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TOTAL COLORING OF INTUITIONISTIC FUZZY FACE ANTI-MAGIC LABELING OF WHEEL GRAPH

L. Naveen [1], N. Uma [2]

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Abstract

In recent times, the practice of involving the concept of fuzzy in graph theory has led to various helpful findings. More precisely, large numbers of real-life problems have been modelled into graph with the involvement of fuzzy concepts. In this paper, some planar graphs like Wheel Graph, Ladder Graph, And Circular Graph are studied with the idea of Fuzzy Anti-Magic Graphs. Moreover, total coloring and an algorithm is discussed to build an Intuitionistic Fuzzy Face Anti-Magic Planar Graph.

Keywords: *Wheel Graph, Total Colouring, Anti-Magic Graphs, Fuzzy Face Anti-Magic Labeling, Intuitionistic Fuzzy Planar Graph.*

[1] Assistant Professor, Department of Mathematics, Dr.SNS Rajalakshmi College of Arts and Science, Coimbatore, Tamil Nadu, India.

[2] Associate Professor, Department of Mathematics, Sri Ramakrishna College of Arts and Science, Coimbatore, Tamil Nadu, India.

[1] lnaveen24@gmail.com, [2] umasnr@gmail.com

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1. Introduction

Earlier around 1960's Lotfi A. Zadeh [1], invented a new concept called Fuzzy Sets. A Fuzzy Set is a generalization of a crisp set, with the elements of membership degrees having upper bound of 1 and lower bound 0. In the fuzzy set, each object's membership grade has a single value, range of [0, 1]. The Fuzzy Sets have been applied to sort out the countless real-life difficulties, which is unpredictable, incomplete, imprecise and uncertain.

An ordered pair of a set of vertices V and a set of edges E is known as a crisp graph. Graph theory has many applications in solving various problems of several domains, including networking, communication, data mining, clustering, image capturing, image segmentation, planning, and scheduling. However, in some situations, certain aspects of a graph-theoretical system may be uncertain. Additionally, the cardinality of the set of vertices and the set of edges is called the order and size of the graph. A bijection f in a crisp graph $G = (V, E)$ mapping from $V \cup E$ to N which assigns a completely unique natural number to each vertex and/or edge is known as a Labeling.

Later, the basic format of the Fuzzy Graph and some of its properties were expanded by Asriel Rosenfield [2], and ideology led to the development in several fields with long-established mathematical models, especially in the fields of scientific modeling, telecommunications and so on. The notion of connectivity in Fuzzy Graphs began with R.T. Yeh and S.Y. Bang [3] launched a new type of graph in 1964 as Magic Graph, when the graph has an edge-labeling, internal set of real numbers, such that the sum of an edge's labels and their two end points is equal to a constant. If the sum of the labels related to the vertex is consistent, the impartial of the selection of vertex is called vertex-magic. Predominantly, there is a saying that "Every connected graph other than Magic Graphs are Anti-Magic Graphs". Hence, it is necessary to know about the Magic Graphs to understand the Anti-Magic Graphs better.

2. Preliminaries

Planar Graph

In graph theory, a graph can be embedded in the plane, so that no edges cross each other. Such a graph is called a **planar graph** or planar embedding.

Wheel Graph

A **Wheel graph** is formed by connecting a single vertex to all the vertices of a cycle. For $n \geq 4$ the wheel W_n is defined to be the graph $K_1 + C_{n-1}$. It has $n + 1$ vertices, $2n$ edges and $n + 1$ faces. Here, as an example Wheel graph W_5 is given below **(Figure 2.1).**

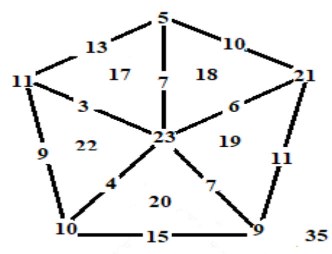


Figure 2.1: Wheel Graph W_5

Fuzzy Graph

A **fuzzy graph** $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v, f \in V$, we have $\mu(u, v, f) \leq \sigma(u) \wedge \sigma(v) \wedge \sigma(f)$.

Fuzzy Wheel Graph

A wheel graph is said to be a **fuzzy wheel graph** in which all the vertices, edges and face has membership function and the graph K_1 with a label U attaching all the vertices v_j of P_n through the edges e_j along the face f_j such the $\mu(U, v_i, f_i) > 0$ and $\mu(v_i, v_{i+1}, f_i) < 1$ and $|e_i| > 1$, where $1 \leq j \leq n$.**(Figure 2.2)**

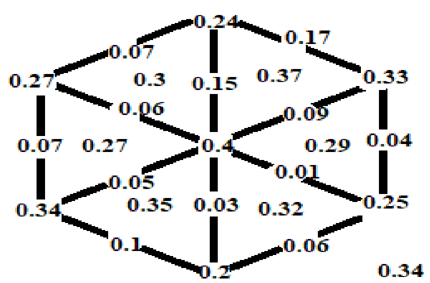


Figure 2.2: Fuzzy Wheel Graph W_6

Coloring Function

The graph $G = (V, E)$ is a crisp graph, a **coloring function** is a mapping C: V$(G) \rightarrow N$(where N is set of positive integers) such that $C(u) \neq C(v)$ if u and v are adjacent in G.

The graph $G = (V, E)$ is a crisp graph, a **k-coloring function** is a mapping $C^k: V(G) \rightarrow \{1,2, \dots, k\}$ such that $C^k(u) \neq C^k(v)$ if u and v are adjacent in G. A graph G is k-colorable if it admits k-coloring.

The **chromatic number** (G), of a graph G is the minimum k for whichG is k-colorable.

Total Coloring of a Planar Graph

Graph coloring is to assign a color to elements (vertex, edge, and face) of graph such that two adjacent elements have different color.

- **(i) Vertex coloring** of a planar graph is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color.
- **(ii) Edge coloring** of a planar graph is an assignment of colors to the edges of the graph so that no two incident edges have the same color.
- **(iii) Face coloring** of a planar graph assigns a color to each face so that no two faces that share an edge have the same color.

Total coloring of a Planar graph is an assignment of colors to all the elements of vertices, edges and faces such that

- (i) no two adjacent vertices are of the same color
- (ii) no two incident edges have the same color and
- (iii) no two faces that share an edge have the same color.

Chromatic number of a graph

The chromatic number of a graph is the smallest numbers of color needed in graph coloring.

Chromatic Number of the Fuzzy Graph

If $G = (V, \mu)$ is such a fuzzy graph where $V = \{1,2,3, \dots, n\}$ and μ is a fuzzy number on the set of all subsets of $V \times V$. Assume $I = A \cup \{0\}$ where $A = \{\alpha_1 < \alpha_2 < \dots < \alpha_k\}$ is the fundamental set (level set) of G. For each $\alpha \in I$, G_α denote the crisp graph $G_\alpha = (V, E_\alpha)$ where $= \{(i, j)/ 1 \leq i < j \leq n, (i, j) \geq \alpha\}$ and $\chi_\alpha = \chi(G_\alpha)$, denote the chromatic number of crisp graph. By this definition the **chromatic number of the fuzzy graph** G is the fuzzy number $\chi(G) = \{(i, v(i))/i \in X\}$ where $v(i) = \max\{\alpha \in I/i \in A_\alpha\}$ and $A_\alpha = \{1, \dots, \chi_\alpha\}$.

Basic Notation

The **fuzzy vertex chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy vertex coloring of G. It is denoted by $\boldsymbol{\chi(G)}$.

The **fuzzy edge chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy edge coloring of G. It is denoted by $\boldsymbol{\chi'(G)}$.

The **fuzzy face chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy face coloring of G. It is denoted by $\boldsymbol{\chi''(G)}$.

3. Total Coloring of Planar Graph

Procedure for the Total Coloring of a Planar Graph

The following are the steps involved to color the vertices, edges and faces of the planar graph.

Step 1: Exhibit the vertices of the graph in the same order.

Step 2: Choose the first vertex and provide it with the first color for vertex.

Step 3: Choose the next vertex and color it with the lowest numbered color that has not been colored on any vertices adjacent to it. If all the adjacent vertices are colored with this color, assign a new color to it.

Step 4: Connect the two vertices with an edge and provide it with the first color for edge. If new edge is attained color, it with the lowest numbered color that has not been colored on any edges adjacent to it. If all the adjacent edges are colored with this color, assign a new color to it.

Step 5: Repeat Step 3 and Step 4 until the graph configure a face. If new face is attained color it with the lowest numbered color that has not been colored on any faces adjacent to it. If all the adjacent faces are colored with this color, assign a new color to it.

Step 6: Repeat the above steps until all the vertices, edges and faces are colored.

Total Coloring of Wheel Graph

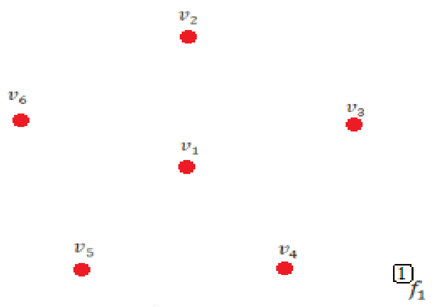
Let $n \geq 4$ and W_n be the wheel graph. Then total coloring of wheel graph has main three as follows,

(i) $\chi(W_n) = \begin{cases} 3; & for\ n\ is\ even \\ 4; & for\ n\ \ is\ odd \end{cases}$.

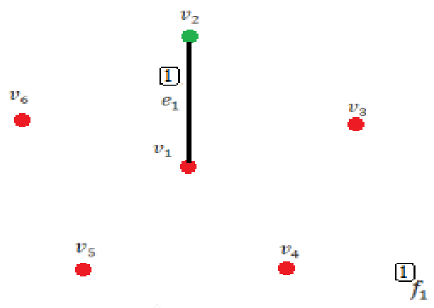
(ii) $\chi'(W_n) = n, \quad for\ all\ n.$

(iii) $\chi''(W_n) = \begin{cases} 3; & for\ n\ is\ even \\ 4; & for\ n\ \ is\ odd \end{cases}$.

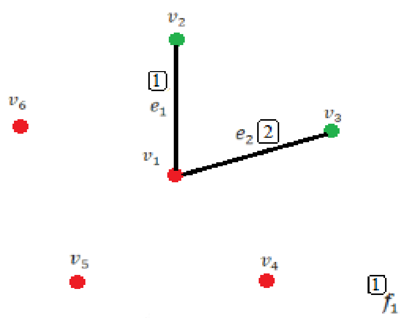
Here, we considered the specific type of planar graph, say Wheel Graph W_5 as an example to perform the total coloring.



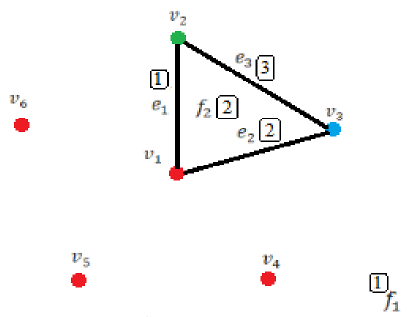
$G_{1,0}$; $\chi_{1,0} = 1$; $\chi'_{1,0} = 0$; $\chi''_{1,0} = 1$



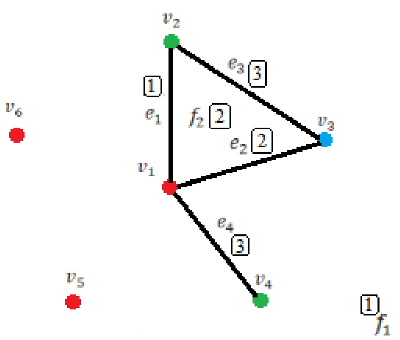
G_{e_1} ; $\chi_{e_1} = 2$; $\chi'_{e_1} = 1$; $\chi''_{e_1} = 1$



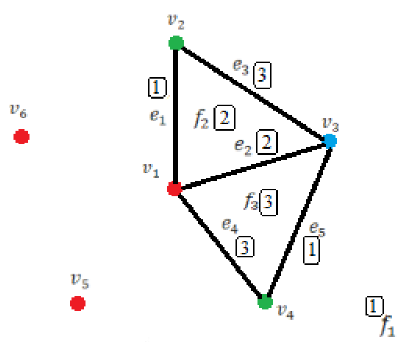
G_{e_2} ; $\chi_{e_2} = 2$; $\chi'_{e_2} = 2$; $\chi''_{e_2} = 1$



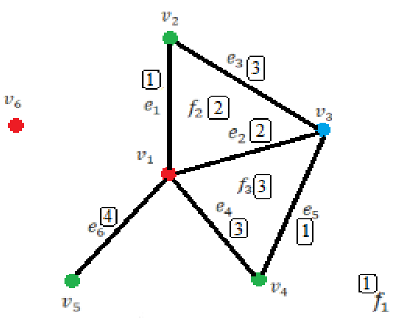
G_{e_3} ; $\chi_{e_3} = 3$; $\chi'_{e_3} = 3$; $\chi''_{e_3} = 2$



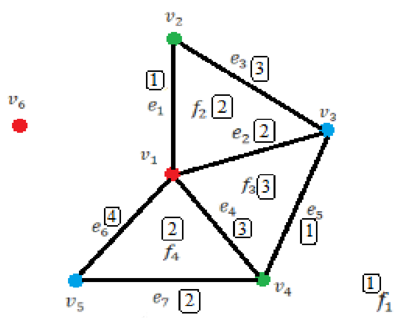
G_{e_4} ; $\chi_{e_4} = 3$; $\chi'_{e_4} = 3$; $\chi''_{e_4} = 2$



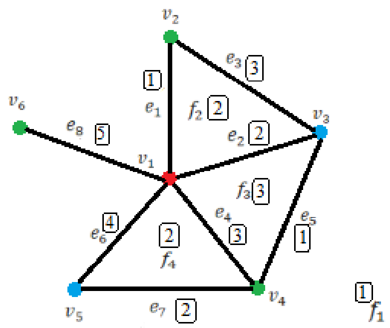
G_{e_5} ; $\chi_{e_5} = 3$; $\chi'_{e_5} = 3$; $\chi''_{e_5} = 3$



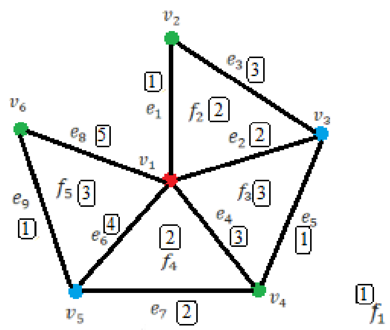
G_{e_6} ; $\chi_{e_6} = 3$; $\chi'_{e_6} = 4$; $\chi''_{e_6} = 3$



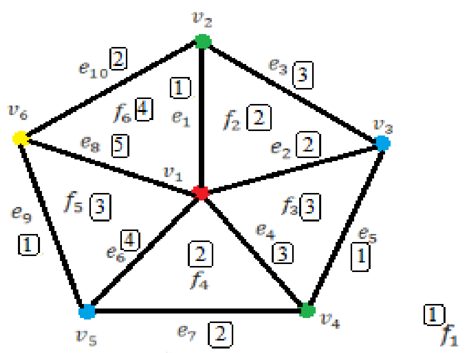
G_{e_7} ; $\chi_{e_7} = 3$; $\chi'_{e_7} = 4$; $\chi''_{e_7} = 3$

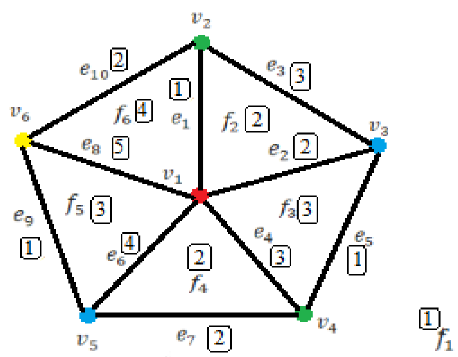


G_{e_8} ; $\chi_{e_8} = 3$; $\chi'_{e_8} = 5$; $\chi''_{e_8} = 3$



G_{e_9} ; $\chi_{e_9} = 3$; $\chi'_{e_9} = 5$; $\chi''_{e_9} = 3$





$G_{e_{10}}$; $\chi_{e_{10}} = 4$; $\chi'_{e_{10}} = 5$; $\chi''_{10} = 4$

Thus, the total (vertex, edge and face) coloring of the Wheel graph is shown in the following fig 3.1.

W_5 ; $\chi_{W_5} = 4$; $\chi'_{W_5} = 5$; $\chi''_{W_5} = 4$

Figure 3.1: Total Coloring of Wheel Graph W_5

4. Fuzzy Face Anti-Magic Labeling of Planar Graphs

Fuzzy Labelling Graph

A graph $G = (V, \mu, \sigma)$ is said to be a **fuzzy labelling graph** if $\mu: V \rightarrow [0,1]$ and $\sigma : V \times V \rightarrow [0,1]$, is bijective such that the membership value of edges and vertices are distinct and

$$\sigma(x, y) \leq \mu(x) \wedge \mu(y) \text{ for all } x, y \in V.$$

Fuzzy Anti-Magic Graph

Every connected graph other than magic graphs are **anti-magic**. Hence, it is necessary to know about Fuzzy Magic Graphs to understand the Fuzzy Anti-Magic Graphs better.

Fuzzy Magic Graph

A fuzzy graph G = (V, μ, σ) is called a **fuzzy magic graph** if there are two bijective functions

μ:V → [0,1] and $\sigma : V \times V \to [0,1]$, with restricted the conditions σ (u,v) < μ (u) + μ (v) and μ (u)+ σ (uv)+ μ (v) = m(G) ≤ 1 where , m(G) is a real constant for all u,v∈ G

(i) A fuzzy labelling graph is said to be a **fuzzy vertex magic graph** if $\mu(u) + \sigma(uv) + \mu(v)$ has a same magic value for all $u, v \in V$ which is denoted as $m_0(G)$.

(ii) A fuzzy labelling graph is said to be a **fuzzy edge magic graph** if $\mu(u) + \sigma(uv) + \mu(v)$ has a same magic value for all $u, v \in V$ which is denoted as $M_0(G)$.

Fuzzy Face Magic Graph

A fuzzy labelling graph is said to be a **fuzzy face magic graph** if the weights of all s-sided faces are all equal to some constant ω(s) then the labeling is considered to be face-magic where, the weight of a face is the sum of the face's own label and the labels of vertices and edges enclosing the face.

Fuzzy Face Anti-Magic Graph

A fuzzy labelling graph is said to be a **fuzzy face anti-magic graph** if it is not fuzzy face-magic. (i.e.) the weights of all s-sided faces are all not equal to some constant ω(s) then the labeling is considered to be face anti-magic where, the weight of a face is the sum of the face's own label and the labels of vertices and edges enclosing the face.

Fuzzy Face Anti-Magic Labeling of Wheel Graph

Wheel Graph (considered for the Fuzzy Face Anti-Magic Labeling)

The Wheel graph $W_n$is a graph with the vertex set $V = \{v_i : 1 \le i \le n\} \cup \{h\}$ where h is referred to hub, edge set $E = R \cup S$ where $R = \{v_i v_{i+1} : 1 \le i \le n\}$are referred to as rims and $S = \{hv_i : 1 \le i \le n\}$ are referred to as spokes and a set of face, say F.

(i) The vertex set is $V = \{v_1, v_2, \dots . v_5, h\}$ where h is referred as hub.
(ii) The edge set is $E = S \cup R$ where $S = \{s_1, s_2, \dots . s_5\}$ where S is referred as a set of spokes and $R = \{r_1, r_2, \dots . , r_5\}$ where R is referred as a set of rims.
(iii) The face set is $F = \{f_1, f_2, \dots, f_6\}$.

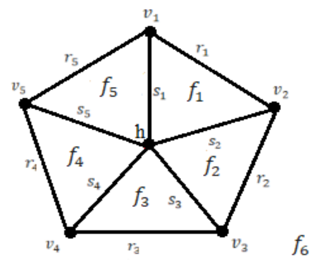


Figure 4.1: Wheel Graph W_5 with the Representations as Vertex, Hub, Spoke, Rim and Face

Construction of Fuzzy Face Anti-Magic Wheel graph

Fuzzy Face Anti-magic Wheel graph can be easily constructed by just not following one or two steps of the algorithm of Fuzzy Face-magic wheel graph. The algorithm to construct the Fuzzy Face Anti-magic Wheel graph is as follows:

Algorithm

Step 1: Begin the labeling with the blank wheel graph.

Step 2: Label the n-interior faces freely across the wheel.

Note: The exterior face label can be assigned later since it does not affect the weight of the face. Yet, it need to be assigned accordingly such that sum of each face weight must be less than or equal to 1. (≤ 1)

Step 3: Observe the face with the highest weight. Calculate weight needed for each rim (to be balanced) and label them accordingly.

Note: Step 3, can be omitted and label each rim freely, but including this step helps us to understand the face weight precisely.

Step 4: Label the remaining edges (spokes) and vertices, randomly. Make sure that each are not in complemented pairs with same weight such that each face weight differs. (If it has balances weight face, then it becomes Fuzzy Face-magic Wheel graph.)

Step 5: Now, label the left label (hub) and the exterior face label freely since it is neighbour with every face.

Note: The hub label can be assigned freely but it is needed to be noticed, such that sum of each face weight must be less than or equal to 1.(≤ 1)

In each and every step, be caution that the face weight must not exceed 1. (Since it is a fuzzy graph).

Example 4.3.2

Here, we considered the specific type of planar graph, say Wheel Graph W_5 as an example to construct a fuzzy face anti-magic wheel graph.

Step 1: Beginning with blank W_5. **(Figure 4.2)**

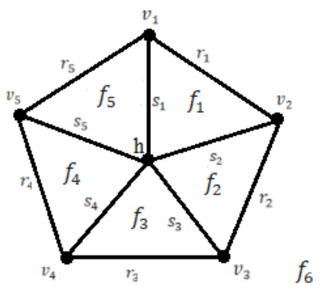


Figure 4.2: Blank W_5

Step 2: Choose any random face labels for interior faces. (Figure 4.3)

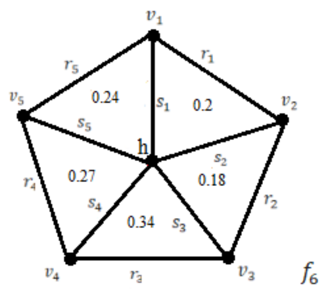


Figure 4.3: Face Labels

Step 3: Observe the face with the highest weight. Calculate weight needed for each rim (to be balanced) and label them accordingly**. (figure 4.4) .** Face with highest face: f_3= 0.34

Weight needed to be balanced: $f_1 = 0.14$; $f_2 = 0.16$; $f_3 = 0$; $f_4 = 0.07$; $f_5 = 0.1$

Labeling rims with the random value, w. (w + weight needed for each rim to be balanced)

Here, $w = 0.09$.

Here, the weight of each face is 0.43, which need to be unbalanced to achieve fuzzy face anti-magic wheel graph.

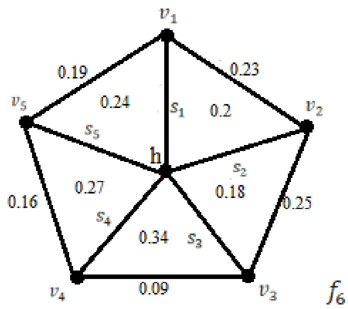


Figure 4.4: Labeling Rims

Step 4: Label the remaining edges (spokes) and vertices, randomly so that each are not in complemented pairs with same weight such that each face weight differs. **(Figure 4.5)**

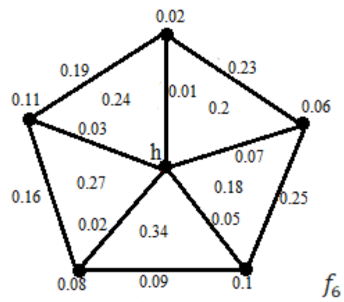


Figure 4.5: Labelling Spokes and Vertices Randomly

Here, the weight of each face is $f_1 = 0.59$; $f_2 = 0.71$; $f_3 = 0.68$; $f_4 = 0.67$; $f_5 = 0.60$.

So far, the face weights are unbalanced and face weights are ≤ 1, thus it makes Fuzzy Face anti-magic Wheel graph.

Step 5: Now label the left labels (hub and exterior face), such that sum of each face weight $\leq$ 1. since, it is a Fuzzy Graph.

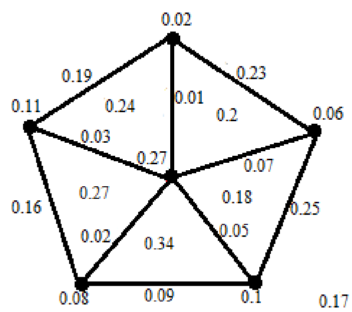


Figure 4.6: Labeling Hub and Exterior Face

Here, the weight of each face is $f_1 = 0.86$; $f_2 = 0.98$; $f_3 = 0.95$; $f_4 = 0.94$; $f_5 = 0.87$.

Hence, the face weights are unbalanced and face weights are ≤ 1, thus it makes Fuzzy Face anti-magic Wheel graph. **(Figure 4.6).**

5. Intuitionistic Fuzzy Face Anti-Magic Labeling of Wheel Graphs

Intuitionistic Fuzzy Set

Intuitionistic fuzzy sets have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov (1983) as an extension of Lotfi Zadeh's [10] notion of fuzzy set, which itself extends the classical notion of a set. The theory of intuitionistic fuzzy sets further extends both concepts by allowing the assessment of the elements by two functions: μ for membership and ν for non-membership, which belong to the real unit interval [0, 1] and whose sum also belongs to the same interval.

Let X be a fixed universe set. Let A be a subset of X. Then **Intuitionistic Fuzzy set** A* in a set X is defined as an object of the form

$A* = \{(x, \mu_A(x), \nu_A(x))/\ x \in X\}$ where $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Intuitionistic Fuzzy Face Anti-Magic Labeling of Planar graph

We know that, the intuitionistic fuzzy set is of the form, $A* = \{(x, \mu_A(x), (x))/\ x \in X\}$

where $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$, for every $x \in X$, $0 \leq \mu(x) + \nu_A(x) \leq 1$.

Intuitionistic Fuzzy Face Anti-Magic Labeling of Wheel graph

Let us first define the degree of membership of the element for the fuzzy face anti-magic labeling of wheel graph. (i.e.) for membership function, Considering the example 4.3.2. **(figure 5.1)**

Similarly, consider another example of fuzzy face anti-magic labeling of the wheel graph for non-membership function. Thus, we get the following graph. **(figure 5.2.)**

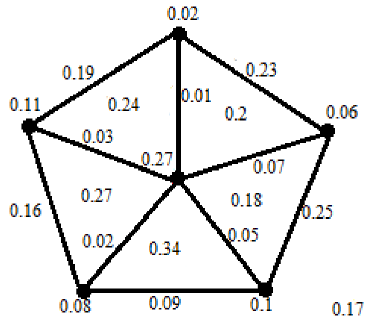


Figure 5.1: Membership Function of Fuzzy Face Anti-Magic Labeling of the Wheel Graph W_5

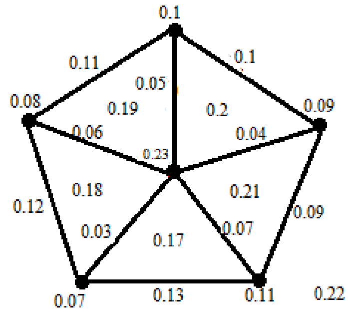


Figure 5.2: Non-Membership Function of Fuzzy Face Anti-Magic Labeling of the Wheel Graph W_5

The weight of each face of Membership function of fuzzy face anti-magic labeling of the wheel graph is $f_1 = 0.86$; $f_2 = 0.98$; $f_3 = 0.95$; $f_4 = 0.94$; $f_5 = 0.87$.

The weight of each face of non-membership function of fuzzy face anti-magic labeling of the wheel graph is $f_1 = 0.81$; $f_2 = 0.84$; $f_3 = 0.81$; $f_4 = 0.77$; $f_5 = 0.82$.

Thus, the intuitionistic fuzzy face anti-magic labeling of the wheel graph W_5 with the above obtained membership and non-membership function is given as, **(figure 5.3).**

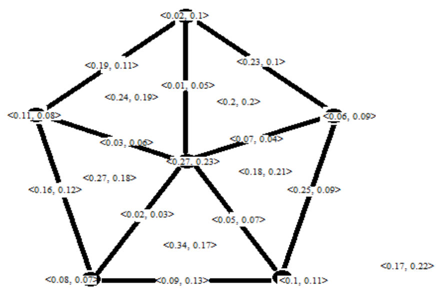


Figure 5.3: Intuitionistic Fuzzy Face Anti-Magic Labeling of the Wheel Graph W_5

Here, the fuzzy face anti-magic value of the intuitionistic Wheel graph is $f_1(0.86, 0.81)$; $f_2(0.98, 0.84)$; $f_3(0.95, 0.81)$; $f_4(0.94, 0.77)$; $f_5(0.87, 0.82)$

6. Applications

Application of Fuzzy Face Anti-Magic Wheel Graph

Fuzzy Face Anti-Magic Wheel graphs has tremendous applications in various fields. In this paper, let us discuss about one of such application, Expatriate Assignment.

Employee or human resource who is sent to get global assignment in overseas is called expatriate. According to McNulty [11], expatriate is defined as a person who lives outside native country, and physically is mobile across international border for professional or personal reasons, for short or long periods of time, organizationally sponsored or not, and crossing an ocean or moving across land.

Expatriate assignments at the branch companies in other countries must be optimised due to the short-term duty. Optimisation of calculation is needed to assign the expatriates including the scheduling system. Various authors conducted several research methods to optimise the scheduling system. One of such methods is using the graph theory.

Here, we use the concept of Graph Coloring, Fuzzy Anti-Magic Graph and a certain type of Planar Graph called Wheel Graph in the field of Graph theory to attain optimised scheduling system of an Expatriate Assignment.

In this concept, primarily several expatriates were assigned to their tasks in a specific area. Secondarily, the assigned tasks of such expatriate were modelled as graph with the division of tasks were labelled as vertices, the connection between two divisions sharing the same expatriate were labelled as edges and the expatriates were labelled as faces. Together, the graph represents expatriate assignment with coloring, represented as a wheel graph. Here, chromatic number of colors is used to minimize the allocation of different times for common expatriates.

This method of using Fuzzy Anti-Magic Wheel Graph and Graph Coloring guaranties that there is no clash in the allocation time in same divisions between expatriates and also provide the optimal allocation time for each expatriate.

7. Conclusion

In this paper, we have discussed about the total (vertex, edge and face) coloring of the specific type of Planar Graph, (Wheel) and also an algorithm for Labeling the Fuzzy Face Anti-Magic of the Wheel Graph is discussed with an example.

Eventually, Intuitionistic fuzzy face anti-magic labeling of the graphs is also defined with the same algorithm. Moreover, some applications of this graph are also discussed which paved a way for further study and development. Thus, knowledge of having fuzzy in the graph theory provides the more stability is helpful in more implicational studies.

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