d -LUCKY LABELING OF HEXAGONAL NETWORKS

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Abstract

In this paper, we present the results on d-lucky labeling of hexagonal mesh and hexagonal mesh torus network and obtained the $\eta_{dl}(G)$ for HX_n and $HMT(n)$.

Keywords: d-lucky labeling, d-lucky number, hexagonal mesh, hexagonal mesh torus.
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1. Introduction

Graph labeling is one of the main areas of research. The method of assignment of labels define the type of labeling. The concept of d -lucky labeling was introduced by Mirka Miller et al. for several networks [2] and graphs [1]. We have studied the d-lucky labeling for honeycomb network and honeycomb torus network [3]. Further we have extended our study of d -lucky labeling to hexagonal networks. The hexagonal network is well defined in the literature [4]. Some of the applications of hexagonal networks include image processing, clustering, etc. One of the major applications are found in communication engineering for GSM distribution.

Preliminaries

Definition 1: A hexagonal mesh of dimension n denoted by HX_n has $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. There are 6 vertices of degree 3 which we call as corner vertices. There is exactly one vertex v at a distance $n-1$ from each of the corner vertices. This vertex is called the centre of HX_n.

Definition 2: A hexagonal mesh torus is obtained by twisting the hexagonal mesh 180° at the centre and then joining the n^{th} vertical line by including wrap around edges. The end vertices of each vertical lines are joined respectively. A hexagonal mesh torus of dimension n denoted by HMT (n) has $3n^2 - 3n + 1$ vertices and $9n^2 - 12n + 6$ edges.

Main Results

Theorem 1. The n −hexagonal mesh network is d-lucky and the d-lucky number for HX_n, $n \leq 4$ is 2 and $n > 4$ is 3.

PROOF

The simplest hexagonal mesh HX_2 is shown in Figure 1.

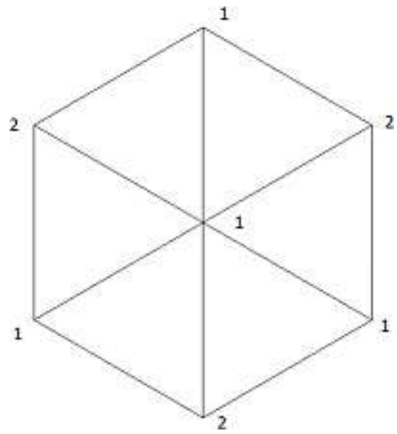

Figure 1: d-lucky labeling of HX_2

We begin the labeling from the centre of the hexagon as 1 and the outer hexagon with 1 and 2 alternatively. In HX_3 all the vertices in the outer hexagon are labeled as 1. See Figure 2.

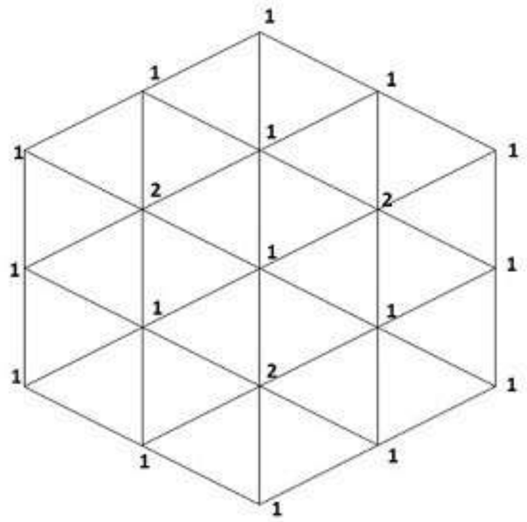

Figure 2: $d-$ lucky labeling of HX_3

Case (i) n ≤ 4

Proceeding with HX_4, the centre and the first layer of hexagon receives the same labels of HX_2. The vertices in the second layer hexagon of HX_4 are labeled as 1 and 2 alternatively. The corner vertices of degree 3 are labeled as 1 and 2 alternatively and the other vertices are labeled as 1 respectively. See Figure 3.

For illustration, take HX_2, Consider the centre vertex as u with label 1 and its adjacent vertices v with labels 1 and 2 alternatively. We claim $c(u) \neq c(v)$, for every pair of vertices u and v which are adjacent in G. Consider the vertex with label 1 as u. This vertex is adjacent to the vertices with labels 1 and 2 which is considered as v in each of the case. The degree of u is 6 and sum of the labels of v is 9. Therefore $c(u)$ is 15. The degree of v with label 1 is 3 and sum of the labels of u is 5. Therefore $c(v)$ is 8. Similarly the degree of v with label 2 is 3 and sum of the labels where v is the vertex with label 1 and $c(v)$ = 6 where v is the vertex with label 2 implies $c(u) \neq c(v)$ where v is the vertex with label 1 and $c(v)$ = 6 where v is the vertex with label 2 implies $c(u) \neq c(v)$.

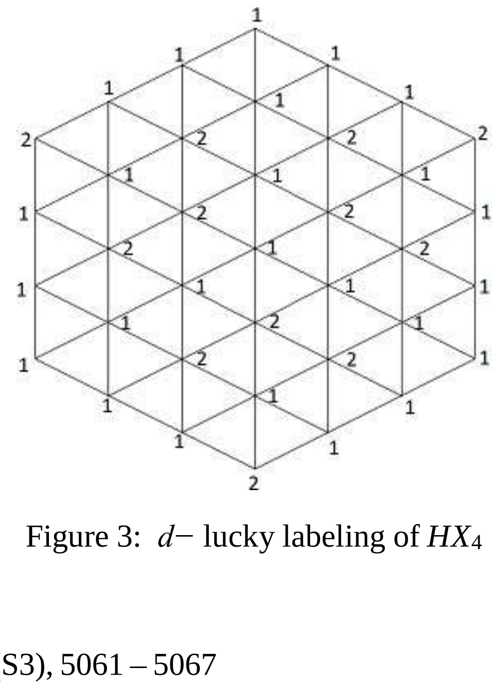

Figure 3: $d-$ lucky labeling of HX_4

Thus HX_n for $n \leq 4$ satisfies the condition $c(u) \neq c(v)$, for every pair of adjacent vertices and $\eta_{dl} = 2$. This proves our claim.

***Case* (*ii*) *n* > 4**

Here we label as in case (i) from the centre of the hexagonal mesh. We call the centre vertical line which has $2n - 1$ number of vertices as the line of symmetry. Beginning from the centre vertex we label 3 alternatively from the line of symmetry. We follow the same procedure as in *case* (*i*) to label the vertices. These vertices are considered as central vertices and the adjacent vertices are labeled 1 and 2. There are n vertical lines including the symmetry line. The n^{th} vertical line will have n number of vertices. For n odd, the corner vertices are labeled as 3. For n even, the corner vertices are labeled as 1 and 2 alternatively. The illustration of d-lucky labeling of HX_5 and HX_6 is shown in Figure 4 and Figure 5

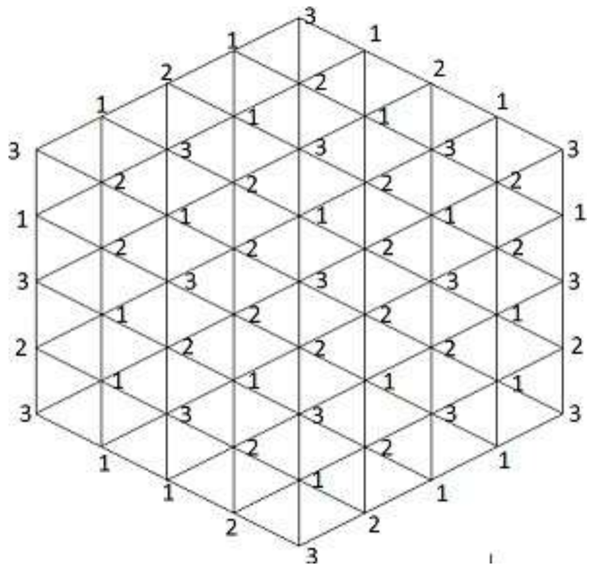

Figure 4: $d-$ lucky labeling of HX_5

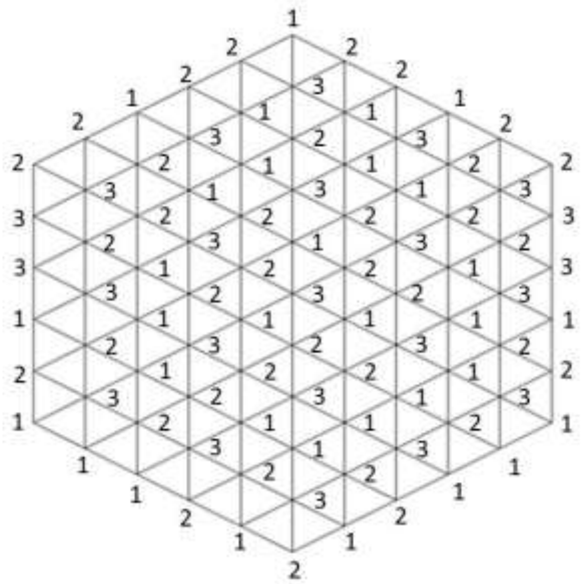

Figure 5: $d-$ lucky labeling of HX_6

For instance, take HX_5. The centre vertex is considered as u with label 3 and its adjacent vertices v with labels 1 and 2. Claim $c(u) \neq c(v)$, The degree of u is 6 and sum of the labels of v is 11 so $c(u)$ is 17. The degree of v with label 1 is 6 and sum of the labels of u is 14. Therefore $c(v)$ is 20. Similarly the degree of v with label 2 is 6 and sum of the labels of u is 12. Therefore $c(v)$ is 18. From this we infer that $c(u) = 17$, $c(v) = 20$ and $c(v) = 18$ which proves our claim $c(u) \neq c(v)$. Similar argument holds for every pair of vertices adjacent in G. We proceed in the same way to label the n-dimensional hexagonal mesh. The d-lucky number $\eta_{dl} = 3$ for $n > 4$.

Theorem 2. The n hexagonal mesh torus network is d-lucky and the d-lucky number for *HMT* (n) is 4.

Proof

The Hexagonal mesh torus network has vertices of degree four, five and six. The connectivity between the vertices are very strong. Here we discuss two types of tori, simple and twisted torus.

***Case (i)* Simple torus**

This torus is formed by wrapping the mesh by including edges horizontally and then joining the end vertices by adding the wraparound edges. This torus is labeled from the centre vertex as 2. The inner vertices are labeled with 1 and 2 satisfying the condition. The outer hexagon is labeled from the set of labels {1, 2, 3, 4}. See Figure 6.

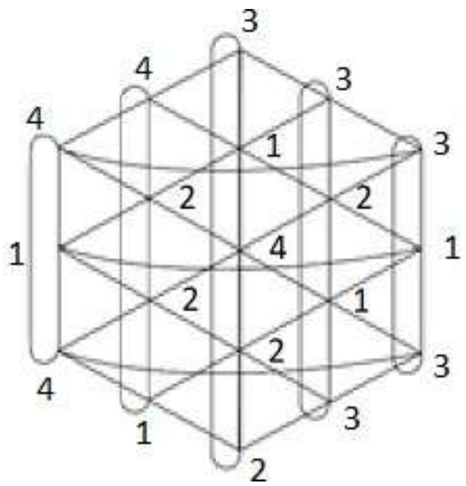

Figure 6: Simple hexagonal mesh torus

For an example, a simple hexagonal mesh torus of dimension 3 is taken. We consider the corner vertex as u with label 4 and its adjacent vertices v with labels 1 on the outer hexagon and 2 in the inner hexagon. We claim $c(u) \neq c(v)$, for every pair of vertices u and v adjacent in G. Consider the adjacent vertices with labels 4 and 1. The degree of u is 5 and sum of the labels of v is 14 so $c(u)$ is 19. The degree of v with label 1 is 5 and sum of the labels of u is 13. Therefore $c(v)$ is 18. Similarly the degree of u is 5 and sum of the labels of v is 14 so $c(u)$ is 19. The degree of v is 6 and sum of the labels of u is 14. Therefore $c(v)$ is 20. Therefore we can say that $c(u) = 19$, $c(v) = 18$ and $c(u) = 19$, $c(v) = 20$ which proves our claim $c(u) \neq c(v)$. Similar argument holds for every pair of vertices adjacent in G. Thus the simple n-dimensional hexagonal mesh torus is d-lucky and $\eta_{dl} = 4$.

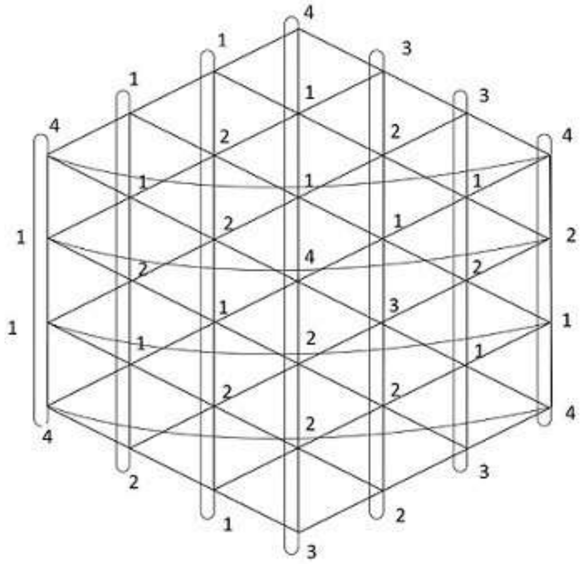

Figure 7: Simple hexagonal mesh torus

Similarly we illustrate for simple hexagonal mesh torus of dimension 4. See Figure 7. We consider the corner vertex as u with label 4 and its adjacent vertices v with labels 2 and 1 on the outer hexagon. We claim $c(u) \neq c(v)$, for every pair of adjacent vertices u and v in G. Consider the adjacent vertices with labels 4 and 2. The degree of u is 5 and sum of the labels of v is 12 so $c(u)$ is 17. The degree of v is 5 and sum of the labels of u is 9. Therefore $c(v)$ is 14. Therefore we can say that $c(u)$ = 17, $c(v)$ = 14 which proves our claim $c(u) \neq c(v)$. Similar argument holds for every pair of vertices adjacent in G. Thus the simple n-dimensional hexagonal mesh torus is d-lucky and $\eta_{dl} = 4$.

Case (ii) Twisted torus

To label the vertices, we begin labeling from the centre vertex and it is 2. The inner hexagon are labeled as 1 and 2 alternatively from the vertex on the central line. We now label the outer hexagon from the set of labels {1, 2, 3, 4} satisfying the condition $c(u) \neq c(v)$.

As an example, a twisted hexagonal mesh torus of n = 3 is taken. We consider the corner vertex as u with label 3 and its adjacent vertices v with labels 1 and 2 on the outer hexagon. We claim $c(u) \neq c(v)$, for every pair of vertices u and v adjacent in G. The degree of u is 5 and sum of the labels of v is 9 so $c(u)$ is 14. The degree of v is 5 and sum of the labels of u is 11. Therefore $c(v)$ is 16. Similarly the degree of u is 5 and sum of the labels of v is 9 so $c(u)$ is 14. The degree of v is 5 and sum of the labels of u is 12. Therefore $c(v)$ is 17. Therefore it proves our claim $c(u) \neq c(v)$. Similar argument holds for every pair of vertices adjacent in G. Thus the twisted n-dimensional hexagonal mesh torus is d -lucky and η_{dl} = 4. See Figure 8.

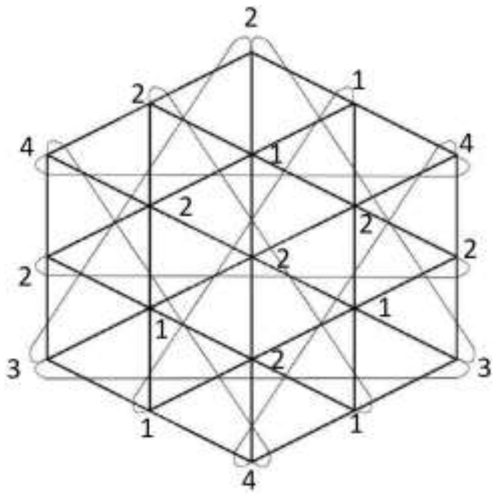

Figure 8: d -lucky labeling of HMT (3)

2. Conclusion

The n-hexagonal mesh network is d-lucky. The d-lucky number for HX_n, $n \leq 4$ is 2 and $n > 4$ is 3. The n hexagonal mesh torus network admits d-lucky labeling and the d-lucky number for HMT (n) is 4. We observed that as the connectivity between the edges increases, the labeling of the vertices with a minimum number becomes challenging.

Acknowledgement

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