b - H_π -OPEN SETS IN HEREDITARY GENERALIZED TOPOLOGICAL SPACE

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Abstract: In this paper we introduce and study the notion of b - H_π -open sets in hereditary generalized topological space.

Keywords: hereditary generalized topology, b - H_π -open, π - H -open.

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1 Introduction and Preliminaries

In the year 2002, Csaszar [2] introduced very usefull notions of generalized topology and generalized continuity. Consider Z be a nonempty set and μ be a collection from the subsets of Z. Then μ is called a *generalized topology* (briefly GT) if $\emptyset \in \mu$ and an arbitrary union of elements from μ belongs to μ. A space Z is called a C_0 -space [13], if $C_0 = Z$, where C_0 is the set of all representative elements of sets of μ . A subset A of a space (Z, μ) is called as μ - α *-open* [4] (resp. μ - σ - open [4], μ - π -open [4], μ - β -open [4], μ - b -open [12]), if $A \subset i_\mu c_\mu i_\mu(A)$ (resp. $A \subset c_\mu i_\mu(A)$, $A \subset i_\mu c_\mu(A)$, $A \subset c_\mu i_\mu c_\mu(A)$, $A \subset c_\mu i_\mu(A) \cup i_\mu c_\mu(A)$). A subset A of Z is μ -locally closed set [6], $A = U \cap V$, where U is μ -open and V is μ -closed. A GTS (Z, μ) is called μ -extremally disconnected [3], if the μ -closure of every μ -open set is μ -open. A nonempty family H of subsets of Z is called as a *hereditary class* [5], if $A \in$ H and $B \subset A$, then $B \in$ H. For each $A \subseteq Z$,
$A^*(H, \mu) = \{z \in Z : A \cap V \notin H$ for $V \in \mu$ such that $z \in V\}$ [5]. For $A \subset Z$,
define $c\mu*(A) = A \cup A^*(H, \mu)$ and $\mu^* = \{A \subset Z : Z - A = c\mu*(Z - A)\}$. If H is a hereditary class on Z then (Z, μ, H) is called a hereditary generalized topological space ($H.G.T.S$).

Definition 1.1. *[9] Consider A be a subset of H.G.T.S.* (Z, μ, H). *Then* $A^*\pi(H, \mu) = \{z \in Z : A \cap V \notin H$ *for All* $V \in \mu$ - π *-open such that* $z \in V\}$.

Definition 1.2. *[5] A subset A of a H.G.T.S.* (Z, μ, H) *is said to be*
1. α - H *-open, if* $A \subseteq i_\mu c^*\mu i_\mu(A)$,
2. σ - H *-open, if* $A \subseteq c^*\mu i_\mu(A)$,
3. π - H *-open, if* $A \subseteq i_\mu c^*\mu(A)$,
4. β - H *-open, if* $A \subseteq c_\mu i_\mu c^*\mu(A)$,
5. *strong* β - H *-open, if* $A \subseteq c^*\mu i_\mu c^*\mu(A)$,
6. μ^* *-closed, if* $c^*\mu(A) \subset A$.

Definition 1.3. *A subset A of a H.G.T.S.* (Z, μ, H) *is said to be* δ - H *-open [7], if* $i_\mu c^*\mu(A) \subseteq c^*\mu i_\mu(A)$.
Definition 1.4. *A subset A of a H.G.T.S.* (Z, μ, H) *is said to be* b - H *-open [10], if* $A \subseteq i_\mu c^*\mu(A) \cup c^*\mu i_\mu(A)$.
Definition 1.5. *[9] A subset A of a H.G.T.S.* (Z, μ, H) *is said to be* $\pi\mu^*$ *-closed, if* $A^*\pi \subseteq A$.
Propositon 1.6. *[9] Let A be a* μ - π *-closed. Then* $A^*\pi \subset A$.
Let (Z, μ, H) be a hereditary generalized topological space. For $A \subset Z$, define $c^*\pi(A) = A \cup A^*\pi$ [9] and $c\pi*(A)$ is enlarging, monotone and idempotent.
Definition 1.7. *[11] A subset L of a H.G.T.S.* (Z, μ, H) *is said to be b* - H_σ *-open set, if* $L \subseteq i_\mu c^*\sigma(L) \cup c^*\sigma i_\mu(L)$.

2 b - H_π -open sets

Definition 2.1. *A subset A of a H.G.T.S.* (Z, μ, H) *is said to be* b - H_π *-open set, if* $A \subseteq i_\mu c^*\pi(A) \cup c^*\pi i_\mu(A)$.

Propositon 2.2. *In H.G.T.S. (Z, μ, H) every μ -open set is b - H_π -open but not conversely.*
Proof. Let a subset A of H.G.T.S. (Z, μ, H) is μ -open. Then $A = i_\mu(A)$. Now $A \subseteq i_\mu(A) \subseteq i_\mu c\pi*(A) \subseteq i_\mu c^*\pi(A) \cup c\pi* i_\mu(A)$. Hence A is b - H_π -open.

Example 2.3. *Consider* $Z = \{a, b, c, d, e\}$ $\mu = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d, e\},\{a, c, d, e\}, \{a, b, c\}, Z\}$, $H = \{\emptyset, \{a\}, \}$. *Then* $A = \{a, c, e\}$ *is* b - H_π *-open but not μ -open.*

Propositon 2.4. *Every b - H_π -open is μ - b -open but not conversely.*
Proof. Let A be a b - H_π -open. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq i_\mu c\mu*(A) \cup c^*\mu i_\mu(A) \subseteq i_\mu c_\mu(A) \cup c_\mu i_\mu(A)$. Hence A is μ - b -open.

Propositon 2.5. *Every b - H_π -open is b - H -open but not conversely.*
Proof. Let A be a b - H_π -open. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq i_\mu c\mu*(A) \cup c^*\mu i_\mu(A)$.
Hence A is b - H -open.

Example 2.6. *Consider* $Z = \{a, b, c, d, e\}$ $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$, $H = \{\emptyset, \{a\}\}$. *Then* $A = \{e\}$ *is* μ - b *-open but not b - H_π -open and $M = \{e\}$ is b - H -open but not b - H_π -open.*

Theorem 2.7. *If $A \subset Z$ is both b - H_π -open and μ - σ -open, then it is β - H -open.*
Proof. Let A be both b - H_π -open and μ - σ -open. Then $A \subseteq i_\mu c^*\pi(A) \cup c\pi* i_\mu(A)$ and $A \subseteq c_\mu i_\mu(A)$. Now $A \subseteq i_\mu c^*\pi(A) \cup c\pi* i_\mu(A) \subseteq c\pi*(A)$, which implies $c_\mu i_\mu(A) \subseteq c_\mu i_\mu c\pi*(A) \subseteq c_\mu i_\mu c^*\mu(A)$ So $A \subseteq c_\mu i_\mu(A) \subseteq c_\mu i_\mu c^*\mu(A)$. Hence A is β - H -open.

Theorem 2.8. *If $A \subset Z$ is both b - H_π -open and μ - σ -open, then it is μ - β -open.*
Proof. Let A be both b - H_π -open and μ - σ -open. Then $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A)$ and $A \subseteq c_\mu i_\mu(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A) \subseteq c\pi*(A)$, which implies $c_\mu i_\mu(A) \subseteq c_\mu i_\mu c^*\pi(A) \subseteq c_\mu i_\mu c\mu*(A) \subseteq c_\mu i_\mu c_\mu(A)$. So $A \subseteq c_\mu i_\mu(A) \subseteq c_\mu i_\mu c_\mu(A)$. Hence A is μ - β -open.

Theorem 2.9. *If $A \subset Z$ is both b - H_π -open and μ^* -closed, then it is σ - H -open.*
Proof. Let A be both b - H_π -open and μ^* -closed. Then $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A)$ and $c\pi*(A) \subseteq A$. Now $A \subseteq i_\mu c^*\pi(A) \cup c\pi* i_\mu(A) \subseteq c\pi* i_\mu(A) \cup i_\mu(A) = c^*\pi\, i_\mu(A) \subseteq c^*\mu i_\mu(A)$. Hence A is σ - H -open.

Theorem 2.10. *If $A \subset Z$ is both b - H_π -open and $\pi\mu^*$ -closed, then it is σ - H - open.*
Proof. Let A be both b - H_π -open and $\pi\mu^*$ -closed. Then $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A)$ and $c\pi*(A) \subseteq A$, which implies $i_\mu c\pi*(A) \subseteq i_\mu(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq c^*\pi i_\mu(A) \cup i_\mu(A) = c\pi* i_\mu(A) \subseteq c^*\mu i_\mu(A)$. Hence σ - H -open.

Theorem 2.11. *If $A \subset Z$ is both b - H_π -open and μ - π -closed, then it is σ - H - open.*
Proof. Let A be both b - H_π -open and μ - π -closed. Then $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A)$ and $c^*\pi(A) \subseteq A$ by Proposition 2.9 of [9]. Which implies $i_\mu c\pi*(A) \subseteq i_\mu(A)$. Now $A \subseteq i_\mu c^*\pi(A) \cup c^*\pi i_\mu(A) \subseteq c\pi* i_\mu(A) \cup i_\mu(A) = c\pi* i_\mu(A) \subseteq c\mu* i_\mu(A)$. Hence σ - H -open.

Theorem 2.12. *If $A \subset Z$ is b - H_π -open such that $i_\mu(A) = \emptyset$, then it is π - H - open.*
Proof. Let A be a b - H_π -open and $i_\mu(A) = \emptyset$. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) = i_\mu c^*\pi(A) \subseteq i_\mu c\mu*(A)$. Hence π - H -open.

Theorem 2.13. *If $A \subset Z$ is b - H_π -open, then it is strong β - H -open.*
Proof. Let A be a b - H_π -open. Then A is b - H -open by Proposition 2.5. Hence A is strong β - H -open by Proposition of 2.26 of [10].

Theorem 2.14. *If $A \subset Z$ is both b - H_π -open and δ - H -open, then it is σ - H - open.*
Proof. Let A is both b - H_π -open and δ - H -open. Then $A \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu(A)$ and $i_\mu c^*\mu(A) \subseteq c^*\mu i_\mu(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c^*\pi\, i_\mu(A) \subseteq i_\mu c\mu*(A) \cup c\mu* i_\mu(A) \subseteq c\mu* i_\mu(A)$. Hence A is σ - H -open.

Theorem 2.15. *If $A \subset Z$ is b - H_π -open and $A \in H$, then it is σ - H -open.*
Proof. Let A be b - H_π -open and $A \in H$. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A)$ and $c^*\pi(A) = A$ by Remark 2.10 of [9]. Now $A \subseteq i_\mu c^*\pi(A) \cup c^*\pi i_\mu(A) = i_\mu(A) \cup c^*\pi i_\mu(A) = c^*\pi i_\mu(A) \subseteq c\mu* i_\mu(A)$. Hence A is σ - H -open.

Theorem 2.16. *If $A \subset Z$ is b - H_π -open and $H = P(Z)$ then it is σ - H -open.*
Proof. Let A be b - H_π -open and $A \in H$. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A)$ and $c^*\pi(A) = A$ by Remark 2.10 of [9]. Now $A \subseteq i_\mu c^*\pi(A) \cup c^*\pi i_\mu(A) = i_\mu(A) \cup c^*\pi i_\mu(A) = c^*\pi i_\mu(A) \subseteq c\mu* i_\mu(A)$. Hence A is σ - H -open.

Theorem 2.17. *If $A \subset Z$ is b - H_π -open and $A \subset A^*\pi$, then it is μ - β -open.*
Proof. Let A be b - H_π -open and $A \subset A\pi*$. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A)$ and $c^*\pi i_\mu(A) \subset c\pi* i_\mu c^*\pi(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu c^*\pi(A) \subseteq c^*\pi i_\mu c^*\pi(A) \subseteq c^*\mu i_\mu c\mu*(A) \subseteq c_\mu i_\mu c_\mu(A)$. Hence A is μ - β -open.

Remark 2.18. *If $A \subset Z$ is b - H_π -open and $A \subset A^*\pi$, then it is strong β - H -open.*
Proof. Let A be b - H_π -open and $A \subset A\pi*$. Then $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A)$ and $c^*\pi i_\mu(A) \subset c\pi* i_\mu c^*\pi(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu c^*\pi(A) \subseteq c^*\pi i_\mu c\pi*(A)$. Hence A is strong β - H -open.

Remark 2.19. *If $A \subset Z$ is b - H_π -open and $A \subset A^*\pi$, then it is β - H -open.*
Proof. Let A be b - H_π -open and $A \subset A^*\pi$. Then $A \subseteq i_\mu c^*\pi(A) \cup c\pi* i_\mu(A)$ and $c^*\pi i_\mu(A) \subset c\pi* i_\mu c^*\pi(A)$. Now $A \subseteq i_\mu c\pi*(A) \cup c\pi* i_\mu(A) \subseteq i_\mu c\pi*(A) \cup c^*\pi i_\mu c^*\pi(A) \subseteq c^*\pi i_\mu c^*\pi(A) \subseteq c\mu* i_\mu c^*\mu(A) \subseteq c_\mu i_\mu c\mu*(A)$. Hence A is β - H -open.

Theorem 2.20. *If $A \subset Z$ is both $\pi\mu^*$ -closed and strong β - H -open, then it is b - H_π -open.*
Proof. Let $A \subset Z$ be both $\pi\mu^*$ -closed and strong β - H -open. Then $c^*\mu(A) \subset A$ and $A \subset c^*\mu i_\mu c\mu*(A)$. Now $i_\mu c\mu*(A) \subset i_\mu(A)$. Which implies $c^*\mu i_\mu c^*\mu(A) \subset c\mu* i_\mu(A)$. So, $A \subset c^*\mu i_\mu c\mu*(A) \subset c^*\mu i_\mu(A) \subset c\mu* i_\mu(A) \cup i_\mu c\mu*(A)$. Hence A is b - H_π -open.

Theorem 2.21. *If $A \subset Z$ is both μ - π -closed and strong β - H -open, then it is b - H_π -open.*
Proof. Let $A \subset Z$ is both μ - π -closed and strong β - H -open. Then $c^*\mu(A) \subset A$ and $A \subset c^*\mu i_\mu c\mu*(A)$. Now $i_\mu c\mu*(A) \subset i_\mu(A)$. Which implies $c^*\mu i_\mu c\mu*(A) \subset c\mu* i_\mu(A)$. So, $A \subset c^*\mu i_\mu c\mu*(A) \subset c^*\mu i_\mu(A) \subset c^*\mu i_\mu(A) \cup i_\mu c\mu*(A)$. Hence A is b - H_π -open.

Theorem 2.22. *Let (Z, μ, H) be a strong H.G.T.S., where Z is C_0 -space and μ - extremally disconnected space, $A \subset Z$. Then the following conditions are equivalent.*
1. *A is μ -open,*
2. *A is b - H_π -open and μ -locally closed set.*

Proof. (1) ⇒ (2) This is obvious from definitions.
(2) ⇒ (1) Let A be b - H_π -open and μ -locally closed set. Then $A \subseteq i_\mu c^*\pi(A) \cup c^*\pi i_\mu(A) \subseteq i_\mu c_\mu(A) \cup c_\mu i_\mu(A)$ and $A = U \cap c_\mu(A)$. Now
$A \subset U \cap c_\mu(A)$.
$\subset U \cap [i_\mu c_\mu(A) \cup c_\mu i_\mu(A)]$
$\subset [U \cap i_\mu c_\mu(A)] \cup [U \cap c_\mu i_\mu(A)]$
$\subset [i_\mu(U) \cap i_\mu c_\mu(A)] \cup [i_\mu(U) \cap c_\mu i_\mu(A)]$
$\subset [i_\mu(U) \cap i_\mu c_\mu(A)] \cup [i_\mu(U) \cap i_\mu c_\mu(A)]$
$\subset [i_\mu(U \cap c_\mu(A))] \cup [i_\mu(U \cap c_\mu(A))]$
$= [i_\mu(A)] \cup [i_\mu(A)]$
$= i_\mu(A)$.
Hence A is μ -open.

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