MAXIMUM ENTROPY MODEL IN THE FIELD OF MARKETING

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Abstract-
Maximum entropy principle has a great importance in various fields of science and engineering viz. Thermodynamics, Business, Economics, Finance, Insurance, Marketing, Operation Research etc. In the present paper, we are discussing Herniter's Model for Brand switching in the field of marketing. Herniter's Model is based on maximum entropy principle. Here the conditional probabilities will be discussed in brand purchasing behavior using Shannon's entropy.

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Introduction-

Herniter used maximum entropy for brand switching in marketing in 1973. He gave the first maximum entropy model involving both discrete and continuous variate probability distributions. he obtained proportions of loyal and wavering customers and probabilities of switching from one brand to another other from the information of market shares of the brand. If none of the market shares of a specified number of brands in the market is prescribed, the entropy will be maximum when the market shares of all these brands are equal.

We shall discuss conditional probabilities in brand purchasing behavior by using Shannon's measure. In this case, we shall use information of market shares of brands to estimate the proportions of customers loyal to each of the brands and of those loyal to two brands.

HERNITER'S MODEL FOR BRAND PURCHASING:

Let us consider three brands 1, 2, 3 of an item. Two persons are interested to purchase different brands of that item. First person purchases one of the three brands. Let p_1, p_2, p_3 be the probabilities of first person purchasing the brand 1, 2 or 3 respectively. Now, second person wants to purchase one brand out of remaining two brands.

If the first person purchases brand 1 then second person may choose brand 2 or 3. Let p_4, p_5 be the probabilities of second person purchasing the brands 2 , 3 respectively and p_6 be the probability of his wavering between the brands 2 and 3. Let his preference random variable for brands 2, 3 be u, (1-u) respectively and corresponding density function be $f_6(u)$.

Again, if the first person purchases brand 2, then second person may choose brands 1or 3. Let p_7, p_8 be the probabilities of second person purchasing the brands 3,1 respectively and p_9 be the probability of his wavering between two brands 3 and 1. Let his preference random variable for brands 3, 1 be u and (1-u) respectively and corresponding density function be $f_9(u)$.

Similarly, if the first person purchases the brand 3 then second person may choose brand 1 or 2. Let p_{10}, p_{11} be the probabilities of second person purchasing the brands 1, 2 respectively and g_{12} be the probability of his wavering between two brands 1 and 2. Let his preference random variable for brands 1, 2 be u, (1-u) respectively and corresponding density function be $f_{12}(u)$.

On the basis of above discussion, probabilities and probability densities for purchasing behavior can be given as:

$$[p_1p_4, p_1p_5, p_1p_6uf_6(u), p_1p_6(1-u)f_6(u), p_2p_7, p_2p_8, p_2p_9uf_9(u), p_2p_9(1-u)f_9(u), \quad p_3p_{10}, p_3p_{11}, p_3p_{12}uf_{12}(u), \quad p_3p_{12}(1-u)f_{12}(u)].$$

Hence, Shannon entropy is given by

$$S = -p_1p_4\ln(p_1p_4) - p_1p_5\ln(p_1p_5) - p_2p_7\ln(p_2p_7) - p_2p_8\ln(p_2p_8) - p_3p_{10}\ln(p_3p_{10}) - p_3p_{11}\ln(p_3p_{11})$$
$$- \int_0^1 [p_1p_6uf_6(u)\ln\{p_1p_6uf_6(u)\} + p_1p_6(1-u)f_6(u)\ln\{p_1p_6(1-u)f_6(u)\}]du$$
$$- \int_0^1 [p_2p_9uf_9(u)\ln\{p_2p_9uf_9(u)\} + p_2p_9(1-u)f_9(u)\ln\{p_2p_9(1-u)f_9(u)\}]du$$
$$- \int_0^1 [p_3p_{12}uf_{12}(u)\ln\{p_3p_{12}uf_{12}(u)\} + p_3p_{12}(1-u)f_{12}(u)\ln\{p_3p_{12}(1-u)f_{12}(u)\}]du$$
$$\Rightarrow S = -p_1p_4\ln(p_1p_4) - p_1p_5\ln(p_1p_5) - p_2p_7\ln(p_2p_7) - p_2p_8\ln(p_2p_8) - p_3p_{10}\ln(p_3p_{10})$$
$$- p_3p_{11}\ln(p_3p_{11}) - \int_0^1 p_1p_6f_6(u)[u\ln p_1 + u\ln p_6 + u\ln u + (1-u)\ln p_1 + (1-u)\ln p_6$$
$$+ (1-u)\ln(1-u) + u\ln f_6(u) + (1-u)\ln f_6(u)]du - \int_0^1 p_2p_9f_9(u)[u\ln p_2 + u\ln p_9$$
$$+ u\ln u + (1-u)\ln p_2 + (1-u)\ln p_9 + (1-u)\ln(1-u) + u\ln f_9(u) + (1-u)\ln f_9(u)]du$$
$$- \int_0^1 p_3p_{12}f_{12}(u)[u\ln g_3 + u\ln g_{12} + u\ln u + (1-u)\ln g_3 + (1-u)\ln g_{12} + (1-u)\ln(1-u)$$
$$+ u\ln f_{12}(u) + (1-u)\ln f_{12}(u)]du$$
$$\Rightarrow S = -p_1p_4\ln(p_1p_4) - p_1p_5\ln(p_1p_5) - p_1p_6\ln(p_1p_6) - p_2p_7\ln(p_2p_7) - p_2p_8\ln(p_2p_8)$$
$$- p_2p_9\ln(p_2p_9) - p_3p_{10}\ln(p_3p_{10}) - p_3p_{11}\ln(p_3p_{11}) - p_3p_{12}\ln(p_3p_{12})$$
$$- p_1p_6\int_0^1 f_6(u)[u\ln u + (1-u)\ln(1-u) + \ln f_6(u)]du - p_2p_9\int_0^1 f_9(u)[u\ln u$$
$$+ (1-u)\ln(1-u) + \ln f_9(u)]du - p_3p_{12}\int_0^1 f_{12}(u)[u\ln u + (1-u)\ln(1-u) + \ln f_{12}(u)]du$$
$$--(1)$$

We have to maximize the entropy subject to the normal constraints.

$$p_1(p_4 + p_5 + p_6) + p_2(p_7 + p_8 + p_9) + p_3(p_{10} + p_{11} + p_{12}) = 1 \qquad --(2)$$

$$\int_0^1 f_i(u)du = 1 \text{, where } i = 6, 9, 12 \qquad --(3)$$

Now, we consider two cases:
(a) When the market shares are prescribed
(b) When the market shares are not prescribed

(a) WHEN THE MARKET SHARES ARE PRESCRIBED:
Market shares have been given as:-

$$p_2p_8 + p_2p_9 \int_0^1 (1-u)\, f_9(u)\, du + p_3p_{10} + p_3p_{12} \int_0^1 u\, f_{12}(u)\, du = m_1$$
$$p_1p_4 + p_1p_6 \int_0^1 u\, f_6(u)\, du + p_3p_{11} + p_3p_{12} \int_0^1 (1-u)\, f_{12}(u)\, du = m_2$$
$$p_1p_5 + p_1p_6 \int_0^1 (1-u)\, f_6(u)\, du + p_2p_7 + p_2p_9 \int_0^1 u\, f_9(u)\, du = m_3 \qquad --(4)$$

Here m_1, m_2 and m_3 are market shares for brands 1, 2 and 3 respectively.

The Lagrangian using equations (1), (2), (3) & (4) is written as:

$$L = - p_1p_4\ln(p_1p_4) - p_1p_5\ln(p_1p_5) - p_1p_6\ln(p_1p_6) - p_2p_7\ln(p_2p_7) - p_2p_8\ln(p_2p_8) - p_2p_9\ln(p_2p_9) - p_3p_{10}\ln(p_3p_{10}) - p_3p_{11}\ln(p_3p_{11}) - p_3p_{12}\ln(p_3p_{12}) - p_1p_6 \int_0^1 f_6(u)[u \ln u + (1-u) \ln(1-u) + \ln f_6(u)]du - p_2p_9 \int_0^1 f_9(u)[u \ln u + (1-u) \ln(1-u)+\ln f_9(u)]du - p_3p_{12} \int_0^1 f_{12}(u)[u \ln u +(1-u) \ln(1-u)+ \ln f_{12} (u)]du - \eta[p_1(p_4+p_5+p_6) + p_2(p_7+ p_8+ p_9) + p_3(p_{10}+p_{11}+p_{12})-1] - \lambda_6 [\int_0^1 f_6(u)du-1] - \lambda_9[\int_0^1 f_9(u)du-1] - \lambda_{12}[\int_0^1 f_{12}(u)du-1] - \xi_1[p_2p_8 + p_2p_9 \int_0^1 (1-u)f_9(u)du + p_3p_{10} + p_3p_{12} \int_0^1 uf_{12}(u)du - m_1] - \xi_2[p_1p_4 + p_1p_6 \int_0^1 u\, f_6(u)du + p_3p_{11} + p_3p_{12} \int_0^1 (1-u)\, f_{12}(u)du - m_2] - \xi_3[p_1p_5 + p_1p_6 \int_0^1 (1-u)\, f_6(u)du + p_2p_7 + p_2p_9 \int_0^1 u\, f_9(u)du - m_3] \qquad --(5)$$

Differentiating equation (5) with respect to g_i and equating to zero, where i = 1,2, _ _ _ ,12.

$$-p_4- p_5- p_6- p_4\ln(p_1p_4) - p_5\ln(p_1p_5) - p_6 \ln(p_1p_6) - \eta (p_4 + p_5 + p_6) - \xi_2p_4 -\xi_3p_5 -p_6 \int_0^1 f_6(u)[u \ln u + (1-u) \ln (1-u) + \ln f_6 (u) + \xi_2u +\xi_3 (1-u)]du=0 \qquad --(6)$$
$$-p_7 - p_8 - p_9 - p_7\ln(p_2p_7) - p_8 \ln(p_2p_8) - p_9 \ln(p_2p_9) - \eta (p_7 + p_8 + p_9) -\xi_3p_7 - \xi_1p_8 - p_9 \int_0^1 f_9(u)[u \ln u + (1-u) + \ln (1-u) + \ln f_9 (u) + (1-u)\, \xi_1 + u\xi_3]du=0 \qquad --(7)$$
$$-p_{10} - p_{11} - p_{12} - p_{10}\ln(p_3p_{10}) - p_{11}\ln(p_3p_{11}) - p_{12}\ln(p_3p_{12}) - \eta(p_{10} + p_{11} + p_{12}) -\xi_1p_{10}-\xi_2p_{11} - p_{12} \int_0^1 f_{12}(u)[u \ln u +(1-u) \ln (1-u) + \ln f_{12} (u) + (1-u)\, \xi_2 +\xi_1u]du=0 \qquad --(8)$$

$$-p_1\ln(p_1p_4) - p_1 - \eta p_1 - \xi_2p_1=0$$
$$\Rightarrow \quad \xi_2= - \ln(p_1p_4) - \eta - 1 \qquad --(9)$$
$$- p_1\ln(p_1p_5) - p_1 - \eta p_1 - \xi_3p_1 = 0$$
$$\Rightarrow \quad \xi_3 = - \ln(p_1p_5) - \eta - 1 \qquad --(10)$$
$$-\ln(p_1p_6) - \eta - 1 - \int_0^1 f_6(u)[u \ln u + (1-u) \ln (1-u) + \ln f_6 (u) + \xi_2u +\xi_3(1-u)]du =0 \qquad --(11)$$

$$\xi_3 = -\ln(p_2p_7) - \eta - 1 \qquad --(12)$$
$$\xi_1 = -\ln(p_2p_8) - \eta - 1 \qquad --(13)$$
$$-\ln(p_2p_9) - \eta - 1 - \int_0^1 f_9(u)[u \ln u + (1-u) \ln (1-u) + \ln f_9 (u) +\xi_1 (1-u) + \xi_3u]du = 0 \qquad --(14)$$
$$-p_3\ln(p_3p_{10}) - p_3 - \eta p_3 - \xi_1p_3 = 0$$
$$\xi_1 = - \ln(p_3p_{10}) - \eta - 1 \qquad --(15)$$
$$\xi_2 = -\ln(p_3p_{11}) - \eta - 1 \qquad --(16)$$
$$-\ln(p_3p_{12}) - \eta- 1 - \int_0^1 f_{12}(u)[u \ln u + (1-u) \ln (1-u) + \ln f_{12} (u) + \xi_1u + \xi_2 (1-u)]du = 0 \qquad --(17)$$

Differentiating equation (5) with respect to fj and equating to zero, where j = 6, 9, 12.

$$-p_1p_6 [u \ln u + (1-u) \ln (1-u) + \ln f_6 (u)] - p_1p_6 - \lambda_6 - \xi_2 p_1p_6 u - \xi_3 p_1p_6 (1-u) = 0$$

$$\Rightarrow -p_1p_6 [u \ln u + (1-u) \ln (1-u) + \ln f_6 (u) + \xi_2 u + \xi_3(1-u)] - p_1p_6 - \lambda_6 = 0 \quad --(18)$$

$$-p_2p_9 [u \ln u + (1-u) \ln (1-u) + \ln f_9 (u) + \xi_1(1-u) + \xi_2 u] - p_2p_9 - \lambda_9 = 0 \quad --(19)$$

$$-p_3p_{12} [u \ln u + (1-u) \ln (1-u) + \ln f_{12} (u) + \xi_1 u + \xi_2(1-u)] - p_3p_{12} - \lambda_{12} =0 \quad --(20)$$

using equations (11) and (18)

$$-\ln(p_1p_6) - \eta - 1 + \frac{\lambda_6 + p_1p_6}{p_1p_6} =0$$

$$\Rightarrow \lambda_6 = p_1p_6 [\ln(p_1p_6) + \eta] \quad --(21)$$

by putting the values of ξ_2, ξ_3 and λ_6 in equation (18) from equations (9), (10), (21) respectively, we obtain

$$-p_1p_6 [u \ln u+(1-u) \ln (1-u) + \ln f_6 (u) – u \ln(p_1p_4) - u\eta - u - (1-u) \ln(p_1p_5)$$

$$- (1-u) \eta - (1-u)]- p_1p_6 - p_1p_6 [\ln(p_1p_6) + \eta] = 0$$

$$\Rightarrow u \ln u + (1-u) \ln (1-u) + \ln f_6 (u) – u \ln(p_1p_4) - (1-u) \ln(p_1p_5) + \ln(p_1p_6)= 0$$

$$\Rightarrow \ln f_6 (u) = - u \ln u - (1-u) \ln (1-u) + u \ln(p_1p_4) + (1-u) \ln(p_1p_5) - \ln(p_1p_6)$$

$$\Rightarrow f_6 (u) = p_6^{-1} u^{-u} (1-u)^{-(1-u)} p_4^{u} p_5^{(1-u)} \quad --(22)$$

Similarly, from equations (14), (19)

$$\lambda_9 = p_2p_9 [\ln(p_2p_9) + \eta] \quad --(23)$$

from equations (12), (13), (19) and (23), we get the following expression

$$f_9 (u) = p_9^{-1} u^{-u} (1-u)^{-(1-u)} p_7^{u} p_8^{(1-u)} \quad --(24)$$

And, from equations (17), (20)

$$\lambda_{12} = p_3p_{12} [\ln(p_3p_{12}) + \eta] \quad --(25)$$

using equations (15), (16), (20) and (25), we get

$$f_{12} (u) = p_{12}^{-1} u^{-u} (1-u)^{-(1-u)} p_{10}^{u} p_{11}^{(1-u)} \quad --(26)$$

Substituting values of $f_j (u)$, j = 6, 9, 12 in equations (3) and (4), we can obtain equations to find the values of g_i.

(b) WHEN THE MARKET SHARES ARE NOT PRESCRIBED:

The Lagrangian is

$$L = -p_1p_4 \ln(p_1p_4) - p_1p_5 \ln(p_1p_5) - p_1p_6 \ln(p_1p_6) - p_2p_7 \ln(p_2p_7) - p_2p_8 \ln(p_2p_8)$$
$$- p_2p_9 \ln(p_2p_9) - p_3p_{10} \ln(p_3p_{10}) - p_3p_{11} \ln(p_3p_{11}) - p_3p_{12} \ln(p_3p_{12})$$
$$- p_1p_6 \int_0^1 f_6(u) [u \ln u + (1-u) \ln (1-u) + \ln f_6 (u)]du - p_2p_9 \int_0^1 f_9(u)[u \ln u$$
$$+ (1-u) \ln (1-u) + \ln f_9 (u)]du - p_3p_{12} \int_0^1 f_{12}(u)[u \ln u + (1-u) \ln (1-u)$$
$$+ \ln f_{12} (u)]du - (\eta-1)[p_1(p_4 + p_5 + p_6) + p_2(p_7 +p_8+p_9) + p_3(p_{10} + p_{11} + p_{12})-1]$$
$$- \lambda_6 [\int_0^1 f_6(u)du - 1] - \lambda_9 [\int_0^1 f_9(u)du - 1] - \lambda_{12} [\int_0^1 f_{12}(u)du - 1] \quad --(27)$$

Differentiating equation (27) with respect to g_i and equating to zero, where I = 1, 2, _ _ _ ,12.

$$-p_4\ln(p_1p_4) - p_5\ln(p_1p_5) - p_6\ln(p_1p_6) - \eta p_4 - \eta p_5 - \eta p_6$$
$$-p_6 \int_0^1 f_6(u)[u \ln u + (1-u) \ln (1-u) + \ln f_6 (u)]du \quad --(28)$$

$$-p_7\ln(p_2p_7) - p_8\ln(p_2p_8) - p_9\ln(p_2p_9) - \eta p_7 - \eta p_8 - \eta p_9$$
$$-p_9 \int_0^1 f_9(u)[u \ln u + (1-u) \ln (1-u) + \ln f_9 (u)]du \quad --(29)$$

$$-p_{10}\ln(p_3p_{10}) - p_{11}\ln(p_3p_{11}) - g_{12}\ln(p_3p_{12}) - \eta p_{10} - \eta p_{11} - \eta p_{12}$$
$$-p_{12} \int_0^1 f_{12}(u)[u \ln u + (1-u) \ln (1-u) + \ln f_{12} (u)]du \quad --(30)$$

$$-p_1\ln(p_1p_4) - p_1 - \eta p_1 + p_1 = 0$$

$$\Rightarrow \ln(p_1p_4) = - \eta \Rightarrow p_1p_4 = e^{-\eta} \quad --(31)$$

$$\ln(p_1p_5) = - \eta \Rightarrow p_1p_5 = e^{-\eta} \quad --(32)$$

$$-\ln(p_1p_6) - \int_0^1 f_6(u)[u \ln u + (1-u) \ln (1-u) + \ln f_6 (u)]du - \eta =0 \quad --(33)$$

$\ln(p_2p_7) = -\eta \Rightarrow p_2p_7 = e^{-\eta}$ - -(34)

$\ln(p_2p_8) = -\eta \Rightarrow p_2p_8 = e^{-\eta}$ - -(35)

$-\ln(p_2p_9) - \int_0^1 f_9(u)[u\ln u + (1-u)\ln(1-u) + \ln f_9(u)]du - \eta = 0$ - -(36)

$\ln(p_3p_{10}) = -\eta \Rightarrow p_3p_{10} = e^{-\eta}$ - -(37)

$\ln(p_3p_{11}) = -\eta \Rightarrow p_3p_{11} = e^{-\eta}$ - -(38)

$-\ln(p_3p_{12}) - \int_0^1 f_{12}(u)[u\ln u + (1-u)\ln(1-u) + \ln f_{12}(u)]du - \eta = 0$ - -(39)

Differentiating equation (27) with respect to fj and equating to zero, where j = 6, 9,12 .

$-p_1p_6[u\ln u + (1-u)\ln(1-u) + \ln f_6(u)] - p_1p_6 - \lambda_6 = 0$ - -(40)

$-p_2p_9[u\ln u + (1-u)\ln(1-u) + \ln f_9(u)] - p_2p_9 - \lambda_9 = 0$ - -(41)

$-p_3p_{12}[u\ln u + (1-u)\ln(1-u) + \ln f_{12}(u)] - p_3p_{12} - \lambda_{12} = 0$ - -(42)

Using equations (33) and (40)

$$-\ln(p_1p_6) + \frac{\lambda_6}{p_1p_6} - \eta + 1 = 0$$

$\lambda_6 = [\eta - 1 + \ln(p_1p_6)]\, p_1p_6$ - -(43)

by substituting the value of λ_6 in equation (40), we get

$-p_1p_6[u\ln u + (1-u)\ln(1-u) + \ln f_6(u)] - p_1p_6 - [\eta - 1 + \ln(p_1p_6)]\, p_1p_6 = 0$

$\Rightarrow u\ln u + (1-u)\ln(1-u) + \ln f_6(u) + \ln(p_1p_6) + \eta = 0$ - -(44)

$\Rightarrow f_6(u) = e^{-\eta} u^{-u} (1-u)^{-(1-u)} p_1^{-1} p_6^{-1}$ - -(45)

Similarly, using equations (36) and (41)

$\lambda_9 = [\eta - 1 + \ln(p_2p_9)]p_2p_9$ - -(46)

again, from equations (41) and (46), we get

$f_9(u) = e^{-\eta} u^{-u} (1-u)^{-(1-u)} p_2^{-1} p_9^{-1}$ - -(47)

And, by using equations (39) and (42)

$\lambda_{12} = [\eta - 1 + \ln(p_3p_{12})]\, p_3p_{12}$ - -(48)

from equations (42) and (48)

$f_{12}(u) = e^{-\eta} u^{-u} (1-u)^{-(1-u)} p_3^{-1} p_{12}^{-1}$ - -(49)

Now, from equation (44)

$\ln(p_1p_6) = -u\ln u - (1-u)\ln(1-u) - \ln f_6(u) - \eta$

Let $\mu = -u\ln u - (1-u)\ln(1-u) - \ln f_6(u)$

$\Rightarrow \ln(p_1p_6) = \mu - \eta \Rightarrow p_1p_6 = e^{\mu-\eta}$ - -(50)

Similarly $p_2p_9 = e^{\nu-\eta}$, $p_3p_{12} = e^{\gamma-\eta}$

Now, from equations (45) and (50)

$$f_6(u) = e^{-\eta}u^{-u}(1-u)^{-(1-u)}e^{\eta-\mu}$$

$\Rightarrow f_6(u) = e^{-\mu}u^{-u}(1-u)^{-(1-u)}$ - -(51)

Similarly, $f_9(u) = e^{-\nu}u^{-u}(1-u)^{-(1-u)}$ - -(52)

And $f_{12}(u) = e^{-\gamma}u^{-u}(1-u)^{-(1-u)}$ - -(53)

where,

$$\nu = -u\ln u - (1-u)\ln(1-u) - \ln f_9(u)$$
$$\gamma = -u\ln u - (1-u)\ln(1-u) - \ln f_{12}(u)$$

On integrating equation (51) with respect to u within the limits 0 to 1.

$$\int_0^1 f_6(u)du = e^{-\mu}\int_0^1 u^{-u}(1-u)^{-(1-u)}\,du$$
$$1 = e^{-\mu}\int_0^1 u^{-u}(1-u)^{-(1-u)}\,du$$
$$\Rightarrow \mu = .5165$$

Similarly, from equations (52) and (53), we get

$$\nu = \gamma = .5165$$

Now, given constraint is

$$p_1(p_4 + p_5 + p_6) + p_2(p_7 + p_8 + p_9) + p_3(p_{10} + p_{11} + p_{12}) = 1$$
$$\Rightarrow p_1p_4 + p_1p_5 + p_1p_6 + p_2p_7 + p_2p_8 + p_2p_9 + p_3p_{10} + p_3p_{11} + p_3p_{12} = 1$$

On putting values of p_ip_j, where i = 1, 2, 3 ; j = 4,5,_ _ _,12

$$e^{-\eta} + e^{-\eta} + e^{\mu-\eta} + e^{-\eta} + e^{-\eta} + e^{\nu-\eta} + e^{-\eta} + e^{-\eta} + e^{\gamma-\eta} = 1$$
$$\Rightarrow 6e^{-\eta} + e^{\mu-\eta} + e^{\nu-\eta} + e^{\gamma-\eta} = 1$$
$$\Rightarrow e^{-\eta}(6 + e^{\mu} + e^{\nu} + e^{\gamma}) = 1 \text{ [Since } \mu=\upsilon=\gamma=0.5165]$$
$$\Rightarrow e^{-\eta}(6 + 1.6722 + 1.6722 + 1.6722) = 1$$
$$\Rightarrow e^{-\eta}(11.0166) = 1$$
$$\Rightarrow e^{-\eta} = 0.0908$$

So,

$$p_1p_4 = p_1p_5 = p_2p_7 = p_2p_8 = p_3p_{10} = p_3p_{11} = e^{-\eta} = 0.0908 \qquad \text{- -(54)}$$
$$p_1p_6 = e^{\mu-\eta}$$
$$= (1.6722)(0.0908)$$
$$= .1518$$

Similarly, $p_2p_9 = p_3p_{12} = .1518$ - -(55)

Now, we can prove that when market shares are not prescribed then all market shares are equal.
Market share can be given as:

$$p_2p_8 + p_2p_9\int_0^1 (1-u)f_9(u)\,du + p_3p_{10} + p_3p_{12}\int_0^1 uf_{12}(u)\,du = m_1 \qquad \text{- -(56)}$$

Now, making use of the symmetry in density function

$$\int_0^1 (1-u)f_9(u)\,du = \int_0^1 uf_{12}(u)\,du = \frac{1}{2}$$

So, from equation (56)

$$p_2p_8 + \frac{1}{2}p_2p_9 + p_3p_{10} + \frac{1}{2}p_3p_{12} = m_1$$

by putting the values from equations (54) and (55)

$$0.0908 + \frac{1}{2}(.1518) + 0.0908 + \frac{1}{2}(.1518) = m_1$$

by solving $m_1 = \frac{1}{3}$

Similarly, other market shares are given as:

$$p_1p_4 + \frac{1}{2}p_1p_6 + p_3p_{11} + \frac{1}{2}p_3p_{12} = m_2$$

and

$$p_1p_5 + \frac{1}{2}p_1p_6 + p_2p_7 + \frac{1}{2}p_2p_9 = m_3$$

from above equations , we obtain

$$m_2 = m_3 = \frac{1}{3}$$
$$\Rightarrow m_1 = m_2 = m_3 = \frac{1}{3}$$

Hence the three market shares are equal.
Now, maximum entropy is given as

$$S_{max} = -(p_1p_4)\ln(p_1p_4) - (p_1p_5)\ln(p_1p_5) - (p_1p_6)\ln(p_1p_6) - (p_2p_7)\ln(p_2p_7) - (p_2p_8)\ln(p_2p_8) - (p_2p_9)\ln(p_2p_9) - (p_3p_{10})\ln(p_3p_{10}) - (p_3p_{11})\ln(p_3p_{11}) - (p_3p_{12})\ln(p_3p_{12})$$

entropy is maximum when all market shares are equal. So, we use equations (54) & (55)
$S_{max} = 2.1655$

Hence from equations (54) and (55), we conclude that 9% customers are loyal to single brand and 15% customers are wavering between two brands.

Concluding Remarks:
We discussed the brand purchasing behavior of customers in conditional sense and estimated the proportions of customers loyal to each of the brands and of those loyal to two brands. So, we conclude that

1. 9% customers are loyal to single brand and 15% customers are wavering between two brands.
2. The entropy is maximum when all market shares are equal.
3. If market shares are not prescribed then all market shares are equal.
4. As we receive the information about market shares, entropy decreases.
5. The total switching is maximum for equal market shares.

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