Neighbourhood Connected and Disconnected Domination in Circular-Arc Graphs using Algorithms

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Abstract: A dominating set S of a connected graph G is called a neighborhood connected dominating set (ncd-set) if the induced sub-graph $\langle N(S)\rangle$ is connected. If the induced sub-graph $\langle N(S)\rangle$ is disconnected, then S is called neighbourhood disconnected dominating set.

In this paper we develop algorithms to find neighborhood connected dominating set and neighborhood disconnected dominating set for circular-arc graphs.

Key words: dominating set, neighborhood set, connected dominating set, disconnected dominating set, neighborhood connected dominating set

1. Introduction

For a dominating set S of G it is natural to look at how $N(S)$ behaves. For example, for the cycle $C_6 = (v_1, v_2, v_3, v_4, v_5, v_6, v_1)$, $S_1 = \{v_1, v_4\}$ and $S_2 = \{v_1, v_2, v_4\}$ are dominating sets, $\langle N(S_1)\rangle$ is not connected and $\langle N(S_2)\rangle$ is connected. Motivated by this example, Arumugam[1] introduced the concept of neighborhood connected domination and initiated a study of the corresponding parameter. A dominating set S of a connected graph G is called a neighbourhood connected dominating set (ncd-set) if the induced sub-graph $\langle N(S)\rangle$ is connected. The minimum cardinality of a ncd-set of G is called the neighbourhood connected domination number of G and is denoted by $\gamma_{\mathrm{nc}}(G)$. And this concept motivated us to initiate the study of neighbourhood disconnected domination.

A dominating set S of a connected graph G is called a neighborhood disconnected dominating set (ndcd-set) if the induced sub-graph $\langle N(S)\rangle$ is disconnected. The minimum cardinality of a ndcd-set of G is called the neighborhood disconnected domination number of G and is denoted by $\gamma_{\mathrm{ndc}}(G)$.

Very few types of disconnected domination have been defined and studied by the authors, S. Balamurugan related the parameter of disconnected domination parameter with others[3]. C.Y Ponnappan studied the concept of perfect disconnected domination in fuzzy graphs, where A dominating set D of a fuzzy graph $G = (\sigma, \mu)$ is a $D_{pd}(G)$ of V is said to be a perfect disconnected dominating set if $D_{pd}(G)$ is perfect and $\langle D_{pd}(G)\rangle$ is disconnected[4]. P. Nataraj, R. Sundareswaran[2] introduced Complementary Equitably Totally Disconnected Equitable domination in graphs and obtain some interesting results. Also, they discussed some bounds of this new domination parameter.

In this paper we develop algorithms to find neighborhood connected dominating set and neighborhood disconnected dominating set for circular-arc graphs. Here, we are using circular-arc graphs which are having vertex degree not more than 5.

Notations

- NCD - neighbourhood connected dominating set.
- $\gamma_{nc}\ (G)$ - neighbourhood connected domination number.
- $NDCD$ - neighbourhood disconnected dominating set.
- $\gamma_{ndc}\ (G)$ - neighbourhood disconnected domination number.
- $A = \{c_1, c_2, c_3, \dots, c_n\}$ - circular-arc family.
- $nrd[x]$ - collection of all neighbours of x.
- $nrd^{-}[x]$ - collection of all succeeding neighbours of x.
- $nrd^{+}\ [x]$ - collection of all preceding neighbours of x.
- $NI(d)$ - first non-intersecting arc of d.

2. Algorithm for neighbourhood connected domination in circular-arc graphs

Algorithm 1: Algorithm to find a neighborhood connected dominating set for a Circular-Arc family

Input: $A = \{c_1, c_2, c_3, \dots, c_n\}$
Output: NCD is the required neighborhood connected dominating set for the given Circular-Arc family.

1. NCD = { }
2. $x = c_1$
3. $S = nrd[x]$
4. $S_1 = \{y \in S / y$ is intersect to all other arcs in $S\}$
5. $a = \max\ (S_1)$
6. If a is the right intersecting arc to x and has no left and right arcs, which are intersecting to each other then
7. $\quad S_1 = S_1 - \{a\}$
8. $\quad a = \max\ (S_1)$
9. $NCD = NCD \cup \{a\}$
10. If there exists a pendent arc $c_p \in N(a)$ then
11. $\quad NCD = NCD \cup \{c_p\}$
12. $d = \max\ (NCD)$
13. $x = NI(d)$
14. $S_{it} = nrd^{-}[x]$
15. $S_{it_1} = \{y \in S_{it}\ /\ y$ is intersect to all other arcs in $S_{it}\}$
16. $S_2 = \{z \in S_{it_1}/z$ having left and right arcs which are intersecting each other$\}$
17. $a = \max\ (S_2)$
18. $NCD = NCD \cup \{c_p\}$
19. If there exists a pendent arc $c_p \in N(a)$ then
20. $\quad NCD = NCD \cup \{c_p\}$
21. $d = \max\ (NCD)$
22. If $x = NI(d) \notin nrd[NCD]$ then
23. go to step 13
24. Else
25. end

Note: If $S_2 = \emptyset$ then, neighbourhood connected dominating set does not exists for the ciercular-arc graph. Because it leads to a disconnected induced subgraph.

Illustration 1:

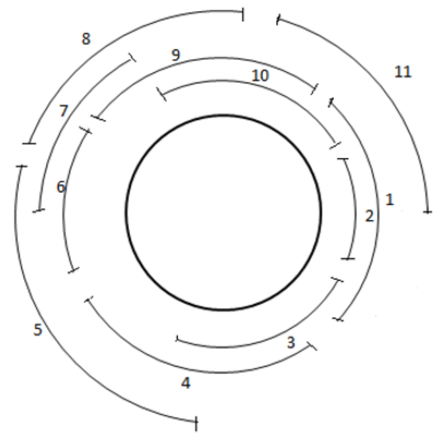

Figure 1: circular-arc family

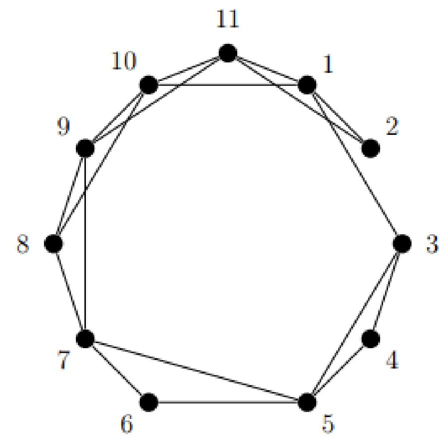

Figure 2: circular-arc graph

Input: $A = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$

1. $NCD = \{\}$
2. $x = 1$
3. $S = nrd[1] = \{1, 2, 3, 10, 11\}$
4. $S_1 = \{1\}$
5. $a = max(\{1\}) = 1$
6. a is not the right intersecting arc to x
9. $NCD = \{\} \cup \{1\} = \{1\}$
10. there exists no pendent arc $c_p \in N(a)$
12. $d = max(\{1\}) = 1$

13. $x = NI(1) = 4$
14. $S_{it} = nrd^{-}[4] = \{4, 5\}$
15. $S_{it_1} = \{4, 5\}$
16. $S_2 = \{4\}$
17. $a = max(\{4\}) = 4$
18. $NCD = \{1\} \cup \{4\} = \{1, 4\}$
19. there exists no pendent arc $c_p \in N(a)$
21. $d = max(\{1, 4\}) = 4$
22. $x = NI(4) = 6 \notin nrd[NCD]$
23. go to step 13

13. $x = NI(4) = 6$
14. $S_{it} = nrd^{-}[6] = \{6, 7\}$
15. $S_{it_1} = \{6, 7\}$
16. $S_2 = \{6\}$
17. $a = max(\{6\}) = 6$
18. $NCD = \{1, 4\} \cup \{6\} = \{1, 4, 6\}$
19. there exists no pendent arc $c_p \in N(a)$
21. $d = max(\{1, 4, 6\}) = 6$

22. $x = NI(6) = 8 \notin nrd[NCD]$
23. go to step 13

13. $x = NI(6) = 8$
14. $S_{it} = nrd^{-}[8] = \{8, 9, 10\}$
15. $S_{it_1} = \{8, 9, 10\}$
16. $S_2 = \{8, 9, 10\}$
17. $a = max(\{8, 9, 10\}) = 10$
18. $NCD = \{1, 4, 6\} \cup \{10\} = \{1, 4, 6, 10\}$
19. there exists no pendent arc $c_p \in N(a)$
21. $d = max(\{1, 4, 6, 10\}) = 10$
22. $x = NI(10) = 2 \in nrd[1]$
25. end

Output: $NCD = \{1, 4, 6, 10\}$ is the required neighbourhood connected dominating set for the given circular-arc family, figure 1.

Manually: Let $S = \{1, 4, 6, 10\}$ be a dominating set for the circular-arc graph figure 2. The induced sub-graph, of $N(S)$, i.e., $\langle N(S) \rangle$ is given by,

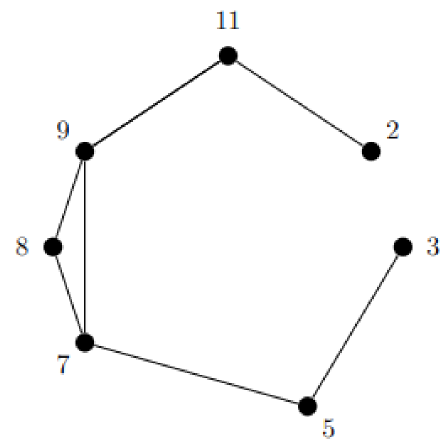

Figure 3: Induced sub-graph

The above induced sub-graph, figure 3 is connected.
Therefore, the dominating set $S = \{1, 4, 6, 10\}$ is a neighbourhood connected dominating set for the circular-arc family, figure 1.

3. Algorithms for neighbourhood disconnected domination in circular-arc graphs

Let the set $D = \{x/x$ has no left and right arcs which are intersecting each other$\}$. On the basis of $|D|$, the algorithm follows the cases are;

1. $|D| = 0$
2. $|D| = 1$
3. $|D| = 2$
4. $|D| > 2$

Case 1: $|D| = 0$

Algorithm 2: Algorithm to find a Neighbourhood Disconnected Dominating Set for Circular-Arc Family of Graphs when $|D| = 0$

Input: $A = \{c_1, c_2, c_3, \dots, c_n\}$
Output: $NDCD$ is the required neighborhood disconnected dominating set for the given Circular-Arc family

1. $NDCD = \{\ \}$
2. $count = 0$
3. $x = c_1$
4. $S = nrd[x]$
5. $S_1 = \{y/y$ is intersecting all other arcs in $S\}$
6. $b = \max(S_1)$
7. $a = nrd^{-1}(b)$
8. $NDCD = NDCD \cup \{a, b\}$
9. $count = count + 1$
10. $d = max(NDCD)$
11. $x = NI(d) \neq \emptyset \notin nrd[NDCD]$
12. If $count \geq 2$ then
13. $\quad S = nrd^{-}[x]$ 1
14. $\quad S_1 = \{y/y$ is intersecting all other arcs in $S\}$
15. $\quad a = max(S_1)$
16. $\quad NDCD = NDCD \cup \{a\}$
17. else
18. $\quad$ go to step 4
19. end

Case 2: $|D| = 1$

Algorithm 3: Algorithm to find a Neighbourhood Disconnected Dominating Set for Circular-Arc Family of Graphs when $|D| = 1$

Input: $A = \{c_1, c_2, c_3, \dots, c_n\}$
Output: $NDCD$ is the required neighborhood disconnected dominating set for the given Circular-Arc family

1. $NDCD = \{\ \}$
2. $count = 1$
3. $x = c_1$
4. $S = nrd[x]$
5. $S_1 = \{y/y$ is intersecting all other arcs in $S\}$
6. $b = \max(S_1)$
7. $a = nrd^{-1}(b)$
8. $NDCD = NDCD \cup \{a, b\}$
9. $count = count + 1$
10. $x = NI(a)$
11. $S = nrd^{-}[x]$
12. If $nrd^{-}[x] \subseteq nrd[D]$ then
13. $\quad x = NI(D) \notin nrd[NDCD]$
14. $\quad S = nrd^{-}[x]$
15. $S_1 = \{y/y$ is intersecting all other arcs in $S\}$
16. $a = max(S_1)$
17. $NDCD = NDCD \cup \{a\}$
18. If $x = NI(a) \notin nrd[NDCD]$ then

19. go to step 11
20. else
21. end

Case 3: $|D| = 2$

Algorithm 4: Algorithm to find a Neighbourhood Disconnected Dominating Set for Circular-Arc Family of Graphs when $|D| = 2$

Input: $A = \{c_1, c_2, c_3, \dots, c_n\}$

Output: $NDCD$ is the required neighborhood disconnected dominating set for the given Circular-Arc family

1. $NDCD = \{\,\}$
2. $count = 2$
3. $x = c_1$
4. $S = nrd^{-}[x]$
5. If $nrd^{-}[x] \subseteq nrd[D(i)]$ then
6. $x = NI(D(i))$
7. $S = nrd^{-}[x]$
8. $S_1 = \{y/y$ is intersecting all other arcs in $S\}$
9. $a = max(S_1)$
10. $NDCD = NDCD \cup \{a\}$
11. If $x = NI(a) \notin nrd[NDCD]$ then
12. go to step 4
13. else
14. end

Illustration for Case 2:

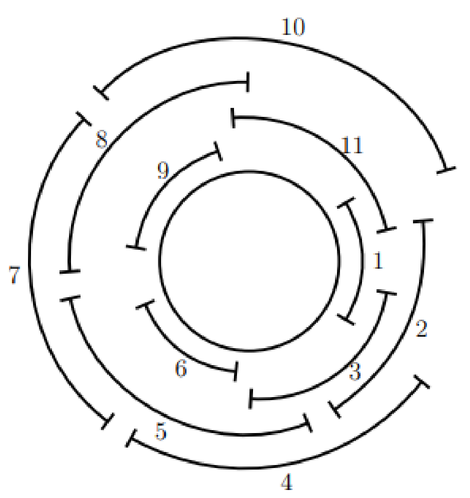

Figure 4: circular-arc family

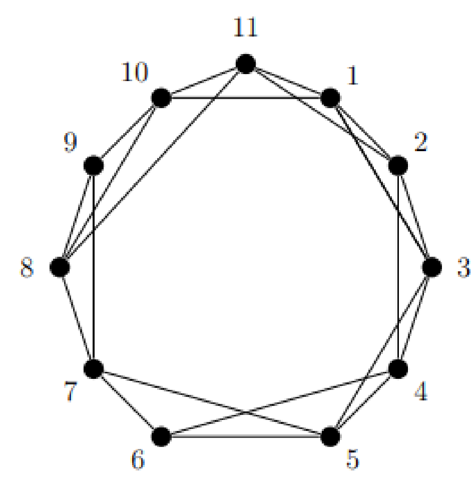

Figure 5: circular-arc graph

For the above graph figure 5,
$D = \{7\} \Rightarrow |D| = 1$
Hence, we proceed with case 2.

Input: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

1. $NDCD = \{7\}$
2. $count = 1$
3. $x = 1$
4. $S = nrd[1] = \{1, 2, 3, 10, 11\}$
5. $S_1 = \{1\}$
6. $b = max(\{1\}) = 1$
7. $a = nrd^{-}(1) = 2$
8. $NDCD = \{7\} \cup \{1, 2\} = \{1, 2, 7\}$
9. $count = 1 + 1 = 2$

10. $x = NI(2) = 5$
11. $S = nrd - [5] = \{5, 6, 7\}$
12. $nrd - [5] \subseteq nrd[7]$
13. $x = NI(7) = 10 \in nrd[1]$
21. end

Output: $NDCD = \{1, 2, 7\}$ is the required neighbourhood disconnected dominating set for the given circular arc family, figure 4.

Manually: Let $S = \{1, 2, 7\}$ be a dominating set for the circular-arc graph figure 5. The induced sub-graph of $N(S)$, i.e., $\langle N(S) \rangle$ is given by,

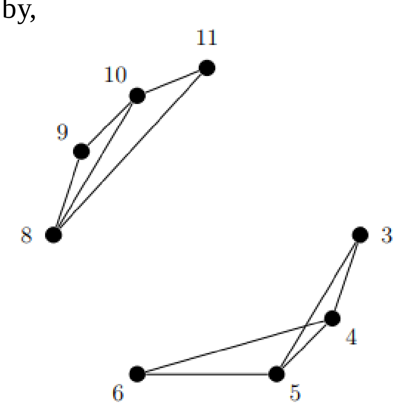

Figure 6: Induced sub-graph

The above induced sub-graph figure 6 is disconnected.
Therefore, the dominating set $S = \{1, 2, 7\}$ is a neighbourhood disconnected dominating set for the circular-arc family, figure 4.

Illustration for Case 3:

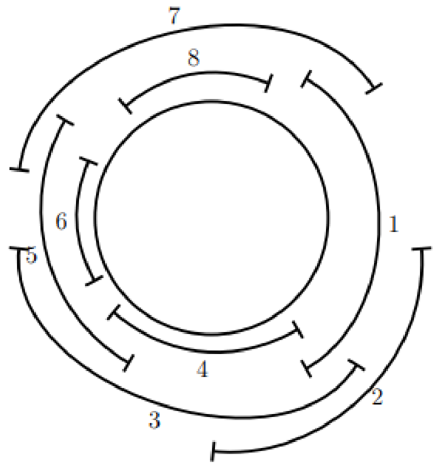

Figure 7: circular-arc family

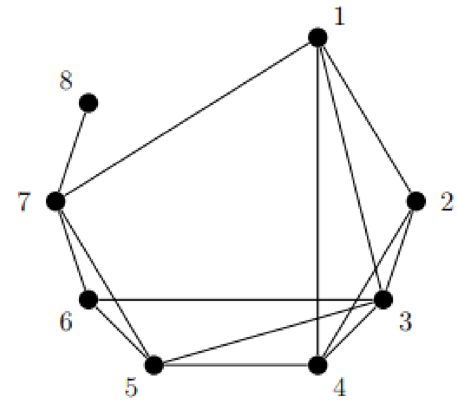

Figure 8: circular-arc graph

For the above graph figure 8,
$D = \{1,7\} \Rightarrow |D| = 2$
Hence, we proceed with case 3.

Input: $A = \{1,2,3,4,5,6,7,8\}$
1. $NDCD = \{1,7\}$
2. $count = 2$
3. $x = 1$
4. $S = nrd^{-}[1] = \{1,2,3,4\}$
5. $nrd^{-}[1] \subseteq nrd[D(0)] = nrd[1]$
6. $x = NI(1) = 5$
7. $S = nrd^{-}[5] = \{5,6,7\}$
8. $S_1 = \{5,6,7\}$
9. $a = max(\{5,6,7\}) = 7$
10. $NDCD = \{1,7\} \cup \{7\} = \{1,7\}$
11. $x = NI(7) = 2 \in nrd[1]$
14. end

Output: $NDCD = \{1,7\}$ is the required neighbourhood disconnected dominating set for the circular-arc family, figure 7.

Manually: Let $S = \{1,7\}$ be a dominating set for the circular-arc graph figure 8. The induced sub-graph of $N(S)$ i.e., $\langle N(S)\rangle$ is given by,

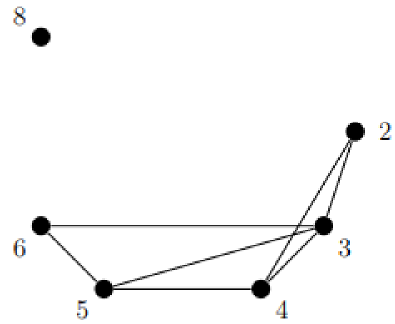

Figure 9: Induced sub-graph

The above induced sub-graph figure 9 is disconnected.
Therefore, the dominating set $S = \{1, 7\}$ is a neighbourhood disconnected dominating set for the circular-arc family, figure 7.

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