BIOLOGICAL MULTIPLE ATTRIBUTE GROUP DECISION MAKING APPLICATIONS FOR AGRO FORESTRY WITH BAMBOO IN PACHAIMALAI HILLS (EASTERN GHATS) USING SINGULAR PERTURBED DIFFERENTIAL EQUATIONS AND LINEAR PROGRAMMING TECHNIQUES

John Robinson P[1*], Samuel Nirmalsingh S[2], Indhumathi M[3]

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Abstract

The Pachaimalai Hills, a set of hills in the Eastern Ghats of India, are well-known for their dense scrubby forest and historical distribution of sandalwood over the hill area. Bamboo is also grown in the area at the foot of the hills, close to Sobanapuram. This study outlines a strategy that can support decision-makers in understanding, articulating, and capturing the value of biodiversity and ecosystem services. Multiple Attribute Group Decision Making (MAGDM) has emerged as an effective methodology due to its ability to combine quantitative and qualitative criteria for selection of the best alternative. Concurrently, Intuitionistic Fuzzy Logic is gaining importance due to its flexibility in handling imprecise subjective data. In the present study, an Intuitionistic fuzzy MAGDM method based on different ranking methods are adopted and developed as a Intuitionistic Fuzzy Decision System (IFDS) and applied to a case study of a Model Agroforestry with Bamboo plantations in the surroundings of Pachaimalai Hills (Eastern Ghats), Tamilnadu, India, for selecting the best-performing Bamboo tree plantation. A System of Singularly Perturbed Differential Equations (SSPDEs) and linear programming problems are employed for determining the expert weights which will provide adequate information on the attribute information provided by the decision makers. It is concluded that application of Intuitionistic fuzzy logic methodology for real-world decision-making problems especially in deciding the close association between plant species is found to be effective.

Keywords: Pachaimalai Hills, Agro forestry, Bamboo, MAGDM, Intuitionistic Fuzzy Sets (IFS)**,** Linear Programming, Singularly Perturbed Differential Equations, Shishkin Mesh.

[1*]Assistant Professor, Department of Mathematics, Bishop Heber College, Affiliated to Bharathidasan University, Tiruchirappalli, India.
[2]Research Scholar, Department of Mathematics, Bishop Heber College, Affiliated to Bharathidasan University, Tiruchirappalli, India.
[3]Assistant Professor, Department of Mathematics, Government Arts and Science College, Kangeyam, Affiliated to Bharathidasan University, Tiruchirappalli, India.

Corresponding Author:
[1*]John Robinson P
[1*]Assistant Professor, Department of Mathematics, Bishop Heber College, Affiliated to Bharathidasan University, Tiruchirappalli, India.

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1. Introduction

India has long been regarded as the repository for plant genetic resources due to its diverse agro-climatic zones and regional geography. As a result, India is regarded as one of the top 12 countries with the greatest diversity. Over 8,000 types of plants make up our herbal richness, which accounts for over 50% of all higher flowering plant species in India. The Western Ghats' tropical woods also house about 70% of the nation's medicinal plants. But according to the information that is currently accessible, 1,800 species are used in traditional Indian medical systems. Ayurvedic medicine makes use of 1,200, Siddha 900, Unani 700, Amchi 600, and Tibetan 450. A significant opportunity exists for the economic growth of the Indian region in the newly emerging field of herbal goods business. Herbs are used for food, medicinal, fragrance, flavour, dyes and other purposes [26],[31]. Numerous plant species are rapidly approaching extinction due to factors such as population growth, the rapid development of agricultural land used for food and other purposes, deforestation, urbanisation, and the establishment of rural enterprises. Due to its rapid growth, bamboo is becoming a valuable cash crop and a top provider of biomass for power plants. Recently, its beneficial impact on the environment has also been recognised.

Agroforestry is an integrated approach of using the interactive benefits from combining trees and shrubs with crops and/or livestock. It combines agricultural and forestry technologies to create more diverse, productive, profitable, healthy, and sustainable land-use systems [27],[18],[16]. A narrow definition of agroforestry is "trees on farms". Agroforestry has a lot in common with intercropping. Both have two or more plant species in close interaction, both provide multiple outputs, as a consequence, higher overall yields and, because a single application or input is shared, costs are reduced. Beyond these, there are gains specific to agroforestry. With two or more interacting plant species in a given land area, it creates a more complex habitat that can support a wider variety of birds, insects, and other animals.

Pachaimalai Hills, also known as the Pachais are hills which are part of Eastern Ghats and are spread across Salem and Tiruchirappalli district of Tamil Nadu, located near 11°11′N 78°21′E / 11.18°N 78.35°E / 11.18; 78.35. The altitude of ranges from 900 MSL to 1200 MSL. They are much greener than some of the other hills in the vicinity. Veera Ramar Dam is located in these hills on Kallar. Rivers include Kallar and Sweata Nadi. Waterfalls include Mangalam Aruvi, Koraiyar Falls and Mayil Uthu Falls. There are also indigenous tribes, such as the Malayalis, who trade some of their surplus agricultural products, which grow in the hills, to towns below on the plains, trading for items not available in the hills [29].

In the context of the rapid degradation of soil, forest vegetation, water resources and the impoverishment of farmer's economy, the role of agro-forestry has become vital in terms of balancing the conflicting issues of conservation of natural resources and their usage in sustaining agricultural development and rural area livelihood. The appropriate management of Multi-Purpose Trees (MPTs) in agro-forestry systems can lead to a sustained production of such basic needs as fuel wood, building materials, food, fodder, medicaments as well as indirectly reduce the pressure on forests. Unfortunately, very little is known about the many MPTs and agro-forestry systems that are practiced in the various agro-ecological zones of South India, particularly in terms of their dynamics and functioning, economic and ecological contribution. Andreopoulou et al. [2] worked on the agricultural and environmental emerging research applications based on the recent developing e-innovative techniques. Sonia et al. [29] conducted a survey in Pachaimalai hill, Eastern Ghats, India regarding the plant association with sandalwood and used fuzzy Bio-statistical techniques along with data mining techniques for their study. Many authors [4][6][9][15][25][28][30] have done extensive research on soil, agriculture and various techniques to improve the quality of agricultural methods.

As a generalization of zadeh's fuzzy set [32], Atanassov, [3] firstly introduced the concept of an intuitionistic fuzzy set (IFS), which is characterized by a membership function and a non-membership function. Doolan et.al. [5] introduces the uniform numerical methods of a singularly perturbed differential equations of initial and boundary layers. Hatzimichailidis et al. [7] proposed a novel distance measure and used in pattern recognition problems. Liu & Wang, [12], Liu et al. [11] and Li, [10] developed various decision-making methods based on correlation theory in intuitionistic fuzzy sets and game theory. Singular perturbations are widespread in nature. A singular perturbation problem is said to be of convection-diffusion type, if the order of the differential equation is reduced by one when the perturbation parameter ε is set equal to zero. If the order reduces by two, it is known as a reaction-diffusion type problem. Future, if the order of the differential equation in an singular perturbation problem is greater than two, it is said to be a higher order SPP. Many authors have devoted their work on SPP, numerical methods to solve differential equations and their applications [1][8][13][14][17][20][21][22][23].
Robinson et al. [24]and Nirmalsingh et al. [19]have proposed new methods to derive decision maker's weights for MAGDM using linear programming techniques. In this work, the unknown weights are derived using a system of singularly perturbed differential equations. We have analysed the MAGDM problem with intuitionistic fuzzy set for ranking the alternatives together with IFOWG and IFHG operators. The feasibility of the proposed method is shown through a MAGDM Problem applied in association of different plant species with Bamboo in Pachaimalai Hills, Eastern Ghats, India, with a numerical illustration.

2. Materials & Methods

In this paper, MAGDM problems are investigated, where both the attribute weights and the expert weights are derived from Singular Perturbation Problems (SPP); attribute values take the form of intuitionistic and interval valued intuitionistic fuzzy numbers. Different ranking methods are presented and a MAGDM model with *OWG* operator is proposed with a numerical illustration on the selection of the appropriate tree plantation with Bamboo as the main crop in the Pachaimalai hills, Tamilnadu, India. The problem of weight determination among decision-makers is investigated by using the numerical solution of System of Singularly Perturbed Differential Equations (SSPDEs) through defuzzifying function. The confidence intervals are constructed using the numerical solution and thereby giving a triangular fuzzy number from which the weighting vectors are obtained. Firstly, for the decision process, the Intuitionistic Fuzzy Ordered Weighted Geometric (IFOWG) operator and Intuitionistic Fuzzy Hybrid Geometric (IFHG) operators are utilized. Using these two operators, all the intuitionistic fuzzy decision matrices are converted into a column matrix. Finally we utilize the Euclidean distance function, correlation coefficient and score function for ranking the order of the best alternatives and select the most desirable one(s). Numerical examples are provided to illustrate the validity and feasibility of our proposed approaches. A suggestion of plant association identified with Bamboo for effective agroforestry model is derived from the proposed decision making methods. The order of preference of the plant species which are very closely related to Bamboo in Pachaimalai hills is identified and suggested for the same pattern of agro-farming in the Pachaimalai hills.

Basic Concepts of Intuitionistic Fuzzy Sets and Aggregation Operators

Let $\bar{A}_0 = \{(x_1, 0.2, 0.4), (x_2, 0.3, 0.5)(x_3, 0.3, 0.6)\}$ be a intuitionistic fuzzy set on the universal set $X_0 = \{x_1, x_2, x_3\}$.the explaining of $\bar{A}_0$ is interpreted as follow: the membership degree of the element x_1 belonging to $\bar{A}_0$ is 0.2 whereas the non- membership degree is 0.4, i.e, $\mu_A(x_1)$ =0.2 and $\gamma_A(x_1)$=0.4; the membership degree of the element x_2 belonging to $\bar{A}_0$ is 0.3 whereas the non- membership degree is 0.5, i.e., $\mu_A(x_2)$= 0.3 and $\gamma_A(x_2)$ = 0.5; the membership degree of the element x_3 belonging to $\bar{A}_0$ is 0.2 whereas the non- membership degree is 0.3, i.e, $\mu_A(x_3)$ = 0.3 and $\gamma_A(x_3)$= 0.6; thus, the intuitionistic fuzzy set $\bar{A}_0$ may also be expressed as follows: $\bar{A}_0 = \{(x_1, 0.2, 0.4) + (x_2, 0.3, 0.5) + (x_3, 0.3, 0.6)\}$.

Sometimes, an intuitionistic fuzzy set $\bar{A}$ on the finite universal set $X = \{x_1, x_2, ..., x_n\}$ may be expressed as below:

$$\bar{A} = \sum_{j=1}^{n} \frac{\langle \mu_{\bar{A}}(x_j), \gamma_{\bar{A}}(x_j) \rangle}{x_j},$$

$$\bar{A}_0 = \left\langle \frac{0.2, 0.4}{x_1} \right\rangle + \left\langle \frac{0.3, 05}{x_2} \right\rangle + \left\langle \frac{0.3, 0.6}{x_3} \right\rangle.$$

Different Types of Measures for IFS

Definition: Score Function

Let $\tilde{a}_j = (\mu_j, \gamma_j)$, for all j = 1,2,…,n be intuitionistic fuzzy numbers. A score function is defined as follows: $S(A) = \mu_j - \gamma_j$.

Definition: Accuracy Function

Let $\tilde{a} = ([a, b])$ be an interval-valued intuitionistic fuzzy number, an accuracy function H of an interval-valued intuitionistic fuzzy value can be represented as follows:

$$H(\tilde{a}) = \frac{a+b}{2}, \ H(\tilde{a}) \in [0,1].$$

The larger value of $H(\tilde{a})$, the more the degree of accuracy of the degree of membership of the intuitionistic fuzzy value $\tilde{a}$.

Definition: Euclidean Distance

For any two intuitionistic fuzzy sets A and B, the Euclidean distance is given as follows:

$$d(A, B) = \sqrt{\frac{1}{2}\left(\sum_{j=1}^{n}\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\gamma_A(x_j) - \gamma_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right)}. \quad (1)$$

Different Types of Aggregation Operators

Definition: Intuitionistic Fuzzy Ordered Weighted Geometric Operator (IFOWG)

Let $\tilde{a}_j = (\mu_j, \gamma_j)$, for all $j = 1, 2, ..., n$ be a collection of intuitionistic fuzzy values. The Intuitionistic Fuzzy Weighted Geometric (IFOWG) operator, $IFOWG : Q^n \rightarrow Q$ is defined as:

$$IFOWG_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \tilde{a}_{\sigma(j)}{}^{\omega_j} = \left(\prod_{j=1}^{n} \mu_{\sigma(j)}{}^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \gamma_{\sigma(j)})^{\omega_j} \right), \quad (2)$$

Where $w = (w_1, w_2, ..., w_n)^T$ is the associated weight vector such that $w_i > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1, 2,…, n), such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$ for all j = 2,…, n.

Definition: Intuitionistic Fuzzy Hybrid Geometric Operator (IFHG)

Let $\tilde{a}_j = (\mu_j, \gamma_j)$, for all $j = 1, 2, ..., n$ be a collection of intuitionistic fuzzy values. The Intuitionistic Fuzzy Hybrid Geometric (IFHG) operator, $IFHG : Q^n \rightarrow Q$ is defined as:

$$IFHG_{\omega,w}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \tilde{a}_{\sigma(j)}{}^{w_j} = \left[\prod_{j=1}^{n} (\mu_{a_{\sigma(j)}})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \gamma_{a_{\sigma(j)}})^{w_j} \right], \quad (3)$$

Where $w = (w_1, w_2, ..., w_n)^T$ is the associated vector such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, and where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of a_j, for all $j = 1, 2, ..., n$ such that $\omega_j > 0$ and $\sum_{j=1}^{n} \omega_j = 1$. Furthermore $a_{\sigma(j)}$ is the j^{th} largest of the weighted IFNs $a_j = a_j^{n\omega_j}$, $j = 1, 2, ..., n$.

Correlation Coefficient of Intuitionistic Fuzzy Sets (IFSS)

In this paper, the correlation coefficient of IFSs is used, taking the membership, non-membership and hesitancy grades are taken into account.

Let $X = \{x_1, x_2, ..., x_n\}$ be the finite universal set and A, $B \in$ IFS(X) be given by

$A = \left\{ \left\langle x, \left[\mu_A(x), \gamma_A(x) \right] \right\rangle / x \in X \right\}$, $B = \left\{ \left\langle x, \left[\mu_B(x), \gamma_B(x) \right] \right\rangle / x \in X \right\}$.

And the length of the intuitionistic fuzzy value values are given by:

$\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$, $\pi_B(x) = 1 - \mu_B(x) - \gamma_B(x)$.

Now for each $A \in$ IFS(X), the informational intuitionistic energy of A is defined as follows:

$E_{IFS}(A) = \frac{1}{n} \sum_{i=1}^{n} \left[\mu_A^2(x_i) + \left(1 - \gamma_A(x_i)\right)^2 + \pi_A^2(x) \right]$, and for each $B \in$ IFS(X), the informational intuitionistic energy of B is defined as follows:

$$E_{IFS}(B) = \frac{1}{n} \sum_{i=1}^{n} \left[\mu_B^2(x_i) + \left(1 - \gamma_B(x_i)\right)^2 + \pi_B^2(x) \right], \quad (4)$$

The correlation between the IFSs A and B is given by the formula:

$$C_{IFS}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[\mu_A(x_i) \mu_B(x_i) + \left(1 - \gamma_A(x_i)\right)\left(1 - \gamma_B(x_i)\right) + \pi_A(x) \pi_B(x) \right], \quad (5)$$

Furthermore, the correlation coefficient between the IFSs A and B is defined by the formula:

$$K_{IFS}(A,\ B) = \frac{C_{IFS}(A,\ B)}{\sqrt{E_{IFS}(A)\ .\ E_{IFS}(B)}}, \qquad (6)$$

Where $0 \le K_{IFS}(A,B) \le 1$.

Proposition 1:

For A, $B \in$ IFS(X), we have:

$i)\ 0 \le C_{IFS}(A,B) \le 1,\ ii)\ C_{IFS}(A,B) = C_{IFS}(B,A),\ iii)\ K_{IFS}(A,B) = K_{IFS}(B,A).$

Theorem 1: For A, $B \in$ IFS(X), then $0 \le K_{IFS}(A,\ B) \le 1$.

Theorem 2: $K_{IFS}(A,B) = 1 \Leftrightarrow A = B.$

Theorem 3: $C_{IFS}(A,B) = 0 \Leftrightarrow A$ and B are non-fuzzy sets and satisfy the condition $\mu_A(x_i) + \mu_B(x_i) = 1$ or $\gamma_A(x_i) + \gamma_B(x_i) = 1$ or $\pi_A(x_i) + \pi_B(x_i) = 1,\ \forall x_i \in$ X.

Theorem 4: $C_{IFS}(A,A) = 1 \Leftrightarrow A$ is a non-fuzzy set.

Algorithm for MAGDM with IFS:

An Approach to Group Decision Making with Intuitionistic Fuzzy Information is given below:

Step: 1 Use IFOWG operator to aggregate all individual intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n} (k = 1,2,3,4)$ into a collective intuitionistic fuzzy decision matrix.

Step: 2 Use IFHG operator to derive the collective overall preference intuitionistic fuzzy values $\tilde{r}_i\,(i = 1,2,\ldots..m)$ of the alternative A_i, where $v = (v_1, v_2 \ldots v_n)$ be the weighting vector of decision makers, with: $V_k \in [0,1],\ \sum_{k=1}^{t} V_k = 1;\ w = (w_1, w_2 \ldots w_n)$ is the associated weighting vector of the IFHG operator with $w_j \in [0,1],\ \sum_{j=-1}^{n} w_j = 1$.

Step: 3 Calculate the score, accuracy, distance and correlation coefficient between the collective overall preference values $\tilde{r}_i$ and the positive ideal value $\tilde{r}^+$, where $\tilde{r}^+ = (1,0)$.

Step: 4 Rank all the alternatives $A_i\,(i = 1,2,\ldots,m)$ and select the most desirable one(s).

Singular Perturbation Problems

A differential equation with a small positive parameter multiplying the highest derivative term subject to boundary conditions belongs to a class of problems known as singular perturbation problems. The solutions of these problems have singularities related to boundary layers. The justification for the name "singular perturbation" is that the nature of the differential equations changes completely in the limit case when singular perturbation parameter is equal to zero. For example, the conservation of momenta and the conservation of energy equations change from being nonlinear parabolic equations to nonlinear hyperbolic equations. The difference between the regular perturbation problems and the singular perturbation problems is given as below. Consider a family of boundary value problems (BVPs) P_ε, depending on a small parameter ε. Under certain conditions, the solution $y_\varepsilon(x)$ of P_ε can be constructed by a well-known 'method of perturbation'; that is, as a power series in ε with its first term y_0 being the solution of the problem P_0 (obtained by putting ε =0 in P_ε). When such a power series expansion converges as $\varepsilon \to 0$ uniformly in x then it is a regular perturbation problem. When $y_\varepsilon(x)$ does not have a uniform limit in x as $\varepsilon \to 0$ this regular perturbation method fails and is called a singular perturbation problem. A numerical method, for a system of 3x3 singularly perturbed reaction-diffusion equations, involving an appropriate layer-adapted Shishkin piecewise uniform mesh is constructed.

The following two-point boundary value problem is considered for the singularly perturbed linear system of 3×3 second order differential equations

$-Eu'' + A(x)u(x) = f(x), \quad x \in (0,1), u(0)$ and $u(1)$ are given. For all $x \in [0,1]$, $\vec{u}(x) = (u_1(x), u_2(x), u_3(x))^T$, $\vec{f}(x) = (f_1(x), f_2(x), f_3(x))^T$, E and $A(x)$ are 3×3 matrices. $E = diag(\vec{\varepsilon})$, $\vec{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ with $0 \prec \varepsilon_1 \le \varepsilon_2 \le \varepsilon_3 \le 1$, The ε_i are assumed to be distinct and, for convenience, to have the ordering $\varepsilon_1 \prec \varepsilon_2 \prec \varepsilon_3$.

Shishkin Mesh:

For the case $\varepsilon_1 \prec \varepsilon_2 \prec \varepsilon_3$, a piecewise uniform Shishkin mesh $\vec{\Omega}^N$ with N mesh-intervals is constructed on $\vec{\Omega} = [0,1]$ as follows. The interval $[0,1]$ is subdivided into following sub-intervals

$[0,\tau_1] \cup (\tau_1,\tau_2] \cup (\tau_2,\tau_3] \cup (\tau_3, 1-\tau_3] \cup (1-\tau_3, 1-\tau_2] \cup (1-\tau_2, 1-\tau_1] \cup [1-\tau_1, 1].$

Where,

$$\tau_3 = \min\left\{\frac{1}{4}, 2\sqrt{\frac{\varepsilon_3}{\alpha}}\ln N\right\},\ \tau_2 = \min\left\{\frac{\tau_3}{2}, 2\sqrt{\frac{\varepsilon_2}{\alpha}}\ln N\right\},\ \tau_1 = \min\left\{\frac{\tau_2}{2}, 2\sqrt{\frac{\varepsilon_1}{\alpha}}\ln N\right\}.$$

Clearly $0 < \tau_1 < \tau_2 < \tau_3 \le \frac{1}{4}$. Then, on the subinterval $(\tau_3, 1-\tau_3]$ a uniform mesh with $\frac{N}{2}$ mesh points is placed and on each of the sub-intervals $(0,\tau_1]$, $(\tau_1,\tau_2], (\tau_2,\tau_3], (1-\tau_3, 1-\tau_2], (1-\tau_2, 1-\tau_1]\, and\, [1-\tau_1, 1]$ a uniform mesh of $\frac{N}{12}$ points is placed. Note that, when both the parameters $\tau_r, r = 1,2,3$ take on their left hand value, the shishkin mesh becomes a classical uniform mesh on $[0,1]$.

In the case, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ a simple construction with just one parameter τ is sufficient.

Discrete Problem:

The discrete two-point boundary value problem is now defined on any mesh by the finite difference method: $-E\delta^2 U + A(x)U = f(x), u(0) = u(0)$ and $u(1) = u(1)$.

This is used to compute numerical approximation to the exact solution can be written in the operator form: $L^N U = f, U(0) = u(0), U(1) = u(1)$.

Where $L^N = -E\delta^2 + A(x)$ and $\delta^2, D^+ \text{and}\, D^- are$ the difference operators

$$\delta^2 U(x_j) = \left(\frac{D^+U(x_j) - D^-U(x_j)}{\bar{h}_j}\right),\ \bar{h}_j = \frac{h_j + h_{j+1}}{2}, h_j = x_j - x_{j-1.}$$

$$D^+U(x_j) = \frac{U(x_{j+1}) - U(x_j)}{h_{j+1}} \quad \text{and} \quad D^-U(x_j) = \frac{U(x_j) - U(x_{j-1})}{h_j}.$$

Determining Experts Weights for MAGDM Problems Using a System of Singular Perturbtion Problem

Problem Proposed By Decision Maker-1:

The decision maker represents weighting vector regarding the plant species information in the form of the following system of second order singularly perturbed differential equation,

$$-E\vec{u}''(x)+A(x)\vec{u}(x)=\vec{f}(x),\ for\ x\in(0,1),\vec{u}(0)=\vec{0},\vec{u}(1)=\vec{0}$$

$$where\ E=diag(\varepsilon_1,\varepsilon_2,\varepsilon_3),\ \ A=\begin{pmatrix} 6 & -2 & 0 \\ -y^2 & 5(1+y) & -1 \\ -1 & -(1+y) & 6+y \end{pmatrix},\ \vec{f}=(1+e^y,1,2+y).$$

The numerical solution can be calculated using the classical finite difference scheme which is given above. The numerical solutions of the system of equations are given in table.

Table 1: Numerical Solution of $-E\vec{u}''(x)+A(x)\vec{u}(x)=\vec{f}(x)$.

x	$u_1(x)$	$u_2(x)$	$u_3(x)$
0.0087610	0.52622	0.31569	0.48033
0.99124	0.74391	0.24105	0.60644
0.017522	0.44592	0.28744	0.45677
0.98248	0.69551	0.22852	0.59181
0.026283	0.43446	0.28399	0.45674
0.97372	0.68468	0.22680	0.58881

The weighting vectors can be calculated using the method of confidence intervals described in the following:
Suppose the fuzzy system of plant associations during N time periods is observed and find that there have been n_i times that i systems (plant species), (i = 0,1,2,....) grow near Bamboo. Let $p(i)$ be the probability that i plant species occur in time period t. Then a point estimate of $p(i)$ is simply n_i/N. However to show the uncertainty in this estimate, a confidence interval for $p(i)$ can also be computed. Hence the $(1-\beta)100\%$ confidence intervals for $p(i)$, for all $0.01\le\beta<1$ is denoted by $[p(i)_1(\beta), p(i)_2(\beta)]$. Starting at 0.01 is arbitrary, and one can choose to begin at 0.001 or 0.005. Then $[n_i/N, n_i/N]$ is the 0% confidence interval for $p(i)$ whose α -cuts $p(i)[\alpha]=[p(i)_1(\alpha), p(i)_2(\alpha)]$ can be formed for $0.01\le\alpha<1$.
Suppose $N=500,\ n_i=100,$ then a $(1-\beta)100\%$ confidence interval for $p(1)$ is given by:

$$\left[0.2-Z_{\beta/2}\sqrt{\frac{0.2(1-0.2)}{500}},0.2+Z_{\beta/2}\sqrt{\frac{0.2(1-0.2)}{500}}\right].$$

Where $Z_{\beta/2}$ is defined by $\int_{-\infty}^{Z_{\beta/2}} N(0,1)dx=1-\beta/2.$

$N(0,1)$ denotes normal density with mean zero and unit variance.
This type of confidence intervals are applied to the numerical solutions of the above system of equations with different α values, namely 0.2, 0.4, 0.6, 0.8, 1.0 and the results are given in

Table **2** to

Table **7**.

Table 2: Confidence interval at even α values creating triangular fuzzy numbers

x	$u_1(x)$	0.0	0.2	0.4	0.6	0.8	1.0
τ_1 =0.0087610	0.52622	0.52622	0.52175	0.51729	0.51282	0.50836	0.50389
		0.52622	0.53069	0.53515	0.53962	0.54408	0.54855
	Defuzzified Values		0.87107	0.86513	0.85917	0.85322	0.84726
	Normalised Defuzzified Values		0.20277	0.20138	0.20000	0.19861	0.19722
τ_2 =0.99124	0.74391	0.74391	0.74001	0.73610	0.73220	0.72829	0.72439
		0.74391	0.74781	0.75172	0.75562	0.75953	0.76343
	Defuzzified Values		1.2346	1.2294	1.2242	1.2190	1.2138
	Normalised Defuzzified Values		0.20169	0.20084	0.20000	0.19915	0.19830
τ_3 =0.017522	0.44592	0.44592	0.44147	0.43703	0.43258	0.42814	0.42369
		0.44592	0.45037	0.45481	0.45926	0.46370	0.46815
	Defuzzified Values		0.73727	0.73135	0.72541	0.71949	0.71356
	Normalised Defuzzified Values		0.20326	0.20163	0.19999	0.19836	0.19673
$1-\tau_1$ =0.98248	0.69551	0.69551	0.69139	0.68728	0.68316	0.67905	0.67493
		0.69551	0.69963	0.70374	0.70786	0.71197	0.71609
	Defuzzified Values		1.1537	1.1482	1.1427	1.1372	1.1317
	Normalised Defuzzified Values		0.20192	0.20096	0.20000	0.19903	0.19807
$1-\tau_2$ =0.026283	0.43446	0.43446	0.43003	0.42559	0.42116	0.41673	0.41229
		0.43446	0.43889	0.44333	0.44776	0.45219	0.45663
	Defuzzified Values		0.71819	0.71227	0.70637	0.70046	0.69454
	Normalised Defuzzified Values		0.20334	0.20167	0.20000	0.19832	0.19665
$1-\tau_3$ =0.97372	0.68468	0.68468	0.68052	0.67637	0.67221	0.66806	0.66390
		0.68468	0.68884	0.69299	0.69715	0.70130	0.70546
	Defuzzified Values		1.1356	1.1301	1.1245	1.1190	1.1134
	Normalised Defuzzified Values		0.20197	0.20099	0.19999	0.19901	0.19802

Table 3: Confidence interval at odd α values creating triangular fuzzy numbers

x	$u_1(x)$	0.0	0.1	0.3	0.5	0.7	0.9

τ_1 =0.0087610	0.52622	0.52622	0.52399	0.51952	0.51506	0.51059	0.50612
		0.52622	0.52845	0.53292	0.53738	0.54185	0.54632
	Defuzzified Values		0.87406	0.86810	0.86215	0.85619	0.85023
	Normalised Defuzzified Values		0.33562	0.33333	0.33104		
	Normalised Defuzzified Values				0.33565	0.33333	0.33101
τ_2 =0.99124	0.74391	0.74391	0.74196	0.73805	0.73415	0.73025	0.72634
		0.74391	0.74586	0.74977	0.75367	0.75757	0.76148
	Defuzzified Values		1.2372	1.2320	1.2268	1.2216	1.2164
	Normalised Defuzzified Values		0.33474	0.33333	0.33192		
	Normalised Defuzzified Values				0.33475	0.33333	0.33191
τ_3 =0.017522	0.44592	0.44592	0.44370	0.43925	0.43481	0.43036	0.42591
		0.44592	0.44814	0.45259	0.45703	0.46148	0.46593
	Defuzzified Values		0.74024	0.73431	0.72839	0.72245	0.71652
	Normalised Defuzzified Values		0.33602	0.33333	0.33064		
	Normalised Defuzzified Values				0.33607	0.33333	0.33059
$1-\tau_1$ =0.98248	0.69551	0.69551	0.69345	0.68934	0.68522	0.68110	0.67699
		0.69551	0.69757	0.70168	0.70580	0.70992	0.71403
	Defuzzified Values		1.1564	1.1510	1.1455	1.1400	1.1345
	Normalised Defuzzified Values		0.33490	0.33333	0.33175		
	Normalised Defuzzified Values				0.33494	0.33333	0.33172
$1-\tau_2$ =0.026283	0.43446	0.43446	0.43224	0.42781	0.42338	0.41894	0.41451
		0.43446	0.43668	0.44111	0.44554	0.44998	0.45441
	Defuzzified Values		0.72040	0.71449	0.70859	0.70267	0.69676

	Normalised Defuzzified Values		0.33608	0.33333	0.33057		
	Normalised Defuzzified Values				0.33614	0.33333	0.33052
$1-\tau_3$ =0.97372	0.68468	0.68468	0.68260	0.67845	0.67429	0.67013	0.66598
		0.68468	0.68676	0.69091	0.69507	0.69923	0.70338
	Defuzzified Values		1.1384	1.1328	1.1273	1.1217	1.1162
	Normalised Defuzzified Values		0.33497	0.33333	0.33170		
	Normalised Defuzzified Values				0.33498	0.33333	0.33168

Table 4: Confidence interval at even α values creating triangular fuzzy numbers

x	$u_2(x)$	0.0	0.2	0.4	0.6	0.8	1.0
τ_1 =0.0087610	0.31569	0.31569	0.31153	0.30738	0.30322	0.29906	0.29490
		0.31569	0.31985	0.32400	0.32816	0.33232	0.33648
	Defuzzified Values		0.52060	0.51507	0.50952	0.50398	0.49843
	Normalised Defuzzified Values		0.20434	0.20217	0.20000	0.19782	0.19564
τ_2 =0.99124	0.24105	0.24105	0.23722	0.23340	0.22957	0.22575	0.22192
		0.24105	0.24488	0.24870	0.25253	0.25635	0.26018
	Defuzzified Values		0.39664	0.39155	0.38644	0.38135	0.37624
	Normalised Defuzzified Values		0.20527	0.20264	0.19999	0.19736	0.19471
τ_3 =0.017522	0.28744	0.28744	0.28339	0.27934	0.27530	0.27125	0.26720
		0.28744	0.29149	0.29554	0.29958	0.30363	0.30768
	Defuzzified Values		0.47367	0.46827	0.46288	0.45748	0.45208
	Normalised Defuzzified Values		0.20466	0.20233	0.20000	0.19766	0.19533
$1-\tau_1$ =0.98248	0.22852	0.22852	0.22476	0.22101	0.21725	0.21350	0.20974
		0.22852	0.23228	0.23603	0.23979	0.24354	0.24730
	Defuzzified Values		0.37585	0.37085	0.36584	0.36084	0.35583
	Normalised Defuzzified Values		0.20547	0.20273	0.19999	0.19726	0.19452

$1-\tau_2$ =0.026283	0.28399	0.28399	0.27996	0.27592	0.27189	0.26786	0.26382
		0.28399	0.28802	0.29206	0.29609	0.30012	0.30416
	Defuzzified Values		0.46794	0.46256	0.45718	0.45181	0.44642
	Normalised Defuzzified Values		0.20470	0.20235	0.19999	0.19764	0.19529
$1-\tau_3$ =0.97372	0.22680	0.22680	0.22305	0.21931	0.21556	0.21182	0.20807
		0.22680	0.23055	0.23429	0.23804	0.24178	0.24553
	Defuzzified Values		0.37300	0.36801	0.36301	0.35803	0.35303
	Normalised Defuzzified Values		0.20550	0.20275	0.19999	0.19725	0.19449

Table 5: Confidence interval at odd α values creating triangular fuzzy numbers

x	$u_2(x)$	0.0	0.1	0.3	0.5	0.7	0.9
τ_1 =0.0087610	0.31569	0.31569	0.52399	0.51952	0.51506	0.51059	0.50612
		0.31569	0.52845	0.53292	0.53738	0.54185	0.54632
	Defuzzified Values		0.87406	0.86810	0.86215	0.85619	0.85023
	Normalised Defuzzified Values		0.33562	0.33333	0.33104		
	Normalised Defuzzified Values				0.33565	0.33333	0.33101
τ_2 =0.99124	0.24105	0.24105	0.74196	0.73805	0.73415	0.73025	0.72634
		0.24105	0.74586	0.74977	0.75367	0.75757	0.76148
	Defuzzified Values		1.2372	1.2320	1.2268	1.2216	1.2164
	Normalised Defuzzified Values		0.33474	0.33333	0.33192		
	Normalised Defuzzified Values				0.33475	0.33333	0.33191
τ_3 =0.017522	0.28744	0.28744	0.44370	0.43925	0.43481	0.43036	0.42591
		0.28744	0.44814	0.45259	0.45703	0.46148	0.46593
	Defuzzified Values		0.74024	0.73431	0.72839	0.72245	0.71652
	Normalised Defuzzified Values		0.33602	0.33333	0.33064		

	Normalised Defuzzified Values				0.33607	0.33333	0.33059
$1-\tau_1$ =0.98248	0.22852	0.22852	0.69345	0.68934	0.68522	0.68110	0.67699
		0.22852	0.69757	0.70168	0.70580	0.70992	0.71403
	Defuzzified Values		1.1564	1.1510	1.1455	1.1400	1.1345
	Normalised Defuzzified Values		0.33490	0.33333	0.33175		
	Normalised Defuzzified Values				0.33494	0.33333	0.33172
$1-\tau_2$ =0.026283	0.28399	0.28399	0.43224	0.42781	0.42338	0.41894	0.41451
		0.28399	0.43668	0.44111	0.44554	0.44998	0.45441
	Defuzzified Values		0.72040	0.71449	0.70859	0.70267	0.69676
	Normalised Defuzzified Values		0.33608	0.33333	0.33057		
	Normalised Defuzzified Values				0.33614	0.33333	0.33052
$1-\tau_3$ =0.97372	0.22680	0.22680	0.68260	0.67845	0.67429	0.67013	0.66598
		0.22680	0.68676	0.69091	0.69507	0.69923	0.70338
	Defuzzified Values		1.1384	1.1328	1.1273	1.1217	1.1162
	Normalised Defuzzified Values		0.33497	0.33333	0.33170		
	Normalised Defuzzified Values				0.33498	0.33333	0.33168

Table 6: Confidence interval at even α values creating triangular fuzzy numbers

x	$u_3(x)$	0.0	0.2	0.4	0.6	0.8	1.0
τ_1 =0.087610	0.48033	0.48033	0.47586	0.47139	0.46692	0.46246	0.45799
		0.48033	0.48480	0.48927	0.49374	0.49820	0.50267
	Defuzzified Values		0.79459	0.78863	0.78267	0.77672	0.77076
	Normalised Defuzzified Values		0.20304	0.20152	0.19999	0.19847	0.19695
	0.60644	0.60644	0.60207	0.59770	0.59333	0.58896	0.58459
		0.60644	0.61081	0.61518	0.61955	0.62392	0.62829

τ_2 =0.99124	Defuzzified Values		1.0049	0.99928	0.99345	0.98763	0.98180
	Normalised Defuzzified Values		0.20231	0.20118	0.20000	0.19883	0.19766
τ_3 =0.017522	0.45677	0.45677	0.45231	0.44786	0.44340	0.43895	0.43449
		0.45677	0.46123	0.46568	0.47014	0.47459	0.47905
	Defuzzified Values		0.75534	0.74940	0.74346	0.73752	0.73158
	Normalised Defuzzified Values		0.20319	0.20159	0.20000	0.19840	0.19680
$1-\tau_1$ =0.98248	0.59181	0.59181	0.58741	0.58302	0.57862	0.57423	0.56983
		0.59181	0.59621	0.60060	0.60500	0.60939	0.61379
	Defuzzified Values		0.98048	0.97463	0.96876	0.96291	0.95704
	Normalised Defuzzified Values		0.20241	0.20121	0.19999	0.19879	0.19757
$1-\tau_2$ 0.026283	0.45674	0.45674	0.45228	0.44783	0.44337	0.43892	0.43446
		0.45674	0.46120	0.46565	0.47011	0.47456	0.47902
	Defuzzified Values		0.75529	0.74935	0.74341	0.73747	0.73153
	Normalised Defuzzified Values		0.20319	0.20159	0.20000	0.19840	0.19680
$1-\tau_3$ =0.97372	0.58881	0.58881	0.58441	0.58001	0.57561	0.57121	0.56680
		0.58881	0.59321	0.59761	0.60201	0.60641	0.61082
	Defuzzified Values		0.97548	0.96962	0.96375	0.95778	0.95200
	Normalised Defuzzified Values		0.20243	0.20121	0.20000	0.19878	0.19756

Table 7: Confidence interval at odd α values creating triangular fuzzy number

x	$u_3(x)$	0.0	0.1	0.3	0.5	0.7	0.9
τ_1 =0.0087610	0.48033	0.48033	0.47810	0.47363	0.46916	0.46469	0.46022
		0.48033	0.48256	0.48703	0.49150	0.49597	0.50044
	Defuzzified Values		0.79758	0.79162	0.78566	0.77970	0.77374
	Normalised Defuzzified Values		0.33584	0.33333	0.33082		
	Normalised Defuzzified Values				0.33588	0.33333	0.33078

τ_2 =0.99124	0.60644	0.60644	0.60426	0.59989	0.59552	0.59115	0.58678
		0.60644	0.60862	0.61299	0.61736	0.62173	0.62610
	Defuzzified Values		1.0078	1.0022	0.99637	0.99055	0.98472
	Normalised Defuzzified Values		0.33522	0.33333	0.33141		
	Normalised Defuzzified Values				0.33529	0.33333	0.33137
τ_3 =0.017522	0.45677	0.45677	0.45454	0.45009	0.44563	0.44118	0.43672
		0.45677	0.45900	0.46345	0.46791	0.47236	0.47682
	Defuzzified Values		0.75831	0.75238	0.74643	0.74050	0.73455
	Normalised Defuzzified Values		0.33596	0.33333	0.33070		
	Normalised Defuzzified Values				0.33600	0.33333	0.33065
$1-\tau_1$ =0.98248	0.59181	0.59181	0.58961	0.58522	0.58082	0.57642	0.57203
		0.59181	0.59401	0.59840	0.60280	0.60720	0.61159
	Defuzzified Values		0.98342	0.97756	0.97170	0.96583	0.95998
	Normalised Defuzzified Values		0.33533	0.33333	0.33133		
	Normalised Defuzzified Values				0.33535	0.33333	0.33131
$1-\tau_2$ =0.026283	0.45674	0.45674	0.45451	0.45006	0.44560	0.44115	0.43669
		0.45674	0.45897	0.46342	0.46788	0.47233	0.47679
	Defuzzified Values		0.75826	0.75233	0.74638	0.74045	0.73450
	Normalised Defuzzified Values		0.33596	0.33333	0.33070		
	Normalised Defuzzified Values				0.33600	0.33333	0.33065
$1-\tau_3$ =0.97372	0.58881	0.58881	0.58661	0.58221	0.57781	0.57341	0.56901
		0.58881	0.59101	0.59541	0.59981	0.60421	0.60861
	Defuzzified Values		0.97842	0.97255	0.96668	0.96082	0.95495
	Normalised Defuzzified Values		0.33534	0.33333	0.33132		

	Normalised Defuzzified Values				0.33536	0.33333	0.33129

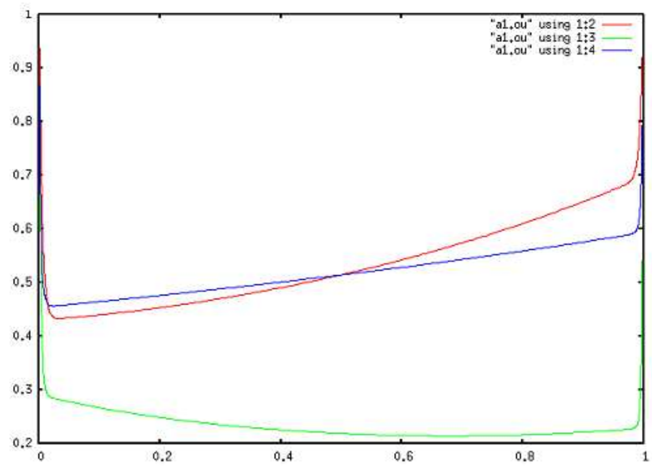
At the point x = 0:0087610 (value of τ_1 ; table 1.2.), the confidence interval is calculated and the weight vector (0.87107, 0.86513, 0.85917, 0.85322, 0.84726) is obtained. The above vector is calculated by using defuzzification function for the α values of the point x. The decision maker weighting vector is normalized and is calculated as (0.20277, 0.20138, 0.20000, 0.19861, 0.19722). The remaining decision maker weighting vectors are calculated in the similar manner for different x values. The same procedure is followed for odd and even values of different α values and whose confidence interval values are displayed in tables 2-7. The numerical computation have been carried out in Linux environment and FORTRAN codes. The maximum point wise error and the order of convergence are calculated using the two mesh algorithm (Table 8).

Table 8: Values of $D_\varepsilon^N, D^N, P^N, P^*$ *and* C_P^N *for* $\varepsilon_1 = \frac{\eta}{64}, \varepsilon_2 = \frac{\eta}{8}$ *and* $\alpha = 3.9$

η	**192**	**384**	**768**	**1536**
2^0	0.239E-02	0.116E-02	0.570E-03	0.283E-03
2^{-2}	0.487E-02	0.267E-02	0.146E-02	0.762E-03
2^{-4}	0.473E-02	0.258E-02	0.141E-02	0.769E-03
2^{-6}	0.466E-02	0`253E-02	0.138E-02	0.751E-03
2^{-8}	0.465E-02	0.251E-02	0.136E-02	0.741E-03
D^N	0.487E-02	0.267E-02	0.146E-02	0.769E-03
P^N	0.868E+00	0.869E+00	0.926E+00	
C_P^N	0.104E+01	0.104E+01	0.103E+01	0.995E+00

The order of convergence=0.8683857E+00. The error constant=0.1035456E+01

Figure 1: Numerical Solution of $-E\vec{u}''(x) + A(x)\vec{u}(x) = f(x)$



Computing Weights for Plant Species MAGDM Problem Using Linear Programming Method

An economy developed through agroforestry falls in three different categories bound to certain level of return and risk. Let the amount invested in the plantations of Bamboo and its associated species be w_1, w_2 and w_3 respectively. The return and risk associated with the process are given in the

Table **9**.

Table 9: Return and Risk of Categories

Level of investment	Return	Risk
L_1	80%	50%
L_2	70%	60%
L_3	40%	80%

The total amount to be invested, the total minimum returns and the maximum combined risk which are given by three decision makers (DM) are given in the Table 10.

Table 10: Conditions of the DM.

Decision Maker	Total Investment	Minimum Returns	Maximum Risk
DM(i)	0.8	60%	40%
DM(ii)	0.9	45%	30%
DM(iii)	0.7	40%	30%

Three decision makers give three possible optimal solutions to the weighting vector for the decision-making problem. The formulation of the decision problem by first Decision Maker is given as follows.

D.M (i): Max $Z = 0.8w_1 + 0.7w_2 + 0.4w_3$

Subject to constraints,

$$w_1 + w_2 + w_3 \leq 0.8; 0.5w_1 + 0.6w_2 + 0.8w_3 \leq 0.32;$$

$$0.8w_1 + 0.7w_2 + 0.4w_3 \geq 0.48;\ w_1,\ w_2,\ w_3 \geq 0. \tag{7}$$

Table 11: Optimal table of DM(i)

c_B	y_B	c_j	0.8	0.7	0.4	0	0	0

		x_B	w_1	w_2	w_3	s_1	s_2	s_3
0	s_1	**0.16**	0	-0.2	-0.6	1	-2	0
0	s_3	**0.032**	0	0.26	0.88	0	1.6	1
0.8	w_1	**0.64**	1	1.2	1.6	0	2	0
	$z_j - c_j$		0	0.26	0.88	0	1.6	0

Hence the weight vector from DM (i) is $w_1 = 0.64$ $w_2 = 0$ $w_3 = 0$.
The formulation of the decision problem by second Decision Maker is given as follows.
D.M (ii): Max $Z = 0.8w_1 + 0.7w_2 + 0.4w_3$

Subject to constraints,

$w_1 + w_2 + w_3 \leq 0.9$; $0.5w_1 + 0.6w_2 + 0.8w_3 \leq 0.27$;

$0.8w_1 + 0.7w_2 + 0.4w_3 \geq 0.4$; $w_1, w_2, w_3 \geq 0$. (8)

Table 12: Optimal table of DM (ii)

c_B	y_B	c_j	0.8	0.7	0.4	0	0	0
		x_B	w_1	w_2	w_3	s_1	s_2	s_3
0	s_1	**0.36**	0	-0.2	-0.6	1	-2	0
0	s_3	**0.032**	0	0.26	0.88	0	1.6	1
0.8	w_1	**0.54**	1	1.2	1.6	0	2	0
	$z_j - c_j$		0	0.26	0.88	0	1.6	0

Hence the weight vector from DM (ii) is $w_1 = 0.54$ $w_2 = 0$ $w_3 = 0$
The formulation of the decision problem by third Decision Maker is given as follows.
D.M (iii): Max $Z = 0.8w_1 + 0.7w_2 + 0.4w_3$

Subject to constraints,

$w_1 + w_2 + w_3 \leq 0.7$; $0.5w_1 + 0.6w_2 + 0.8w_3 \leq 0.21$;

$0.8w_1 + 0.7w_2 + 0.4w_3 \geq 0.28$; $w_1, w_2, w_3 \geq 0$. (9)

Table 13: Optimal table of DM (iii)

c_B	y_B	c_j	0.8	0.7	0.4	0	0	0
		x_B	w_1	w_2	w_3	s_1	s_2	s_3
0	s_1	**0.28**	0	-0.2	-0.6	1	-2	0
0	s_3	**0.056**	0	0.26	0.88	0	1.6	1
0.8	w_1	**0.42**	1	1.2	1.6	0	2	0
	$z_j - c_j$		0	0.26	0.88	0	1.6	0

Hence the weight vector from DM (iii) is $w_1 = 0.42$ $w_2 = 0$ $w_3 = 0$
Construct a matrix with the vector obtained from the three decision makers as follows.

$$w = \begin{bmatrix} 0.64 & 0 & 0 \\ 0.54 & 0 & 0 \\ 0.42 & 0 & 0 \end{bmatrix} \tag{10}$$

Taking column average and dividing each entry of that column, we get,

$$\omega = \begin{bmatrix} 0.4 & 0 & 0 \\ 0.34 & 0 & 0 \\ 0.26 & 0 & 0 \end{bmatrix} \tag{11}$$

Taking row average of the above matrix, we get,

$$\omega_1 = \begin{bmatrix} 0.13 \\ 0.11 \\ 0.09 \end{bmatrix} \tag{12}$$

To find the weight vector, we calculate $v = w \times \omega_1$.

$$\text{v} = \begin{bmatrix} 0.64 & 0 & 0 \\ 0.54 & 0 & 0 \\ 0.42 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.13 \\ 0.11 \\ 0.09 \end{bmatrix} = \begin{bmatrix} 0.083 \\ 0.07 \\ 0.055 \end{bmatrix} \tag{13}$$

Taking column average and dividing each entry of that column, we get, $v = (0.3990, 0.3365, 0.2645)^T$. This will represent the weight vector obtained from the decision makers.

Return and Risk associated with Category 2

The return and risk associated with category 2 are given in the

Table **14**.

Table 14: Return and Risk of Categories.

Level of investment	Return	Risk
L_1	70%	30%
L_2	80%	30%
L_3	60%	20%

The total amount to be invested, the total minimum returns and the maximum combined risk which are given by three decision makers (DM) are given in the

Table **15**.

Table 15: Conditions of the DM.

Decision Maker	Total Investment	Minimum Returns	Maximum Risk
DM(i)	0.9	70%	50%
DM(ii)	0.8	70%	60%
DM(iii)	0.7	60%	40%

Three decision makers give three possible optimal solutions to the weighting vector for the decision-making problem. Similarly the weight vector $v = (0.375, 0.3333, 0.2917)^T$ is obtained from the decision makers.

Numerical Illustration with IFS Data Set

Pachaimalai hills though not known for its wild population of Bamboo cultivation, the Forest department of Tamilnadu has taken steps in the past to cultivate and improve the bamboo plantations which can be seen on the foot of the hills. A thick vegetation of bamboo plantation can be observed in the Keelakarai village and Sobanapuram Check-post (Maavidayaan Koil) in the vicinities of Pachaimalai hills. The distribution of bamboo plantation in the foot of the hills is mostly even or predictable, and hence surveying the frequency of plant associations with bamboo is a not a tedious job. Hence we identified fifty potential spots in the region of bamboo plantations in the hills where the distribution of the same is considerably notable and the surveyed frequency data is converted into fuzzy numbers due to the irregular distribution of the plant communities. Fuzzy membership functions are used for the purpose of data extraction. Five different plant species are found to be closely associated to bamboo in Pachaimalai hills and they seem to recur in all the surveyed spots. The five plant species found to be distributed around bamboo in the hills are *Drypetes sepiaria* (Wight & Arn.) Pax & K.Hoffm., *Citrus sp*, *Acacia pennata* (L.)Willd., *Atalantia monophylla* Hook. & Arn. and *Phoenix sylvestris* (L.) Roxb. The present study was conducted in a place called Keelakarai village and Sobanapuram Check-post in Pachaimalai hills which are a green hill range just 80 kms north of Tiruchirappalli via Thuraiyur, South India. The altitude of study is about 100 to 200metres where the plantations of Bamboo were long ago promoted by the forest department, Tamilnadu. The hills is spread over an area of 13,500 square km and only the Bamboo plantation in the Keelakarai village were covered for the study.

Suppose the Forest Department of Pachaimalai Hills-Eastern Ghats, is planning to conduct agro forestry. From our study and survey, we have identified a panel with five possible alternatives of different plant species which can be identified as very closely associated to grow around Bamboo in that area. They are;

(1) f_1 - Drypetes sepiaria (Wight & Arn.) Pax & K.Hoffm.;
(2) f_2 – Citrus sp;
(3) f_3 – Acacia pennata (L.)Willd.;
(4) f_4 – Atalantia monophylla Hook & Arn.;
(5) f_5 – Phoenix sylvestris (L.)Roxb.

The Forest Department must take a decision according to the following five attributes:

(1) C_1 is the Economic Value;
(2) C_2 is the Soil Type;
(3) C_3 is the Temperature & Humidity;
(4) C_4 is the Rainfall;
(5) C_5 is the Plant Communities (Associations).

A Note on the Attributes

Economic Value: The economic value of biodiversity is measured in the numerous benefits that are derived from it: both tangible and intangible. These range from the things that are produced and sold, which are derived both directly and indirectly from biodiversity, to the non-marketed things that contribute to both our well-being and to the economy. These benefits can be demonstrated to already be significant in the areas where they are measured in market activities.

Soil Type: The soils are classified as hill soils. Soil types of the area are more important since it is the main criteria in the agricultural production and in the recharge of ground water. The soil type usually found in Pachaimalai hills is as follows:

- Black soil
- Alluvial Soil
- Red loam soil

Major crops grown in Pachaimalai are:

- Paddy, Gingelly,
- Thenai, Samai, Ragi, Cumbu (Local Names)
- Different fruit trees
- Dry Paddy is the major crop and only local variety is grown is this hill.

Temperature, Humidity: In Pachamalai hill sub-tropical climate prevails with a maximum temperature ranging between 23 to 31^{0}C, and a minimum temperature range of 12^{0}C to 18^{0}C. A maximum of 1250 mm had been recorded so far in the past ten years.

Rainfall: Generally Maximum amount of rainfall is received only during Northeast monsoon (i.e.,) in the months of September, October and November. Southwest monsoon rains are received during the months of June, and August. The northeast monsoon rains are however more dependable.

Plant Communities (Associations): A grouping of plant species, or a plant community, that recurs across the landscape. Plant associations are used as indicators of environmental conditions such as temperature, moisture, light, etc. Some plants will grow much better if close to companion plants. This natural phenomenon is called allelopathy: as a plant will attract greenflies, it will protect crops from pests and weeds; thus a dense potato plot will be free of bindweed: this means almost no pesticides are needed (or even none at all)

Figure-2: Bamboo with other species in Pachaimalai Hills (Near the location of Maavidayaan Koil)

Figure-3: Drypetes sepiaria (Wight & Arn.) Pax & K. Hoffm.

Figure-4: Citrus sp

Figure-5: Acacia Pennata(L.)Willd.:

Figure-6: Atalantia monophylla Hook & Arn.

Figure-7: Phoenix sylvestris (L.) Roxb.

Figure-8: Linear relationship between the identified species f_1, f_2, f_3, f_4, f_5.

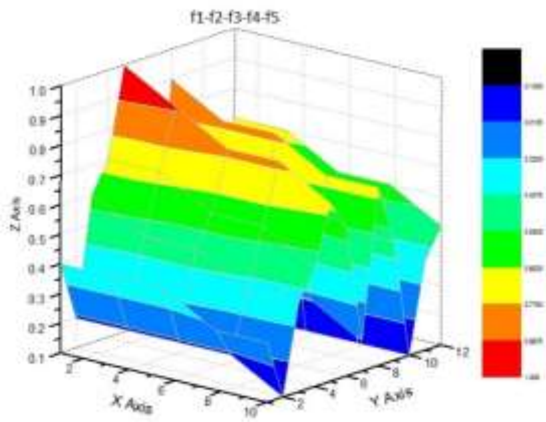


Table 9: Pearson's Correlation coefficient with scatter diagram for 5 variables

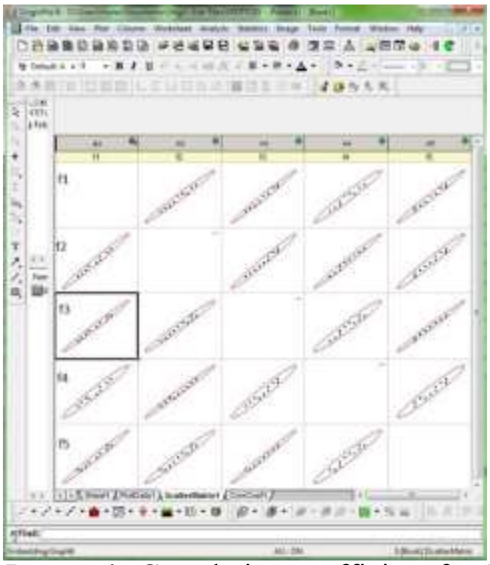


Table 10: Pearson's Correlation coefficient for 5 variables

The five possible alternatives are to be evaluated using the intuitionistic fuzzy number by the three decision matrices provided by one of the decision makers. According to the nature of the plant species distribution in the Pachaimalai Hills, the field of knowledge of the local communities and living environment, three decision matrices given by a decision maker can be expressed, respectively, as follows:

$$R_1 = \begin{bmatrix} (0.3,0.6)(0.1,0.9)(0.2,0.6)(0.8,0.1)(0.1,0.3) \\ (0.7,0.2)(0.3,0.5)(0.1,0.5)(0.2,0.4)(0.4,0.6) \\ (0.2,0.8)(0.1,0.8)(0.6,0.2)(0.1,0.9)(0.1,0.9) \\ (0.4,0.6)(0.5,0.1)(0.2,0.7)(0.8,0.1)(0.1,0.7) \\ (0.5,0.2)(0.7,0.2)(0.2,0.7)(0.3,0.6)(0.1,0.2) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0.2,0.4)(0.1,0.8)(0.3,0.6)(0.1,0.9)(0.5,0.2) \\ (0.1,0.2)(0.5,0.3)(0.6,0.2)(0.1,0.3)(0.3,0.6) \\ (0.3,0.5)(0.1,0.5)(0.2,0.7)(0.4,0.2)(0.7,0.1) \\ (0.7,0.1)(0.1,0.9)(0.4,0.3)(0.5,0.4)(0.2,0.7) \\ (0.6,0.3)(0.8,0.1)(0.7,0.2)(0.3,0.6)(0.5,0.2) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0.4,0.2)(0.2,0.4)(0.8,0.1)(0.1,0.8)(0.3,0.6) \\ (0.5,0.3)(0.1,0.2)(0.1,0.3)(0.6,0.2)(0.5,0.2) \\ (0.1,0.3)(0.5,0.2)(0.4,0.5)(0.7,0.1)(0.3,0.2) \\ (0.4,0.1)(0.2,0.6)(0.5,0.3)(0.6,0.1)(0.9,0.1) \\ (0.5,0.3)(0.3,0.1)(0.4,0.4)(0.5,0.2)(0.2,0.5) \end{bmatrix}$$

By using the above algorithm and the weighting vectors derived from Singular Perturbation Problems and linear programming problem we obtain:

$K(\tilde{r}_1, \tilde{r}^+) = 0.8887$; $K(\tilde{r}_2, \tilde{r}^+) = 0.8043$; $K(\tilde{r}_3, \tilde{r}^+) = 0.8844$;

$K(\tilde{r}_4, \tilde{r}^+) = 0.9354$; $K(\tilde{r}_5, \tilde{r}^+) = 0.9064$.

Rank the alternatives $K_i\,(i = 1,2,3,4,5)$. $K_4 > K_5 > K_1 > K_3 > K_2$.

Hence the best alternatives is A_4.

Table 16: Comparison of Ranking

Ranking Methods	Ranking
Correlation Coefficient	$A_4 > A_5 > A_1 > A_3 > A_2$.
Score	$A_5 > A_2 > A_4 > A_3 > A_1$.
Accuracy	$A_1 > A_4 > A_3 > A_5 > A_2$.
Euclidean Distance	$A_5 < A_4 < A_2 < A_3 < A_1$.

3. Discussion

From the above comparison table, it is observed that, when using Correlation coefficient the best alternative is A_4. When comparisons are made by ranking with Score function, Accuracy function and Distance measure it can be observed that A_4 is the second or third best alternative. Since the Correlation coefficient gives the linear relationship between the variables, the comparison revealed that the ranking with the correlation coefficient gives the best alternative when compared to all other ranking measures. Hence the final decision is consistent for all the ranking methods. Agroforestry can turn to be an Economy raising farming both to the farmers and to the Government. Total employment within the indigenous forest industry will increase and employment generated by new planting, the harvesting and haulage of timber in sawmills, stake plants and pulp mills will also find new horizons.

4. Conclusion

In this study, a system of singularly perturbed differential equations for the decision making process is numerically solved to determine the unknown decision maker weights. The individual intuitionistic fuzzy decision matrices are all combined into a single matrix using the IFOWG and IFHG operators. The correlation coefficient approach, distance measurements, and score functions are all used in the ranking process. The three approaches were used to select the best options. From the study it is concluded that, when bamboo is planted together with Atalantia monophylla Hook & Arn, Phoenix sylvestris (L.)Roxb. and Drypetes sepiaria (Wight & Arn.) Pax & K. Hoffm, it will yield a great economic growth for Agroforestry.

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