*Section A-Research paper*
*ISSN 2063-5346*

# MODELLING OF LAMB WAVES IN A SEMICONDUCTOR MATERIAL PLATE LOADED WITH FLUID

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**DOI: 10.48047/ecb/2023.12.si4.1574**

## ABSTRACT

This paper is aimed at to investigate the propagation of acousto diffusive (ETP) Lamb waves in homogenous isotropic, thermally conducting, semiconductor material in contact with fluid half space or layer. The concept of relaxation of heat and charge carrier fields is also taken. Secular equations, in isolated mathematical conditions and compact form, for the acousto diffusive lamb waves in semiconducting material half space or a layer with thickness d are derived. The different regions of secular equations are obtained and deduced. The secular equations for acousto diffusive lamb waves are also obtained and deduced. Finally, the numerical solution of various secular equations and other relevant relations is carried out for Germanium (Ge) semiconductor material with the help of functional iteration numerical technique. The dispersion curves, attenuation coefficient of the waves are computed and presented graphically, in order to illustrate and compare the analytical results.

***Keywords:*** Semiconductors; Relaxation time; holes; Germanium; Lamb waves; Diffusion; Lifetime; Fluid.

## 1. INTRODUCTION

Nondestructive evaluation and testing for material characterization and condition monitoring is a critical technology for many industries. Recently, resurgent interest in Lamb waves was partially initiated by its application of multisensors [1–2]. Schoch [3] derived the dispersion relation for leaky Lamb waves for an isotropic plate immersed in an inviscid liquid. Incidentally, the dispersion equations also have an interface wave solution whose velocity is slightly less than the bulk sound velocity in the liquid and most of the energy is in the liquid. It is often called the Scholte wave after Scholte [4]. Watkins et al. [5] calculated the attenuation of Lamb waves in the presence of an inviscid liquid using an acoustic impedance method. Wu and Zhu [6] studied the propagation of Lamb waves in a plate bordered with inviscid liquid layers on both sides. The dispersion equations of this case were derived and solved numerically. Zhu and Wu [7-8] derived the dispersion equations of Lamb waves of a plate bordered with viscous liquid layer or half-space viscous liquid on both sides. Maruszewski [9-13] presented theoretical considerations of the simultaneous interactions of elastic, thermal and diffusive of charge carrier fields in semiconductors. Sharma and Thakur [14] studied the plane harmonic elastodiffusive surface wave in semiconductor materials.

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Sharma, et al. [15] have also been studied about the propagation characteristics of thermo-elastodiffusive surface acoustic waves in semiconductor materials. Sharma and Thakur [16] studied the propagation characteristics of elasto-thermodiffusive waves in semiconductor materials half-space. Recently, Sharma, et al. [17] have also been studied about elasto-thermodiffusive surface waves in a semiconductor half-space underlying a fluid with varying temperature. Sharma and Pathania [18] studied the Lamb waves propagation characteristics.

Here we consider the problem of Lamb waves and present a systematic analysis of surface wave propagation in thermoelastic semiconductor materials in contact with fluid half space or a layer of finite thickness d based on the governing equations derived by Maruszewski [12] and non-dimensionalzed by Sharma and Thakur [16]. After deriving the secular equations for Lamb waves generation in semiconductor materials in contact with fluid by considering the effect of relaxation and life time of charge carrier fields in addition to thermal relaxation time the various characteristics of elasto-thermodiffusive (ETP) waves have been investigated. These waves are coupled with each other and get modified due to thermal variations, thermal relaxation time and life/relaxation time of holes charge carrier fields. The analytical results so obtained have been verified numerically and are illustrated graphically in case of Germanium (Ge) semiconductor half space (or a layer).

## 2. ANALYTICAL FARMULATIONS

We start by considering a two dimensional, infinitely large extrinsic, homogeneous, isotropic, thermally conducting, elastic semiconductor plate of thickness 2d medium, initially under undeformed state at uniform temperature $T_0$. The plate is in contact with fluid half space or a layer of thickness $h$ on both top and bottom. We take the origin of coordinate system $(x, y, z)$ on the middle surface of the plate. The $x-y$ plane is chosen to coincide with the middle surface and the z-axis normal to it along the thickness as illustrated in Fig.a below. We take x-z plane as the plane of incidence and assume that all the particles on a line parallel to y-axis are equally displaced so that all the field quantities are independent of y-coordinate.

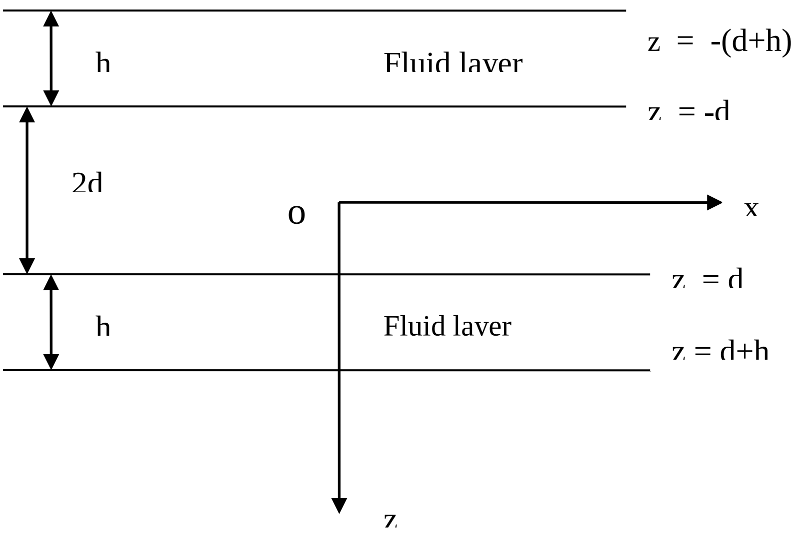


Fig.1. Geometry of the problem

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*ISSN 2063-5346*

The x-axis is taken along the direction of acoustic wave propagation in the semiconductor half-space and the densities of the charge carriers at doping level are assumed to be of such values that the life time $t_p^+$, and the diffusion coefficients $D^p$ are independent of them. Further the disturbance is assumed to be confined to the neighbourhood of the free surface and hence vanishes as $z \to \infty$. In linear theory of thermoelasticity in semiconductors, the non-dimensional governing field equations for temperature $T(x,z,t)$, displacement vector $u(x,z,t)=(u,0,w)$, holes diffusion fields $P(x,z,t)$ respectively; in the absence of body forces and heat sources; are given by Maruszewski [12].

$$\mu \nabla^2 \vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} - \lambda^p \nabla P - \lambda^T \nabla T = \rho \ddot{\vec{u}} \tag{1.1}$$

$$K\nabla^2 T + m^{pq}\nabla^2 P - (1+t^Q \frac{\partial}{\partial t})\left(\rho C_e \dot{T} + \rho T_0 \alpha^p \dot{P} + T_0 \lambda^T \nabla.\dot{\vec{u}}\right) - \rho a_1^p \dot{P} = a_1^P \left(\frac{\rho}{t_p^+}\right) P \tag{1.2}$$

$$\rho D^p \nabla^2 P + m^{qp}\nabla^2 T - \rho\left(1 - a_2^p T_0 \alpha^p + t^p \frac{\partial}{\partial t}\right)\dot{P} - a_2^p\left(\rho C_e \dot{T} + T_0 \lambda^T \nabla.\dot{\vec{u}}\right) = -\left(1 + t^p \frac{\partial}{\partial t}\right)\left(\frac{\rho}{t_p^+}\right) P \tag{1.3}$$

where we have used the notations

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, a_1^p = \frac{a^{Qp}}{a^Q}, a_2^p = \frac{a^{Qp}}{a^p}, P = p - p_0,$$

$$\lambda^T = (3\lambda + 2\mu)\alpha_T, \; T = T_1 - T_0 \tag{2}$$

Here $\lambda$ , $\mu$ are Lame parameters ; $\rho$ is the density of the semiconductor; $\lambda^p$ are the elastodiffusive constants of holes, $\alpha_T$ is the coefficient of linear thermal expansion of the material; K is the thermal conductivity, $\alpha^p$ are thermo diffusive constants of holes; $a^{Qp}, a^Q, a^p$ are the flux like constant; $D^p$ are the diffusion coefficients of holes and electrons. The quantities $m^{pq}, m^{qp}$ are the Peltier-Seebeck-Dufour-Soret like constants; $t^Q, t^p$ are the relaxation times of heat, holes fields, $C_e$ is the specific heat, $t_p^+$, denotes the life times of the charge carriers' fields; $p_0$ , $p$ equilibrium and non-equilibrium values of holes and electrons concentrations, respectively.

Further the equations (1) are subjected to following assumption:

(i) All the considerations are made in the frame work of the phenomenological model.
(ii) The electric neutrality of the semiconductor is satisfied.
(iii) The magnetic field effect is ignored.
(iv) The mass of charge carrier fields is negligible.
(v) The electron field with in the boundary layer is very weak and can be neglected.
(vi) The recombination function of holes is reduced on the basis of the facts that take care of defects and hence concentration values of the charge carrier field [20].

## 3. BOUNDARY CONDITIONS

The boundary conditions at solid-liquid interfaces $z = \pm d$ to be satisfied are:

$$\lambda u_{,x} + (\lambda + 2\mu) w_{,z} - \lambda^T T - \lambda^p P = 0 \tag{3.1}$$

$$u_{,z} + w_{,x} = 0 \tag{3.2}$$

$$KT_{,z}+m^{pq}P_{,z}+K^{S}T+\rho a_1^p s^p P=0 \qquad (3.3)$$

$$m^{qp}T_{,z}+\rho D^p P_{,z}+a_2^p K^S T+\rho s^p\left(1+t^p\frac{\partial}{\partial t}\right)p=0 \qquad (3.4)$$

Where $K^S$, $s^p$ are respectively, the surface heat conduction coefficient and surface recombination velocities.

In the following analysis we shall confine our discussion to the interaction of elastic, thermal and holes charge carrier fields only and consequently complete equilibrium state of electrons concentration is assumed to be established. Where we have used the quantities

$$x'=\frac{\omega^* x}{c_1},\quad z'=\frac{\omega^* z}{c_1},\quad t'=\omega^* t,\quad T'=\frac{T}{T_0},\quad P'=\frac{P}{p_0},$$

$$u'=\frac{\rho\omega^* c_1}{\lambda^T T_0}u,\quad w'=\frac{\rho\omega^* c_1}{\lambda^T T_0}w,\quad \tau^Q=t^Q\omega*,\quad t^{p'}=t^p\omega^*,$$

$$t_p^{+'}=t_p^{+}\omega*,\quad \delta^2=\frac{c_2^{\,2}}{c_1^{\,2}},\quad \in_T=\frac{\lambda^{T^2}T_0}{\rho C_e(\lambda+2\mu)},\quad \omega^*=\frac{C_e(\lambda+2\mu)}{K},$$

$$c_1^{\,2}=\frac{\lambda+2\mu}{\rho},\quad c_2^{\,2}=\frac{\mu}{\rho},\quad \chi=\frac{K}{\rho C_e},\quad \overline{\lambda}_p=\frac{\lambda^p p_0}{\lambda^T T_0},$$

$$\in_p=\frac{a_2^p K T_0}{\rho p_0 D^p},\quad \varepsilon^{pq}=\frac{m^{pq}p_0}{KT_0},\quad a_0^{\,p}=\frac{a_1^{\,p}p_0}{C_e T_0} \qquad (4)$$

where $\omega^*$ and $\varepsilon_T$ are respectively the characteristics frequency and thermoelastic coupling parameters of the semiconductor.

Upon introducing the quantities (4) and setting in the basic equations (1), we obtain

$$\delta^2\nabla^2\vec{u}+(1-\delta^2)\nabla\nabla.\vec{u}-\ddot{\vec{u}}-\overline{\lambda}_p\nabla P-\nabla T=0, \qquad (5.1)$$

$$\nabla^2 T-(\dot{T}+t^Q\ddot{T})+\varepsilon^{pq}\nabla^2 P-\left((a_0^p+\alpha_0^p)\dot{P}+t^Q\alpha_0^p\ddot{P}+\frac{a_0^p}{t_p^+}P\right)-\in_T\nabla\left(\dot{\vec{u}}+t^Q\ddot{\vec{u}}\right)=0, \qquad (5.2)$$

$$\nabla^2 P+\frac{k}{D^p}\left[\frac{1}{t_p^+}P-\left(1-\frac{\in_p\alpha_0^p D^p}{\chi}-\frac{t^p}{t_p^+}\right)\dot{P}-t^p\ddot{P}\right]-\in_p\dot{T}+\varepsilon^{qp}\nabla^2 T-\in_p\in_T\nabla\cdot\dot{\vec{u}}=0 \qquad (5.3)$$

We introduce the scalar point potential function $\phi$ and vector point potential function $\vec{\psi}=(0,-\psi,0)$ through the relations

$$u=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z},\quad w=\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x} \qquad (6)$$

The substitution of equations (6) in equations (5) leads to

$$\nabla^2\phi-\ddot{\phi}-\overline{\lambda}_p P-T=0, \qquad (7.1)$$

$$-\varepsilon_T\nabla^2(\dot{\phi}+t^Q\ddot{\phi})+\varepsilon^{pq}\nabla^2 P-\left\{\frac{t^Q\alpha_0^p\partial^2}{\partial t^2}+\left(a_0^{\,p}+\alpha_0^p\right)\frac{\partial}{\partial t}+\frac{a_0^p}{t_p^{\,+}}\right\}P+\nabla^2 T-\left(\dot{T}+t^Q\ddot{T}\right)=0, \qquad (7.2)$$

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$$-\varepsilon_p\varepsilon_T\nabla^2\dot{\phi}+\nabla^2P-\frac{K}{\rho C_eD^p}\left[-\frac{1}{t_p^+}+\left(1-\frac{\epsilon_p\,\alpha_0^pD^p}{\chi}-\frac{t^p}{t_p^+}\right)\frac{\partial}{\partial t}+\frac{t^p\partial^2}{\partial t^2}\right]P-\varepsilon_p\dot{T}+\varepsilon^{qp}\nabla^2T=0, \tag{7.3}$$

$$\nabla^2\psi = \frac{\ddot{\psi}}{\delta^2} \tag{7.4}$$

The equation (7.4) corresponds to purely transverse waves which get decoupled from rest of the motion and are not affected by the thermal and charge carrier field. Thus elastic wave can travel in space without attenuation. The equations (7.1) and (7.3) in the above system can be simplified under the assumption that considered semiconductor is of relaxation type. For such materials, the diffusion approximation of the physical process ceases to be obligatory and the diffusion / life times $(t^p, t_p^+)$ becomes comparable to each other in their values $(t^p = t_p^+)$

In the case of fluid medium, we have

$$\mathrm{u}_\mathrm{L}=\frac{\partial\phi_L}{\partial x}+\frac{\partial\psi_L}{\partial z},\quad w_L=\frac{\partial\phi_L}{\partial z}-\frac{\partial\psi_L}{\partial x} \tag{8}$$

where $\phi_L$ and $\psi_L$ are respectively, the scalar and vector point velocity potentials. Thus in the fluid medium the governing equations are given by

$$\nabla^2\phi_L-\frac{1}{\left(\delta_L^2\left(1+\varepsilon_L\right)+\frac{4}{3}\nu_L\frac{\partial}{\partial t}\right)}\ddot{\phi}=0 \tag{9.1}$$

$$\nabla^2\psi_L-\frac{1}{\nu_L}\dot{\psi}_L=0 \tag{9.2}$$

$$T_L=-\frac{\varepsilon_L\rho_L\delta_L^2}{\bar{\beta}\rho}\nabla^2\phi_L \tag{9.3}$$

where

$$\varepsilon_L=\frac{\beta^{*^2}T_0^*}{\rho_LC_v^*\lambda_L},\ \delta_L^2=\frac{c_L^2}{c_1^2},\ c_L^2=\frac{\lambda_L}{\rho_L},\ \nu_L=\frac{\mu_L\omega^*}{\rho_Lc_1^2},\ \bar{\beta}=\frac{\beta^*}{\lambda^T},\ \beta^*=3\lambda_L\alpha^*,\ T_L'=\frac{T_L-T_0^*}{T_0} \tag{10}$$

Here, $c_L$ is the velocity of sound in the liquid, $\lambda_L$ is the bulk modulus, $\rho_L$ and $\mu_L$ is respectively the density and viscosity of the liquid; $\alpha^*$ is the coefficient of volume thermal expansion; $T_L$ is the temperature deviation of liquid medium from ambient temperature $T_0^*$. The continuity of stresses, displacements, temperature, holes concentration, heat and holes fluxes leads to the following conditions required to be satisfied at the solid-liquid interface $(z=0)$ in the non-dimensional form. The non-dimensional boundary conditions in equations (3) at the solid-liquid interface $(z=\pm d)$ are given by

$$\frac{\ddot{\phi}}{\delta^2}-2\left(\phi_{,xx}+\psi_{,xz}\right)=\frac{-\rho_L}{\rho\delta^2}\left\{\ddot{\phi}_L-2\nu_L\left(2\dot{\phi}_{L,xx}+\dot{\phi}_{L,zz}-\dot{\psi}_{L,xz}\right)\right\} \tag{11.1}$$

$$\frac{\ddot{\psi}}{\delta^2}-2\left(\psi_{,xx}-\phi_{,xz}\right)=\frac{-\rho_L\nu_L}{\rho\delta^2}\left\{2\dot{\phi}_{L,xz}+\dot{\psi}_{L,zz}-\dot{\psi}_{L,xx}\right\} \tag{11.2}$$

$$\phi_{,z}-\psi_{,x}=\phi_{L,z}-\psi_{L,x} \tag{11.3}$$

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$$\phi_{,x} + \psi_{,z} = \phi_{L,x} + \psi_{L,z} \tag{11.4}$$

$$T_{,z} + h_T(T - T_L) + \varepsilon^{pq}\left(P_{,z} + h_p P\right) = 0 \tag{11.5}$$

$$P_{,z} + h_p \bar{\varepsilon}_{pq}\left(P + t^p \dot{P}\right) + \varepsilon^{qp}\left(T_{,z} + h_T \frac{\varepsilon_p}{\varepsilon^{qp}}(T - T_L)\right) = 0 \tag{11.6}$$

Where $h_T = \dfrac{K^S c_1}{K\,\omega *}$, $h_p = \dfrac{a_0{}^p s^p}{\varepsilon^{pq} c_1}$, $\bar{\varepsilon}_{pq} = \dfrac{m^{pq}}{\rho D^p a_1{}^p}$

Here $h_T$, $h_p \to 0$ correspond to thermally insulated and charge free (no flow of hole flux across the boundary) boundary and $h_T$, $h_p \to \infty$ refers to isothermal and equipotential one.

## 4. FORMAL SOLUTION

We consider the case of time harmonic waves so that the solutions $\phi$, $T$, $P$, $\psi$, $\phi_L$, $\psi_L$ of equations (6) take the form

$$(\phi, T, P, \psi, \phi_L, \psi_L) = \left(\bar{\phi}(z), \bar{T}(z), \bar{P}(z), \bar{\psi}(z), \bar{\phi}_L, \bar{\psi}_L\right)\exp\{ik(x - ct)\} \tag{12}$$

where $c = \dfrac{\omega}{k}$, is the phase velocity, k and $\omega$ are wave number and angular frequency respectively of the waves. Upon using solutions (12) in equations (7), after lengthy but straight forward algebraic reductions and simplifications, we obtain the following formal solution for $(\phi, P, T, \psi, \phi_L)$ that satisfies the radiation conditions $R_e(m_i) \geq 0$ and $\mathrm{Re}(\gamma_i) \leq 0$ $i = 1, 2, 3$ we have

$$\phi = \sum_{i=1}^{3}\left(A_i \sin m_i z + B_i \cos m_i z\right)\exp ik(x - ct) \tag{13.1}$$

$$T = \sum_{i=1}^{3} W_i\left(A_i \sin m_i z + B_i \cos m_i z\right)\exp ik(x - ct) \tag{13.2}$$

$$P = \sum_{i=1}^{3} S_i\left(A_i \sin m_i z + B_i \cos m_i z\right)\exp ik(x - ct) \tag{13.3}$$

$$\psi = \left(A_4 \sin \beta z + B_4 \cos \beta d\right)\exp ik(x - ct) \tag{13.4}$$

$$\phi_{L_1} = \begin{cases} \left[A_5 \sinh \gamma_1 (z - (d + h))\right]\exp\{ik(x - ct)\}, \ \textit{for finite layer}, \quad d < z < d + h, \\ A_5 \exp\{\gamma_1 z + ik(x - ct)\}, \ \textit{for halfspace} \ (d \to \infty) \end{cases} \tag{13.5}$$

$$\phi_{L_2} = \begin{cases} \left[A_6 \sinh \gamma_1 (z + (d + h))\right]\exp\{ik(x - ct)\}, \ \textit{for finite layer}, \ -(d + h) < z < -d, \\ A_6 \exp\{\gamma_1 z + ik(x - ct)\}, \ \textit{for halfspace} \ (d \to \infty) \end{cases} \tag{13.6}$$

$$\psi_{L_1} = \begin{cases} \left[A_7 \sinh \gamma_2 (z - (d + h))\right]\exp\{ik(x - ct)\}, \ \textit{for finite layer}, \quad d < z < d + h, \\ A_7 \exp\{\gamma_2 z + ik(x - ct)\}, \ \textit{for halfspace} \ (d \to \infty) \end{cases} \tag{13.7}$$

$$\psi_{L_2} = \begin{cases} \left[A_8 \sinh \gamma_2 (z + (d + h))\right]\exp\{ik(x - ct)\}, \ \textit{for finite layer}, \ -(d + h) < z < -d, \\ A_8 \exp\{\gamma_2 z + ik(x - ct)\}, \ \textit{for halfspace} \ (d \to \infty) \end{cases} \tag{13.8}$$

$$T_L = s_L \phi_L \tag{13.9}$$

where $S_i = -\dfrac{\varepsilon^{qp}\left(m_i^2-\alpha^2\right)\left(m_i^2-\beta_1^2\right)+i\omega\varepsilon_p\varepsilon_T\left(m_i^2-k^2\right)}{\left(1-\overline{\lambda_p}\varepsilon^{qp}\right)m_i^2-\left(\alpha_p^{*2}-\overline{\lambda_p}\varepsilon^{qp}\beta_1^2\right)}$, $i=1,2,3$

$$W_i = m_i^2-\alpha^2-\overline{\lambda_p}S_i,\ i=1,2,3\ ,\ \gamma_1'^2 = k^2\left[1-\frac{c^2}{\delta_L^2\left(1+\varepsilon_L\right)-\frac{4}{3}i\omega\nu_L}\right]$$

$$m_i^2 = k^2\left(1-a_i^2c^2\right), i=1,2,3,\ \gamma_2'^2 = k^2\left[1-\frac{ic^2}{\omega\nu_L}\right]$$

$$s_L = \frac{\varepsilon_L\ \rho_L\delta_L^2\omega^2}{\overline{\beta}\rho\left(\delta_L^2\left(1+\varepsilon_L\right)-\frac{4}{3}i\omega\nu_L\right)}$$

Here $a_i^2,\ i=1,\ 2,\ 3$ are the roots of the complex cubic equation

$$a^6-Aa^4+Ba^2-C = 0 \tag{14}$$

where $A,\ B$ and $C$ are given by

$$A=\frac{1-\varepsilon^{qp}\varepsilon^{np}+\tau_p^*+\tau^Q(1+\varepsilon_T)-\varepsilon^{qp}\left(\tau_p'+\varepsilon_T\tau^Q\overline{\lambda}_p\right)+i\omega^{-1}\varepsilon_p\left\{\varepsilon_T\overline{\lambda_p}-\varepsilon^{pq}\left(1+\varepsilon_T\right)\right\}}{1-\varepsilon^{pq}\varepsilon^{qp}}$$

$$B=\frac{\tau^Q+\tau_p^*\left\{1+\tau^Q\left(1+\varepsilon_T\right)\right\}-\varepsilon^{qp}\tau_p'-i\omega^{-1}\varepsilon_p\left(\varepsilon^{pq}+\tau_p'\left(1+\varepsilon_T\right)\right)}{1-\varepsilon^{pq}\varepsilon^{qp}}$$

$$C=\frac{\tau^Q\ \tau_p^*-i\omega^{-1}\varepsilon_p\tau_p'}{1-\varepsilon^{pq}\varepsilon^{qp}} \tag{15}$$

$$\tau^Q = t^Q+i\omega^{-1}\quad ,\qquad \tau_p{}' = t^Q\alpha_0^p+i\omega^{-1}(\alpha_0^p+a_0^p)-a_0^p\omega^{-2}/t_p^+$$

$$\tau_p{}^* = \frac{K}{\rho C_eD^p}\left[t^p+i\omega^{-1}\left(1-\frac{\in_p\ \alpha_0^pD^p}{\chi}-\frac{t^p}{t_p{}^+}\right)+\frac{1}{\omega^2t_p{}^+}\right] \tag{16}$$

$$\alpha_p^{*2} = k^2(1-\tau_p^*c^2),\quad \tau^Q = t^Q+i\omega^{-1},$$

$$\beta_1^2 = k^2(1-\frac{i\omega^{-1}\varepsilon_pc^2}{\varepsilon^{qp}}),\quad \alpha^2 = k^2(1-c^2), \tag{17}$$

In case of relaxation type semiconductor life time and relaxation time are comparable $t^p = t_p^+$ and consequently $\tau_p^*$ gets modified. In general, the characteristic roots $m_i$ $(i=1,2,3)$ are complex and as we are considering surface waves only, so without loss of generality we choose only that form of $m_i$ and $\gamma_i$ which satisfies the radiation condition. Hence the solution is a superposition of the plane waves attenuating with depth.

## 5. SECULAR EQUATIONS AND THEIR SOLUTION

We consider the situation in which p-type semiconductor plate is bordered with viscous fluid. Upon applying the required interface boundary conditions (11) at the fluid-solid - interface $\left(z = \pm d\right)$ and using equations (13.1)-(13.9). We obtain a homogenous system of

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twelve equations in twelve unknowns $\left[A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, B_1, B_2, B_3, B_4\right]$.

$$p\sum_{i=1}^{3}\left(A_i s_i + B_i c_i\right) - q\left(A_4 c_4 - B_4 s_4\right) - \left(\omega_L + 2a^* k^2\left(\gamma_1^2 - 2\right)\right)A_5 s_5 - 2ia^* k^2 \gamma_2 c_6 A_7 = 0 \qquad (18.1)$$

$$p\sum_{i=1}^{3}\left(B_i c_i - A_i s_i\right) - q\left(A_4 c_4 + B_4 s_4\right) + \left(\omega_L + 2a^* k^2\left(\gamma_1^2 - 2\right)\right)A_6 s_5 - 2ia^* k^2 \gamma_2 c_6 A_8 = 0 \qquad (18.2)$$

$$p\left(A_4 c_4 + B_4 s_4\right) - \sum_{i=1}^{3} 2im_i k\left(A_i c_i - B_i s_i\right) - 2ia^* k^2 \gamma_1 c_5 A_5 - (\gamma_2^2 + 1)a^* k^2 s_6 A_7 = 0 \qquad (18.3)$$

$$p\left(A_4 c_4 - B_4 s_4\right) - \sum_{i=1}^{3} 2im_i k\left(A_i c_i + B_i s_i\right) - 2ia^* k^2 \gamma_1 c_5 A_5 + (\gamma_2^2 + 1)a^* k^2 s_6 A_7 = 0 \qquad (18.4)$$

$$\sum_{i=1}^{3} m_i k\left(A_i c_i - B_i s_i\right) - ik\left(A_4 s_4 + B_4 c_4\right) - \gamma_1 k A_5 c_5 + iks_6 A_7 = 0 \qquad (18.5)$$

$$\sum_{i=1}^{3} m_i k\left(A_i c_i + B_i s_i\right) - ik\left(B_4 c_4 - A_4 s_4\right) - \gamma_1 k A_6 c_5 + iks_6 A_8 = 0 \qquad (18.6)$$

$$\sum_{i=1}^{3} ik\left(B_i c_i - A_i s_i\right) + \beta k\left(A_4 c_4 - B_4 s_4\right) - iks_5 A_5 - \gamma_2 kc_6 A_7 = 0 \qquad (18.7)$$

$$\sum_{i=1}^{3} ik\left(B_i c_i - A_i s_i\right) + \beta k\left(A_4 c_4 + B_4 s_4\right) + iks_5 A_5 - \gamma_2 kc_6 A_7 = 0 \qquad (18.8)$$

$$\sum_{i=1}^{3}[\left(m_i c_i + h_T s_i\right)W_i + \varepsilon^{pq}\left(m_i c_i + h_p s_i\right)S_i]A_i + \left[\left(-m_i s_i + h_T c_i\right)W_i + \varepsilon^{pq}\left(-m_i s_i + h_p c_i\right)S_i\right]B_i - s_L h_T s_5 A_5 = 0 \qquad (18.9)$$

$$\sum_{i=1}^{3}[\left(m_i c_i - h_T s_i\right)W_i + \varepsilon^{pq}\left(m_i c_i - h_p s_i\right)S_i]A_i + \left[\left(m_i s_i + h_T c_i\right)W_i + \varepsilon^{pq}\left(m_i s_i + h_p c_i\right)S_i\right]B_i + s_L h_T s_5 A_6 = 0 \qquad (18.10)$$

$$\begin{aligned}&\sum_{i=1}^{3}[\varepsilon^{qp}\left(m_i c_i + h_T \frac{\varepsilon_p}{\varepsilon^{qp}} s_i\right)W_i + \left(m_i c_i + h_p \bar{\varepsilon}_{pq} s_i\right)S_i]A_i + \\ &\left[\varepsilon^{qp}\left(-m_i s_i + h_T \frac{\varepsilon_p}{\varepsilon^{qp}} c_i\right)W_i + \varepsilon^{pq}\left(-m_i s_i + h_p \bar{\varepsilon}_{pq} c_i\right)S_i\right]B_i - s_L h_T s_5 A_5 = 0\end{aligned} \qquad (18.11)$$

$$\begin{aligned}&\sum_{i=1}^{3}[\varepsilon^{qp}\left(m_i c_i - h_T \frac{\varepsilon_p}{\varepsilon^{qp}} s_i\right)W_i + \left(m_i c_i - h_p \bar{\varepsilon}_{pq} s_i\right)S_i]A_i + \\ &\left[\varepsilon^{qp}\left(m_i s_i + h_T \frac{\varepsilon_p}{\varepsilon^{qp}} c_i\right)W_i + \varepsilon^{pq}\left(m_i s_i + h_p \bar{\varepsilon}_{pq} c_i\right)S_i\right]B_i + s_L h_T s_5 A_6 = 0\end{aligned} \qquad (18.12)$$

Where $p = \beta^2 + k^2$ , $\beta^2 = k^2\left(1 - \dfrac{c^2}{\delta^2}\right)$ , $q = 2i\beta k$

$s_i = \sin m_i d$ , $c_i = \cos m_i d$ $i = 1, 2, 3$

$\omega_L = \dfrac{\omega^2 \rho_L}{k^2 \rho \delta^2}$ , $a^* = \dfrac{i\omega \nu_L \rho_L}{\rho \delta^2}$

The system of equations (18) will have non-trivial solution of the resulting coupled equations, after lengthy but straight forward algebraic reductions and simplifications, the secular equations for Lamb wave is given below

$$\left( p^2 + A_0 + A_1 T_5^{'} T_6^{'} + A_2 \frac{T_6^{'}}{\left[T_4^{'}\right]^{\pm 1}} \right)\left(L_1 - L_2 + L_3\right) + \left( C_0 T_5^{'} + C_1 \frac{T_5^{'} T_6^{'}}{\left[T_4^{'}\right]^{\pm 1}} \right) F^{*}$$
$$= \left( 4\beta \left[ \frac{1}{T_4^{'}} \right]^{\pm 1} + B_0 \frac{1}{\left[T_4^{'}\right]^{\pm 1}} + B_1 T_5^{'} + B_2 \frac{T_5^{'} T_6^{'}}{\left[T_4^{'}\right]^{\pm 1}} \right)\left( m_1 L_1 \left[T_1\right]^{\pm 1} - m_2 L_2 \left[T_1\right]^{\pm 1} + m_3 L_3 \left[T_3\right]^{\pm 1} \right) \quad (19)$$

where

$$A_0 = -4a^{*2}$$
$$A_1 = \left(a^{*}\left(\gamma_2^2 + 1\right) + p\right)\left(\omega_L + 2a^{*}\left(\gamma_1^2 - 2\right) - p\right)$$
$$A_2 = \beta a^{*}(2 - p)\left(\gamma_2^2 + 3\right)$$
$$B_0 = -4\beta a^{*2}$$
$$B_1 = (2 - p)\left(\omega_L + 2a^{*}\left(\gamma_1^2 - 1\right)\right)$$
$$B_2 = -4\beta - 2\beta a^{*}\left(\gamma_2^2 + 1\right) + \beta\left(\omega_L + 2a^{*}\left(\gamma_1^2 - 2\right)\right)\left(2 + a^{*}\left(\gamma_2^2 + 1\right)\right)$$
$$C_0 = (2 - p)\left(2a^{*} + p\right)$$
$$C_1 = (p - 2)\beta\left(2 + a^{*}\left(\gamma_2^2 + 1\right)\right)$$
$$\omega_L = \frac{\omega^2 \rho_L}{k^2 \rho \delta^2}, T_i = \tan\left(m_i d\right), \; i = 1, 2, 3$$
$$T_4 = \tan\left(\beta d\right) \; T_i = \tanh\left(\gamma_i h\right), \; i = 5, 6;$$
$$F^{*} = \left( m_1 \left(S_2 - S_3\right) T_1^{'\pm} - m_2 \left(S_1 - S_3\right) T_2^{'\pm} + m_3 \left(S_1 - S_2\right) T_3^{'\pm} \right)\left( h_p \left( \varepsilon^{pq} \varepsilon_p - \varepsilon_{pq}^{'} \right) \right)$$
$$L_1 = P_2 Q_3 - P_3 Q_2$$
$$P_i = \left\{ \left(h_T - m_i\right) W_i + \varepsilon^{pq} \left(h_p - m_i\right) S_i \right\}, \quad i = 1, 2, 3$$
$$Q_i = \left\{ \left(h_T \varepsilon_p - \varepsilon^{qp} m_i\right) W_i + \left(h_p \bar{\varepsilon}_{pq}^{'} - m_i\right) S_i \right\}, \quad i = 1, 2, 3 \quad (20)$$

Here $L_2$ , $L_3$ can be obtained from $L_1$ by replacing the subscripts permutation (2, 3) with (1, 2), (1,3), respectively. The secular equation (17) governs the motion of modified guided elasto-thermodiffusive (ETP) Lamb waves in the instant analysis. It contains complete information about the phase velocity, attenuation coefficient and other characteristics of the ETP Lamb waves in a thermoelastic semiconductor plate underlying a fluid layer of finite thickness with varying temperature. In case the semiconductor plate is loaded with fluid half-space $\left(h \to \infty\right)$, the secular equation of modified guided ETP Lamb waves is again given by equation (17) with changed values of the coefficients $A_i$ , $B_i$ $\left(i = 0,1, 2\right)$ and $C_i$ $\left(i = 0, 1\right)$ which can be obtained from equation (18) by setting $T_5^{'} = 1 = T_6^{'}$.

In case of inviscid fluid $(\mu_L = 0 \Rightarrow \nu_L = 0 = a^{*})$, the secular equation (19) becomes

$$p^2 \left(L_1 - L_2 + L_3\right)$$
$$= \left( 4\beta \left[ \frac{1}{T_4^{'}} \right]^{\pm 1} + \omega_L (2 - p) T_5^{'} \right)\left( m_1 L_1 \left[T_1^{'}\right]^{\pm 1} - m_2 L_2 \left[T_2^{'}\right]^{\pm 1} + m_3 L_3 \left[T_3^{'}\right]^{\pm 1} \right) + p(2 - p) F^{*} \quad (21)$$

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The secular equation for the surface waves at the interface of inviscid fluid/germanium semiconductor plate $(h \to \infty)$ can be written from (19) by setting $T_5^{'} = 1$ .

In the absence of fluid $\left( \rho_L = 0 = \varepsilon_L = 0 \Rightarrow \omega_L = 0 = s_L \right)$, the secular equations (19) and (21) reduce to

$$p^2 \left( L_1 - L_2 + L_3 \right) - 4\beta \left[ \frac{1}{T_4^{'}} \right]^{\pm 1} \left( m_1 L_1 \left[ T_1^{'} \right]^{\pm 1} - m_2 L_2 \left[ T_2^{'} \right]^{\pm 1} + m_3 L_3 \left[ T_3^{'} \right]^{\pm 1} \right) = 0 \qquad (22)$$

The characteristic roots $m_i, \gamma_i \; (i = 1, 2, 3, 4)$ given by equation (14) are in general complex and therefore, the wave number and hence, phase velocities of the waves are complex quantities,

**Case I**

For isothermal $h_T \to \infty$ and isoconcentration $h_p \to \infty$ is obtained as the values of $P_i, Q_i$ is given by

$$P_i = \left\{ W_i + \varepsilon^{pq} S_i \right\}, \quad i = 1, 2, 3$$
$$Q_i = \left\{ W_i \varepsilon_p + \bar{\varepsilon}_{pq}^{'} S_i \right\}, \quad i = 1, 2, 3 \qquad (23)$$

**Case II**

For insulated $h_T \to 0$ and isoconcentration $h_p \to \infty$ is obtained as the values of $P_i, Q_i$ is given by

$$P_i = \left\{ W_i m_i T_i + \varepsilon^{pq} S_i \right\}, \quad i = 1, 2, 3$$
$$Q_i = \left\{ - m_i W_i T_i + \bar{\varepsilon}_{pq}^{'} S_i \right\}, \quad i = 1, 2, 3 \qquad (24)$$

**Case III**

For isothermal $h_T \to \infty$ and impermeable $h_p \to 0$ is obtained as the values of $P_i, Q_i$ is given by

$$P_i = \left\{ W_i - \left( \varepsilon^{pq} m_i S_i \right) T_i \right\}, \quad i = 1, 2, 3$$
$$Q_i = \left\{ \varepsilon_p W_i + S_i T_i \right\}, \quad i = 1, 2, 3 \qquad (25)$$

**Case IV**

For insulated $h_T \to 0$ and isoconcentration $h_p \to 0$ is obtained as the values of $P_i, Q_i$ is given by

$$P_i = m_i \left\{ W_i + \varepsilon^{pq} S_i \right\}, \quad i = 1, 2, 3$$
$$Q_i = \left\{ W_i \varepsilon^{qp} + S_i \right\}, \quad i = 1, 2, 3 \qquad (26)$$

If we write

$$c^{-1} = V^{-1} + i\,\omega^{-1} Q \qquad (27)$$

so that $k = R + iQ$ , $R = \frac{\omega}{V}$ where $V$ and $Q$ are real. The exponent in the plane wave solution (11) becomes $iR\left( x - V t \right) - Q x$, which shows that $V$ is the propagation speed and $Q$ the attenuation coefficient of the waves. Upon using equation (27) in secular equations (19) along with other relevant relations, the values of phase speed $(V)$ and attenuation coefficient

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$(Q)$ for the propagation of non-leaky and leaky Rayleigh waves can be obtained for different values of the wave number $(R)$. For known values of $m$ these equations can be solved to compute phase velocity $(V)$ and attenuation coefficient $(Q)$ for fixed values of wave number $(R)$ and given $V = V_0$, $Q = Q_0$. We shall use functional iteration method to solve the secular equations for phase velocity $(V)$ and attenuation coefficient $(Q)$ for different values of wave number $(R)$ and procedure adopted is outlined below.

The functional iteration method to solve of an equation of the form $g(V) = 0$, requires to put this equation in the form $V = F(V)$, so that the sequence $\{V_n\}$ of iterations for the desired root can be easily generated as follows: If $V = V_0$ be the initial approximation to the root, then we have $V_1 = F(V_0)$, $V_2 = F(V_1)$, $V_3 = F(V_2), \ldots$ and so on.

In general, $V_{n+1} = F(V_n)$, $n = 0, 1, 2, 3$.. If $|F^{'}(V) << 1|$, for all $V \in I$, then the sequence $\{V_n\}$ of approximations to the root will converge to the actual value $V = V_a$ of the root, provided $V_0 \in I$. Here $I$ is the interval in which roots is expected.

For initial values of $V = V_0$ and $Q = Q_0$, the values of $m_i$, $\gamma_i$ $(i = 1, 2, 3)$ can be obtained from equation (14) and then these values are further used in secular equations (19) obtain current values of $V$ and $Q$ which are then used to generate a new approximation until or unless the sequence of iterations to the values of $V$ or $Q$ converges to the desired level of accuracy. That is the condition $|V_{n+1} - V_n| < \in$, $\in$ being arbitrarily small number to be selected at random in order to achieve the accuracy level, is required to be satisfied. This process is continuously repeated for different values of wave number $(R)$ to obtain phase velocity $(V)$ and attenuation coefficient $(Q)$.

## 6. DISCUSSION OF THE SECULAR EQUATIONS

we may have $\alpha, \beta, m_1, m_2, m_3$ being real, zero or imaginary. Then the frequency equation (19) is correspondingly altered as follows.

### 6.1.1. REGION I

For $k \geq \frac{\omega}{\delta}$ implying that $c < \delta, 1/a_1, 1/a_2, 1/a_3$ and consequently, we have $\beta = i\beta$, $m_i = i\alpha_i$, $i = 1, 2, 3$. In this case the secular equation (19) is written by replacing circular tangent functions of $\beta, m_i$, $i = 1, 2, 3$ with hyperbolic tangent functions of $\beta$, $\alpha_i$, $i = 1, 2, 3$

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$$\left(p^2 + A_0 + A_1 T_5' T_6' + A_2 \frac{T_6'}{[i \tanh \beta d]^{\pm 1}}\right)(L_1 - L_2 + L_3)$$
$$-\left(4\beta\left[\frac{1}{\tanh \beta d}\right]^{\pm 1} + B_0 \frac{1}{[\tanh \beta d]^{\pm 1}} + B_1 (i)^{\pm 1} T_5' + B_2 \frac{T_5' T_6'}{[\tanh \beta d]^{\pm 1}}\right) \times$$
$$\left(m_1 L_1 [\tanh \alpha_1 d]^{\pm 1} - m_2 L_2 [\tanh \alpha_2 d]^{\pm 1} + m_3 L_3 [\tanh \alpha_3 d]^{\pm 1}\right) + \left(C_0 (i)^{\pm 1} T_5' + C_1 \frac{T_5' T_6'}{[\tanh \beta d]^{\pm 1}}\right) F^* = 0$$

Where

$$F^* = s_L h_T\, h_p \left(\varepsilon^{pq} \varepsilon_p - \bar{\varepsilon}'_{pq}\right)\left(m_1 [\tanh \alpha_1]^{\pm 1}(S_2 - S_3) - m_2 [\tanh \alpha_2]^{\pm 1}(S_1 - S_3) + m_3 [\tanh \alpha_3]^{\pm 1}(S_1 - S_2)\right)$$

### 6.1.2. REGION II

For $\frac{\omega}{\delta} > k > \omega$**, it** follows that $\delta < c < 1,$ and the frequency equation in this case is obtained from Eq. 19 by replacing circular tangent functions of $m_i$, $i = 1, 2, 3$ with hyperbolic tangent functions of $\alpha_i$, $i = 1, 2, 3$

$$\left(p^2 + A_0 + A_1 T_5' T_6' + A_2 \frac{T_6'}{[T_4]^{\pm 1}}\right)(L_1 - L_2 + L_3)$$
$$-\left(4\beta\left[\frac{1}{T_4}\right]^{\pm 1} + B_0 \frac{1}{[T_4]^{\pm 1}} + B_1 (i)^{\pm 1} T_5' + B_2 \frac{T_5' T_6'}{[T_4]^{\pm 1}}\right) \times$$
$$\left(m_1 L_1 [\tanh \alpha_1 d]^{\pm 1} - m_2 L_2 [\tanh \alpha_2 d]^{\pm 1} + m_3 L_3 [\tanh \alpha_3 d]^{\pm 1}\right) + \left(C_0 (i)^{\pm 1} T_5' + C_1 \frac{T_5' T_6'}{[T_4]^{\pm 1}}\right) F^* = 0$$

Where

$$F^* = h_p \left(\varepsilon^{pq} \varepsilon_p - \bar{\varepsilon}'_{pq}\right)\left(m_1 [\tanh \alpha_1]^{\pm 1}(S_2 - S_3) - m_2 [\tanh \alpha_2]^{\pm 1}(S_1 - S_3) + m_3 [\tanh \alpha_3]^{\pm 1}(S_1 - S_2)\right)$$

### 6.1.3. REGION III

For $k < \omega,$ it follows that $c > 1$ and the frequency equation is given by Eq.19.

### 6.2 WAVES AT SHORT WAVELENGTH

Some information on the asymptotic behaviour is obtainable by putting $k \to \infty$. If we take $k > \frac{\omega}{\delta},$ it follows that $k > \omega$ and that $c < \delta, 1.$ In this case the roots of the secular equation lie in region I and then we replace $\beta,\ m_1,\ m_2$ and $m_3$ in the frequency equation (19) by $i\beta$ , $i\alpha_1$ , $i\alpha_2$ , $i\alpha_3$ .For $k \to \infty$

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, $\tanh \alpha_i k / \tanh \beta d \to 1, \quad i = 1, 2, 3 \quad \tanh \gamma_i h / \tanh \beta d \to -i \quad , \quad i = 5, 6$, so that the frequency equation reduces to

$$p^2 (L_1 - L_2 + L_3) - F(mL_1 - mL_2 + mL_3) + G = 0 \tag{31}$$

where

$$F = \frac{\left(B_0' + a^*\left(B_1' + a^* B_2'\right)\right)}{\left(A_0' + a^*\left(A_1' + a^* A_2'\right)\right)}$$

$$G = \frac{\left(C_0' + a^* C_1'\right)F^{*'}}{\left(A_0' + a^*\left(A_1' + a^* A_2'\right)\right)}$$

$$A_0' = 1 + \frac{1}{p}(\omega_L - p)\frac{T_5\, T_6}{\gamma_1 \gamma_2}$$

$$A_1' = \frac{1}{p^2}\left[ \left((\omega_L - p)(\gamma_2^2 + 1) + 2p(\gamma_1^2 - 1)\right)\frac{T_5\, T_6}{\gamma_1\gamma_2} + \beta\left(p - \frac{2T_6}{\gamma_2}\right) + \left(\gamma_2^2 + 3\right)\right]$$

$$A_2' = \frac{2}{p^2}\left[ (\gamma_1^2 - 2)(\gamma_2^2 + 1)\frac{T_5\, T_6}{\gamma_1\gamma_2} - 2\right]$$

$$B_0' = 4\beta\left(1 - \frac{T_5\, T_6}{\gamma_1\gamma_2}\right) + \omega_L (2 - p)\frac{T_5}{\gamma_1} + 2\beta\omega_L \frac{T_5\, T_6}{\gamma_1\gamma_2}$$

$$B_1' = 2(2 - p)(\gamma_1^2 - 1) + \left(\beta(\gamma_2^2 + 1)(\omega_L - 2) + 4\beta(\gamma_1^2 - 2)\right)\frac{T_5\, T_6}{\gamma_1\gamma_2}$$

$$B_2' = -4\beta + 2\beta(\gamma_1^2 - 2)(\gamma_2^2 + 1)\frac{T_5\, T_6}{\gamma_1\gamma_2}$$

$$C_0' = p(2 - p)\frac{T_5}{\gamma_1} + 2\beta(p - 2)\frac{T_5\, T_6}{\gamma_1\gamma_2}$$

$$C_1' = 2(2 - p) + \beta(p - 2)(\gamma_2^2 + 1)\frac{T_5\, T_6}{\gamma_1\gamma_2}$$

$$F^{*'} = \left(m_1(S_2 - S_3) - m_2(S_1 - S_3) + m_3(S_1 - S_2)\right)\left(h_p\left(\varepsilon^{pq}\varepsilon_p - \varepsilon'_{pq}\right)\right)$$

The secular equation (30) governs the motion of elasto-thermodiffusive (ETP) Rayleigh (Stoneley) surface waves in the instant analysis. It contains complete information about the phase velocity, attenuation coefficient and other characteristics of the ETP surface waves in a thermoelastic semiconductor half-space underlying a fluid layer of finite thickness with varying temperature. In case the semiconductor half-space is loaded with fluid half-space $(d \to \infty)$, the secular equation of modified guided ETP surface Rayleigh waves is again given by equation (19) with changed values of the coefficients $A_i$ , $B_i$ $(i = 0,1, 2)$ and $C_i$ $(i = 0 , 1)$ which can be obtained from equation (20) by setting $T_5' = 1 = T_6'$.

In case of inviscid fluid $(\mu_L = 0 \Rightarrow \nu_L = 0 = a^*)$, the secular equation (31) becomes

$$p^2 (L_1 - L_2 + L_3) - \left(4\beta + \omega_L (2 - p)\right)(m_1 L_1 - mL_2 + mL_3) + p(2 - p)F^{*'} = 0 \tag{32}$$

The secular equation for the surface waves at the interface of inviscid fluid/germanium semiconductor plate $(h \to \infty)$ can be written from (32) by setting $T_5' = 1$ .

In the absence of fluid $\left( \rho_L = 0 = \varepsilon_L = 0 \Rightarrow \omega_L = 0 = s_L \right)$, the secular equations (31) and (32) reduce to

$$p^2 \left( L_1 - L_2 + L_3 \right) - 4\beta \left( m_1 L_1 - m_2 L_2 + m_3 L_3 \right) = 0 \qquad (33)$$

The equation (33) is the same as obtained and discussed in detail by Sharma and Thakur [10] in case of thermoelastic semiconductor under free surface conditions.

## 7. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. The material chosen for this purpose is germanium (Ge) semiconductor materials the physical data for which is given in Table 1. The values of viscosity for inviscid ($H_2O$) and viscous fluid ($D_2O$) fluids are taken as $\mu_L = 0.0$ and $\mu_L = 1.0$ , respectively. All the considerations presented in this paper were only devoted to the semiconductor of the relaxation type. This means that we are interested in a case when the relaxation times and the life times of the holes are comparable with each other. The corresponding profiles of phase velocity $(V)$, attenuation coefficient $(Q)$, are plotted on linear scales against non-dimensional wave number (R) and liquid layer thickness(h) in Figs.2 to 7. The non-dimensional values of lifetimes of charge carriers are taken as $t_p^+$ = 0.796, 0.0796 which corresponds to their dimensional values $t_p^+$ = 1ps, 0.1ps respectively, have been considered for the computation purpose.

**Table 1:** Physical data of germanium *(Ge)* semiconductor materials

| **Physical Quantities** | **Units** | **Ge** | **References** |
|---|---|---|---|
| $\lambda$ | $Nm^{-2}$ | $0.48 \times 10^{11}$ | [12] |
| $\mu$ | $Nm^{-2}$ | $0.53 \times 10^{11}$ | |
| $\rho$ | $Kgm^{-3}$ | $5.3 \times 10^3$ | |
| $t_n^+$ | s | $< 10^{-5}$ | |
| $t_p^+$ | s | $< 10^{-5}$ | |
| $D^n$ | $m^2s^{-1}$ | $0.5 \times 10^{-2}$ | |
| $D^p$ | $m^2s^{-1}$ | $0.5 \times 10^{-2}$ | |
| $n_0 = p_0$ | $m^{-3}$ | $10^{20}$ | [21] |
| $\alpha^n$ | $m^2/s$ | $3.4 \times 10^{-3}$ | |
| $\alpha^p$ | $m^2/s$ | $1.3 \times 10^{-3}$ | |
| $K$ | $Wm^{-1}K^{-1}$ | 60 | |

| | | | |
|---|---|---|---|
| $C_e$ | $JKg^{-1}K^{-1}$ | 310 | [22] |
| $\alpha_T$ | $K^{-1}$ | $5.8\times10^{-6}$ | |
| $m^{nq}$ | $vk^{-1}$ | $-0.004\times10^{-6}$ | [23] |
| $m^{qn}$ | $vk^{-1}$ | $-0.004\times10^{-6}$ | |

**Table 2 -** Physical data for inviscid fluid *($H_2O$)* and viscous fluid *($D_2O$)*

| **Physical Quantities** | **Units** | **$H_2O$** | **$D_2O$** | **References** |
|---|---|---|---|---|
| $\rho_L$ | $Kgm^{-3}$ | $10^3$ | 1104.36 | [24] or |
| $c_L$ | $ms^{-1}$ | 1500 | 1500 | [25] |
| $\mu_L$ | $Nm^{-2}s$ | 0.0 | 1.0 | |
| $K_L$ | $Wm^{-1}k^{-1}$ | 0.686 | 0.636 | [26] |

**Table 3:** Specific heat of water at constant volume for different temperatures

| | | | | | |
|---|---|---|---|---|---|
| $T_0^*(^0C)$ | 0 | 15 | 35 | 50 | 100 |
| $C_v^*(J/Kg^oC)$ | 1.008 | 1.00 | 0.997 | 0.998 | 1.006 |

The value of half of the plate thickness (h) is taken as unity and the thickness of liquid layers is also kept as unity for the purpose of numerical calculations. From Fig. 2, it is observed that the phase velocity of lowest acoustic symmetric mode increases from zero value at vanishing wave number to become asymptotically closer to the thermoelastic Rayleigh wave velocity at higher wave numbers. It is quite clear that due to the damping effect of the liquid on both side of the plate, the phase velocity of The phase velocity of acoustic symmetric mode gets significantly reduced/ affected in the presence of liquid loading and asymptotically tends to the velocity of thermoelastic Rayleigh wave in the considered material half-space with increasing wave number symmetric and sk-symmetric mode. The phase velocity of higher (optical) symmetric and skew -symmetric modes attains quite large values at vanishing wave number which slashes down to become steady and asymptotic to the shear wave velocity with increasing wave number. All the modes show dispersive behavior in the considered directions of wave propagation. It can be seen that as the wave number increases, the phase velocity of each mode decreases in all the directions of wave propagation. When wave number becomes indefinitely large, the curves asymptotically approach to the Rayleigh wave velocity, because in such a situation a finite thickness plate behaves like a half-space and the transportation of energy takes place mainly across the free surface of the plate. Figs.3 represents the variation

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of non-dimensional attenuation coefficient (Q) of the waves with non-dimensional wave number (R) for acoustic mode of wave propagation in a plate. it is noticed that the attenuation first increases monotonically in the wave number range $0 \le R \le 2$ and then linearly for $R \ge 2$ and also decreases in the range $3.5 \le R \le 5$ It is also noticed that the attenuation coefficient $(Q)$ exhibit dispersive behavior after wave number interval $R \ge 5$.

The attenuation caused by fluid loading is due to the combined effects of radiation loss due to energy leakage into the fluid and dissipative loss due to viscous friction at the interfaces. Fig.4 shows the variation of phase velocity of symmetric and skew symmetric mode for non leaky lamb waves with wave number in the liquid loading. The lowest symmetric mode and skew symmetric mode has zero velocity at vanishing wave number, which increases to become closer to the velocity of Rayleigh wave with increasing wave number but also get reduced and effected due to the presence of the liquid. The phase velocities of higher modes of propagation attain large values at vanishing wave number, which slash down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number. The magnitude of velocity of higher modes is observed to develop at same rate, which approximately n-times, the magnitude of phase velocity of first mode $(n = 1)$. It is quite clear that due to the damping effect of the liquid on both side of the plate, the phase velocity of acoustic symmetric and skew symmetric mode gets significantly reduced/ affected in the presence of liquid loading and asymptotically tends to the velocity of elastic Rayleigh wave in the considered material half-space with increasing wave number symmetric and antisymmetric mode. All the modes show dispersive behavior in the considered directions of wave propagation. It can be seen that as the wave number increases, the phase velocity of each mode decreases in all the directions of wave propagation. When wave number becomes indefinitely large, the curves asymptotically approach to the Rayleigh wave velocity, because in such a situation a finite thickness plate behaves like a half-space and the transportation of energy takes place mainly across the free surface of the plate.

Fig.5 represents the variation of non-dimensional attenuation coefficient (Q) of the waves with non-dimensional wave number for acoustic mode of wave propagation in a plate for symmetric and skew symmetric mode. It is noticed that the attenuation first increases monotonically in the wave number range $0 \le R \le 2.5$ and then linearly in the range $3.5 \le R \le 5$. It is also noticed that the attenuation coefficient $(Q)$ exhibit dispersive behavior after wave number interval $R \ge 5$. The attenuation caused by liquid loading is due to the combined effects of radiation loss due to energy leakage into the liquid and dissipative loss due to viscous friction at the interfaces. The zoomed version of phase velocity profiles of acoustic modes for both symmetric and asymmetric, with respect to liquid layer thickness(h) has been plotted in Fig. 6. It is observed that the values of phase velocity of acoustic modes get suppressed, which is attributed to the shock absorbing nature of the liquid. Moreover, the magnitude of phase velocity increases with decreasing the value of life time of charge carriers. The dispersion of these modes is attributed to the fact that at long wavelengths the disturbance due to wave penetrates deep into the medium, due to which coupling between various interacting fields become highly operative while at short wavelengths, the wave follows the free surface of the plate and so the effect of coupling reduced significantly.

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Fig. 7 represents the variation of non-dimensional attenuation coefficient (Q) of the waves with non-dimensional liquid layer thickness(h) for acoustic mode of wave propagation in a plate for symmetric and skew symmetric mode. It is noticed that the attenuation first decreases monotonically in the liquid layer thickness(h) range $0.1 \leq h \leq 0.5$ and then linearly increases in the range $0.5 \leq h \leq 0.9$. It is also observed that the magnitude of attenuation of acoustic modes for both symmetric and asymmetric is found to be high as compared to the inviscid liquid loading.

## 8. CONCLUSIONS

The propagation of lamb waves in a homogeneous isotropic semiconductor materials plate loaded with viscous liquid layer on both sides, is investigated. The behaviour of dispersion curves of lamb waves is found to be similar except that the magnitude of phase velocity in case of symmetric mode is found to be large. The phase velocity of lowest symmetric (i.e. fundamental mode) become dispersionless and gets significantly reduced and effected in the presence of the liquid and remains close to the velocity of Rayleigh wave with increasing wave number. The influence of the liquid on a lamb wave depends very significantly on the ratio of the phase velocities of these waves and velocity of the liquid. In essentially every and for practically all mode except $A_0$, the condition $c_L < c_T$ is satisfied. It is clear from physical consideration that in this case the propagation of a wave in a plate will involve the radiation of energy from that wave into the liquid and, accordingly, attenuation along the direction of propagation. The radiation into the liquid strongly depends on the ratio of the vertical to the horizontal component of surface displacement in the given mode. The attenuation become maximum when the vertical component of surface displacement is a maximum, and it is generally vanishes when the vertical surface displacement is equal to zero.

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## FIGURE CAPTIONS

**FIGURE2.** Phase velocity profile with wave number of symmetric and skew symmetric lamb wave at different life time .

**FIGURE3.** Attenuation profile with wave number of symmetric and skew-symmetric lamb wave at different life time (inviscid fluid).

**FIGURE4.** Phase velocity profile with wave number of symmetric and skew-symmetric lamb wave at different life time (Isothermal and Isoconcentration Plate).

**FIGURE5.** Attenuation profile with wave number of symmetric and skew-symmetric lamb wave at different life time (Isothermal and Isoconcentration Plate).

**FIGURE6.** Phase velocity profile with liquid layer thickness (h) of symmetric and skew-symmetric lamb wave at different life time.

**FIGURE7.** Attenuation profile with liquid layer thickness (h) of symmetric and skew-symmetric lamb wave at different life time.

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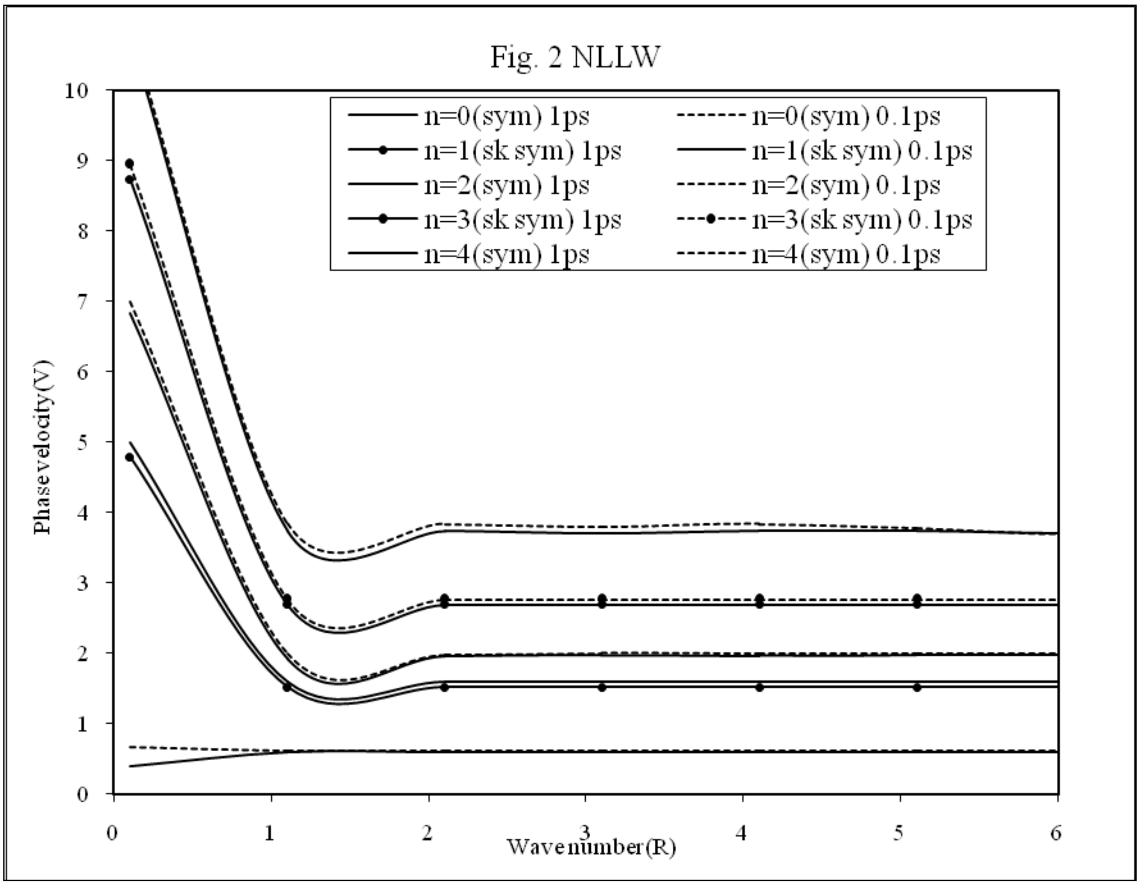
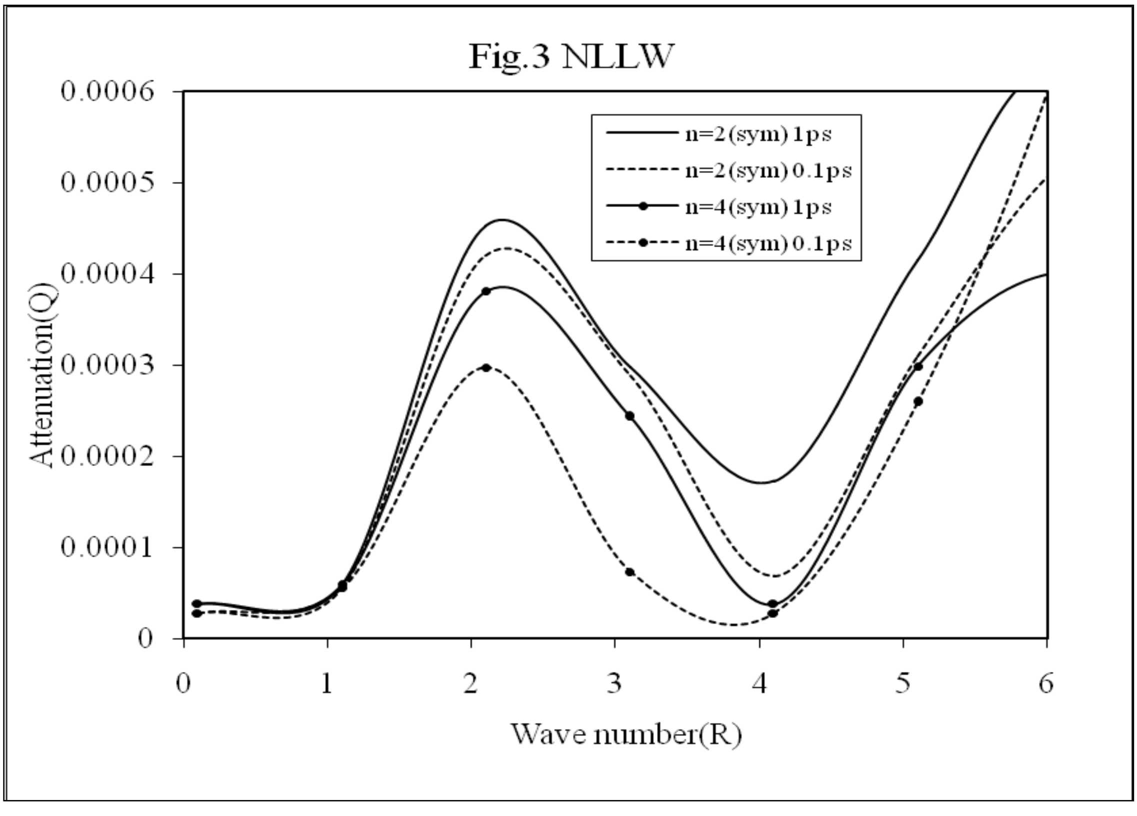
ISSN 2063-5346



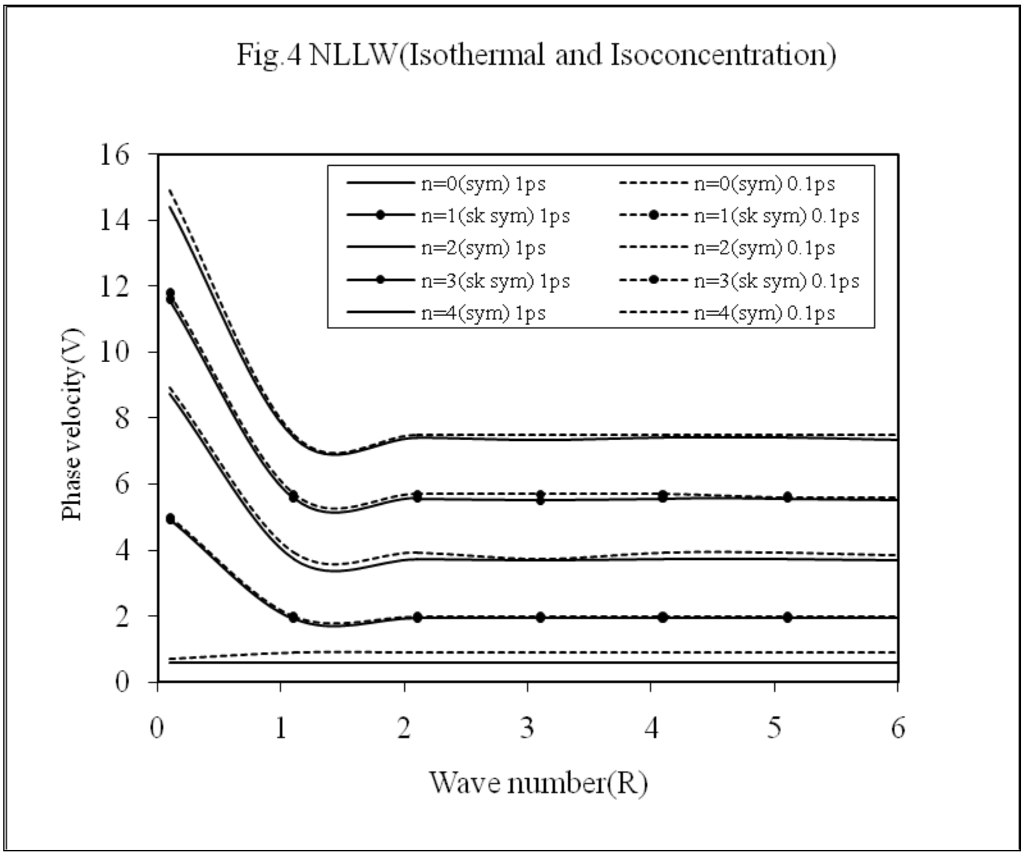


Fig.4 NLLW(Isothermal and Isoconcentration)
n=0(sym) 1ps
n=0(sym) 0.1ps
n=1(sk sym) 1ps
n=1(sk sym) 0.1ps
n=2(sym) 1ps
n=2(sym) 0.1ps
n=3(sk sym) 1ps
n=3(sk sym) 0.1ps
n=4(sym) 1ps
n=4(sym) 0.1ps
Phase velocity(V)
Wave number(R)

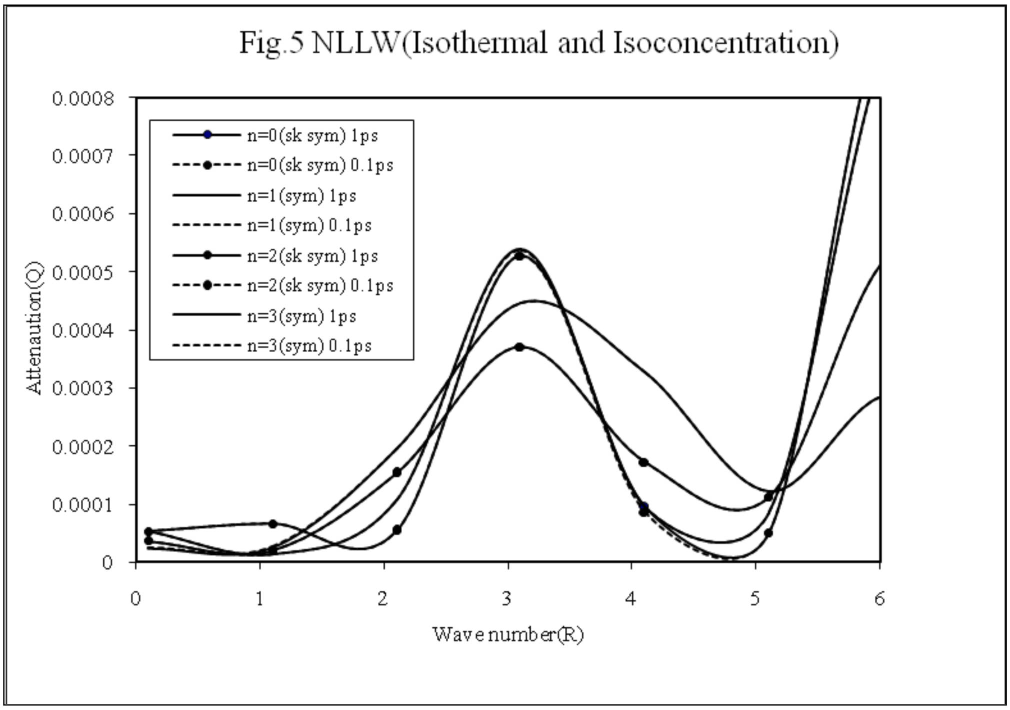


Fig.5 NLLW(Isothermal and Isoconcentration)
n=0(sk sym) 1ps
n=0(sk sym) 0.1ps
n=1(sym) 1ps
n=1(sym) 0.1ps
n=2(sk sym) 1ps
n=2(sk sym) 0.1ps
n=3(sym) 1ps
n=3(sym) 0.1ps
Attenaution(Q)
Wave number(R)

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