Section A-Research paper
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The Radio Circular Distance in Lehmer-3 Mean Number of Some Wheel Related Graphs

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ABSTRACT:

A radio circular distance in lehmer-3 mean labeling of a connected graph G is an injective map f from the vertex set V(G) to the $\mathbb{N}$ *such that for two distinct vertices u and v of G ,* $Cir(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^C(G)$ *where* $Cir(u,v)$ *denotes Circular distance between u and v and* $diam^C(G)$ *denotes the circular diameter of G. The radio circular distance in lehmer – 3 mean number of f,* ${r_{l_3mn}}^C(f)$ *is the maximum label assigned to any vertex of G. The radio circular distance in lehmer -3 mean number of G ,* ${r_{l_3mn}}^C(G)$ *is the minimum value of G. In this paper, we investigate the radio circular Distance in lehmer – 3 mean number of some wheel related graphs.*

Key Words: Circular distance, Circular diameter, radio Circular distance number, Lehmer-3 mean labeling

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1.INTRODUCTION:

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notation $V(G)$ or V is the vertex set of G and $E(G)$ or E is the edge set of $G = G(V,E)$.

Peruri Lakshmi and Janagam Veeranjaneyulu [14] introduced the concept of Circular distance in graph as follows for two distinct vertices u, v in a graph G, Circular distance of a $u - v$ path is defined as $Cir(u,v) = D(u,v) + \mathrm{d}(u,v)$ where $D(u,v)$ is the length of the longest path between u and v. The Circular radius , $r^C(G)$ is the minimum circular eccentricity among all vertices of u and v of G . Similarly the circular diameter , $diam^C(G)$ is the maximum circular eccentricity among all vertices of G. We observe that for any two vertices u,v of G. If G is any connected graph then the circular distance is metric on the set of vertices of G. We can check easily $r^C(G) \leq diam^C(G) \leq 2r^C(G)$.

Chartrand et al. [2] defined the concept of radio labeling of G. Radio labeling of graphs is motivated by restrictions inherent in assigning channel frequencies for radio transmitters [2]. Radio labeling behavior of several graphs are studied by Kchikech et al. [5,6], Khennoufa et al. [7], Liu et al. [8–12], Van den Heuvel et al. [18] and Zhang [20]. Motivated by the radio labelling Ponraj et.al. [15] defined the radio mean labeling of G and found radio mean number of some graphs. The concept of lehmer-3 mean labeling was introduced by S.Somasundaram et al. [17]. Motivated by the radio mean labeling in this paper, We are introduced radio circular distance in lehmer-3 mean number of some wheel related graph.

2.DEFINITIONS:

Definition 2.1: A radio circular distance in lehmer-3 mean labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the $\mathbb{N}$ such that for two distinct vertices u and v of G , $\mathrm{Cir}(\mathrm{u},\mathrm{v}) + \left\lceil \frac{\mathrm{f(u)^3+f(v)^3}}{\mathrm{f(u)^2+f(v)^2}} \right\rceil \geq 1 + \mathrm{diam^C(G)}$ where $Cir(\mathrm{u},\mathrm{v})$ denotes circular distance between u and v and $\mathrm{d^C(G)}$ denotes the circular diameter of G

Definition 2.2: (Double-Wheel Graph) A double-wheel graph DW_n of size n can be composed of $2C_n + K_1$, i.e. it consists of two cycles of size n , where the vertices of the two cycles are all connected to a common hub.

Definition 2.3: (Flower Graph) A Flower graph Fl_n, n ≥ 3, is obtained from a helm H_n by joining each pendent vertex to the central vertex of the helm.

Definition 2.4: (Bowknot graph) A bowknot graph $B_{n,n}$ is the graph by gluing two central vertices of double fan graphs. Obviously, the bowknot graph $B_{n,n}$ is obtained from the wheel W_{2n} by deleting two edges every $n - 1$ edges on the rim of the wheel.

Definition 2.5: (Dutch windmill graph) A Dutch windmill graph D_4^n ,$n \geq 2$ is a graph by gluing a common vertex of n cycles C_4 .

Definition 2.6: (Lotus inside a circle)A lotus inside a circle $LC_n, n \geq 3$, is obtained from a wheel W_n by subdividing every edge forming the outer cycle and joining these new vertices to form a cycle.

3.Main Results

Theorem 3.1: The radio circular distance in lehmer-3 mean number of Double-Wheel Graph,

$$r_{l_3mn}{}^{C}(DW_n) = \begin{cases} 9, n = 3 \\ 3n-1, n \geq 4 \end{cases}$$

Proof:

It is obvious that $diam^{C}(DW_n) = 2n+2\ , n \geq 3$. Let V={u, $u_i, v_i\ /\ i = 1,2,\dots,n\}$ be the vertex set .

We must show that the radio circular distance in lehmer-3 mean condition $Cir(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{C}(DW_n) = 2n+3$, for every pair of vertices (u,v) where $u \neq v$.

For $n \geq 4$, Define the function *f* as

$f(u) = 3n-1,$

$$f(u_i) = \begin{cases} n-1+\left(\dfrac{i+1}{2}\right), i = 1,3,\dots,n, n\ is\ odd \\ n-1+\left(\dfrac{i+1}{2}\right), i = 1,3,\dots,n-1, n\ is\ even \end{cases}$$

$$f(u_i) = \begin{cases} 3\left(\dfrac{n+1}{2}\right)-2+\dfrac{i}{2}, i = 2,4,\dots,n-1, n\ is\ odd \\ 3\left(\dfrac{n}{2}\right)-1+\dfrac{i}{2}, i = 2,4,\dots,n, n\ is\ even \end{cases},$$

$f(v_i) = 2n-1+i, 1 \leq i \leq n-1,$

$f(v_n) = n-1.$

Therefore, the largest label is $3n-1\ , n \geq 4$

$$r_{l_3mn}{}^{C}(DW_n) = \begin{cases} 9, n = 3 \\ 3n-1, n \geq 4 \end{cases}$$ ■

Example:3.1.1:

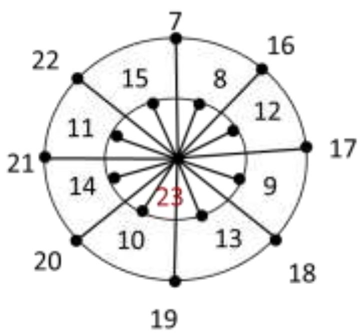

Figure 3.1.1: *Radio circular distance in lehmer – 3 mean number of* DW_8

Theorem 3.2: The radio circular distance in lehmer-3 mean number of Flower Graph, $r_{l_3mn}{}^{C}(Fl_n) = \begin{cases} 7\ ,\ n = 3 \\ 11\ ,\ n = 4 \\ 2n+1\ ,\ n \geq 5 \end{cases}$

Proof:

It is obvious that $diam^{C}(Fl_n) = n + 6\ , n \geq 5$. Let $V = \{\text{u},\ u_i, v_i\ /\ i = 1,2, \dots, n\}$ be the vertex set .

We must show that the radio circular distance in lehmer-3 mean condition $Cir(u, v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{C}(Fl_n) = n + 7$, for every pair of vertices (u, v) where $u \neq v$.

For $n \geq 5$, Define the function *f* as

$f(u) = 2n + 1, f(u_i) = n + i, 1 \leq i \leq n, f(v_i) = i, 1 \leq i \leq n.$

Therefore, the largest label is $2n + 1\ , n \geq 5$

$$r_{l_3mn}{}^{C}(Fl_n) = \begin{cases} 7\ ,\ n = 3 \\ 11\ ,\ n = 4 \\ 2n+1\ ,\ n \geq 5 \end{cases}$$ ■

Example :3.2.2:

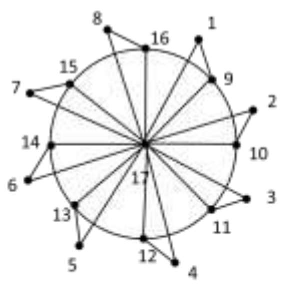

Figure:3.2.2: *Radio circular distance in lehmer – 3 mean number of* Fl_8

Theorem 3.3: The radio circular distance in lehmer-3 mean number of bowknot Graph, $r_{l_3mn}{}^{C}(B_{n,n}) = 3n - 1, n \geq 3$

Proof:

It is obvious that $diam^{C}(B_{n,n}) = 2n + 2\ , n \geq 3$. Let $V = \{u,\ u_i\ /\ i = 1,2, \dots ,2n\}$ be the vertex set .

We must show that the radio circular distance in lehmer-3 mean condition $Cir(u, v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{C}(B_{n,n}) = 2n + 3$, for every pair of vertices (u, v) where $u \neq v$.

For $n \geq 3$, Define the function *f* as

$f(u) = 3n - 1, f(u_1) = n - 1, f(u_{1+i}) = 2n + i, 1 \leq i \leq n - 2, f(u_n) = n + 1,$
$f(u_{n+1}) = n, f(u_{n+1+i}) = n + 2 + i, 1 \leq i \leq n - 2, f(u_{2n}) = n + 2.$
Therefore, the largest label is $3n - 1$, $n \geq 3$
$r_{l_3mn}{}^C(B_{n,n}) = 3n - 1, n \geq 3$ ■

Example:3.3.3:

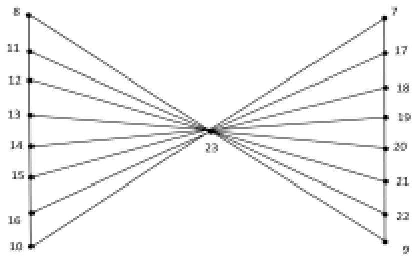

Figure:3.3.3: *Radio circular distance in lehmer – 3 mean number of* $B_{8,8}$.

Theorem 3.4: The radio circular distance in lehmer-3 mean number of Dutch windmill Graph, $r_{l_3mn}{}^C(D_4^n) = 3n + 1, n \geq 3$.

Proof:

It is obvious that $diam^C(D_4^n) = 8$, $n \geq 3$. Let $V = \{u, u_i \,/\, i = 1,2, \dots, 3n\}$ be the vertex set . We must show that the radio circular distance in lehmer-3 mean condition $Cir(u, v) + \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \geq 1 + diam^C(D_4^n) = 9$, for every pair of vertices (u, v) where $u \neq v$.

For $n \geq 3$, Define the function f as
$f(u) = 3n + 1, f(u_{3i-2}) = i, 1 \leq i \leq n$
$f(u_{3i-1}) = 2n + i, 1 \leq i \leq n, f(u_{3i}) = m + i, 1 \leq i \leq n.$
Therefore, the largest label is $3n + 1$, $n \geq 3$
$r_{l_3mn}{}^C(D_4^n) = 3n + 1, n \geq 3$ ■

Example:3.4.4:

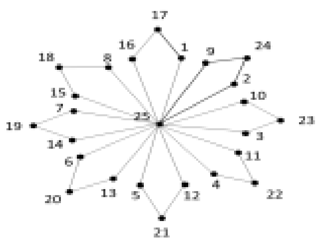

Figure 3.4.4: *Radio circular distance in lehmer – 3 mean number of* D_4^8.

Theorem 3.5: The radio circular distance in lehmer-3 mean number of Lotus inside a circle Graph , ${r_{l_3mn}}^{C}(LC_n) = 2n + 1, n \geq 3$.

Proof:

It is obvious that $diam^C(LC_n) = 2n + 2 , n \geq 3$. Let V={u, u_i, v_i / $i = 1,2, \dots, n$} be the vertex set .

We must show that the radio circular distance in lehmer-3 mean condition $Cir(u, v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^C(LC_n) = 2n + 3$, for every pair of vertices (u, v) where $u \neq v$.

For $n \geq 3$, Define the function *f* as

$f(u) = 1, f(u_i) = i + 1, 1 \leq i \leq n, f(v_i) = n + 1 + i, 1 \leq i \leq n.$

Therefore, the largest label is $2n + 1$, $n \geq 3$

${r_{l_3mn}}^{C}(LC_n) = 2n + 1, n \geq 3$ ■

Example 3.5.5:

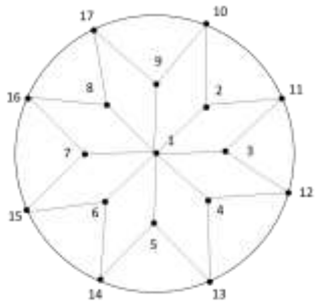

Figure 3.5.5: *Radio circular distance in lehmer – 3 mean number of* LC_8.

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