those edges from e* two edges are to be crossed.

∴ to pebble e', 2^3 pebbles are needed on e*.

Remaining 2 edges which are at distance one from e* are to be pebbled. To reach those edges from e* exactly one edge is to be crossed.

∴ 2^2.2 pebblesare to be placed on e*.

After covering all the edges with one pebble, a pebble has to be placed on e^* to cover e^*.

∴ The cover edge pebbling $CP_E(H_{3,3})$ = 2. 2 + 2^2.2+ 2^3 + 1

= 21

∴ The cover edge pebbling $CP_E(H_{3,3})$ = 21.

THEOREM2.8: Thecoveredgepebblingnumberforcrown graph$H_{n,n}$is 4n(n – 2)+ 9i.e., $CP_E(H_{n,n})$ =4n(n – 2) + 9wheren≥ 3.

Proof:

In crown graph, all the edges are key edges.

Choose any one of the edgesfrom$H_{n,n}$. Let it be e^*.

e^* is exactly adjacent with 2n – 4 edges. To place one pebble on each of these edges, 2. 2n – 4 pebbles are needed on e^*.

And exactly one edge say e' is at the distance two from e*. To pebble e', 2^3 pebbles are needed on e*.

Remaining n^2– 3n + 2 edges are to be pebbled. To reach those edges from e* exactly one edge is to be crossed.

∴ we need 2^2 (n^2– 3n + 2) pebbles to be placed on e*.

After covering all the edges with one pebble, a pebble has to be placed on e^* to cover e^*.

∴ The cover edge pebbling $\boldsymbol{CP_E(H_{n,n})}$ = 2. (2n – 4) + 2^2.(n^2– 3n + 2)+ 2^3 + 1

= 4n (n – 2) + 9

∴ The cover edge pebbling $\boldsymbol{CP_E(H_{n,n})}$ = 4n (n – 2) + 9, n≥ 3.

THEOREM2.9: Thecoveredgepebblingnumberfor pan graph $\boldsymbol{P_{2n+1,1}}$ is $3(2^{n+1}) - 3$. $\boldsymbol{CP_E(P_{2n+1,1})} = 3(2^{n+1}) - 3$where n ≥ 1.

Proof:

In this graph pendant edge and the edge which is on the cycle C_{2n+1} and at maximum distance from the pendant edge are key edges.Choose any one of those key edges.

Let it be e^*. e^* is exactly adjacent with 2 edges on C_{2n+1}.

To place one pebble on each of these 2 edges, 2.2= 4 pebbles are needed on e^*.

Then exactly two edges from the cycleC_{2n+1}, are at distance 1 from e*. To pebble those edges 2.2^2 = 8 pebbles are needed on e*. Also, exactly two edges from the cycle C_{2n+1}, are at distance 2 from e*.To pebble those edges 2.2^3 =16 pebbles are needed on e*. By the property of cycle, and proceeding as above exactly two edges from the cycle C_{2n+1}, are at distance n – 1 from e*.To pebble those edges 2. 2^n= 2^{n+1} pebbles are needed on e*. Remaining one edge is at distance n from e* and to pebble that edge 2^{n+1} pebbles are needed on e*.

Altogether we need $2^2 + 2(2^2) + 2(2^3) +\ldots+2.(2^n) + 2^{n+1}$ pebbles for the above considered cases.

After covering all the edges with one pebble, a pebble has to be placed on e^* to cover e^*.

∴ The cover edge pebbling$\boldsymbol{CP_E(P_{2n+1,1})}$ = $2^2 + 2(2^2) + 2(2^3) +\ldots+ 2(2^n) + 2^{n+1} + 1$

$= 2^{n+1} + 1 + 2(2^n + 2^{n-1} + \cdots + 2)$

$= 2^{n+1} + 1 + 2(2(2^n - 1))$

$= 3(2^{n+1}) - 3$

∴ The cover edge pebbling $\boldsymbol{CP_E(P_{2n+1,1})}$=$3(2^{n+1}) - 3$, n≥ 1.

THEOREM2.10: Thecoveredgepebblingnumberfor pan graph $\boldsymbol{P_{2n,1}}$ is$2^{n+2} - 3$.

i.e., $\boldsymbol{CP_E(P_{2n,1})} = 2^{n+2} - 3$ where n ≥ 2.

Proof:

In this graph pendant edge and the two edges which is on the cycle C_{2n+1} and at maximum distance from the pendant edge are key edges.Choose any one of those key edges.

Let it be e^*. e^* is exactly adjacent with 2 edges on C_{2n}.

To place one pebble on each of these 2 edges, 2.2= 4 pebbles are needed on e^*.

Then exactly two edges from the cycle C_{2n}, are at distance 1 from e*. To pebble those edges $2.2^2 = 8$ pebbles are needed on e*. Also, exactly two edges from the cycle C_{2n+1}, are at distance 2 from e*.To pebble those edges $2.2^3 = 16$ pebbles are needed on e*. By the property of cycle, and proceeding as above exactly two edges from the cycle C_{2n+1}, are at distance n – 1 from e*.To pebble those edges $2.\ 2^n = 2^{n+1}$ pebbles are needed on e*.

Altogether we need $2^2 + 2(2^2) + 2(2^3) +\ldots+ 2(2^n) + 2(2^n)$ pebbles for the above considered cases.

After covering all the edges with one pebble, a pebble has to be placed on e^* to cover e^*.

∴ The cover edge pebbling $\boldsymbol{CP_E(P_{2n,1})} = 2^2 + 2(2^2) + 2(2^3) +\ldots+ 2(2^n) + 2(2^n) + 1$

$$= 2(2^n + 2^{n-1} + \cdots + 2) + 1$$

$$= 2(2(2^n - 1)) + 1$$

$$= 2^{n+2} - 3$$

∴ The cover edge pebbling $\boldsymbol{CP_E(P_{2n,1})} = 2^{n+2} - 3$ where $n \geq 2$.

REFERENCES

[1] F.R.K. Chung, Pebbling in Hypercubes, SIAM J. Disc. Math 2 (1989).

[2] B. Crull, T. Cundif, P. Feltman, G. Hurlbert, L. Pudwell, Z. Szaniszlo, Z. Tuza, The cover pebbling number of graphs, Discrete Math. 296, (2005), 15-23.

[3] Michael E. Subido and Imelda S. Aniversario, The Cover Pebbling Number of the Join of Some Graphs, Applied MathematicalSciences, http://dx.doi.org/10.12988/ams.2014.45377, (2014). Vol. 8, no. 86, (2014), 4275 – 4283.

[4] G. Hulbert, A Survey of Graph Pebbling, CongressusNumerantium 139 (1999), 41-64.

[5]D.A. Mojdeh and A.N. Ghameshlou,Domination in Jahangir graph $J_{2,m}$.Int. J. Contemp. Math. Sciences, 2(24) (2007), 1193– 1199.

[6] A. Priscilla Paul, On Edge Pebbling number and Cover Edge Pebbling number of some graphs, Journal of Information and Computational Science, http://www.joics.org/gallery/ics-3730.pdf, volume 10 issue 6, (2020), 337-344.