Section A-Research paper

An Application of Triangular Fuzzy Numbers in Medical Diagnosis

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Abstract:

The research paper deals with clinical applications, and fields in medical diagnosis that were analyzed with fuzzy logic through well-known triangular fuzzy numbers. In women's real-life situations, polycystic ovarian syndrome (PCOS) plays a vital role in fertility issues. Here we deal with the analysis of (PCOS) under various parameters with numerical examples.

Keywords: Fuzzy logic, Medical Diagnosis, Fuzzy numbers, Defuzzification, Triangular fuzzy number, PCOS.

1. Introduction:

Fuzzy Logic is similar to how people make decisions. It attempts to get rid of ambiguous and inaccurate information. Fuzzy logic is applied in many fields, here we are applying it in the field of medical diagnostic support of laboratory investigations. Additionally, a doctor doesn't perform intricate calculations based on input data with high precision; instead, he bases the majority of his diagnoses on his experience and perception as an authority on the issue.

The term Medical Diagnosis is the act of identifying which disease or condition is responsible for a person's symptoms and a physical indicator (abbreviated Dx, or Ds). The process frequently includes one or more diagnostic procedures like medical tests, ultrasound abdomen, fibro scan, follicular study etc.

Elements in fuzzy sets can have varying degrees of membership. Fuzzy sets are a variation of the traditional idea of a set.

Fuzzy Numbers (FN): FN is a convex and normalized fuzzy set, it is defined in the real numbers that connects the grouped possible values rather than a single value, with each potential value weighing 0 and 1. The membership function is the weight of a fuzzy number. Real numbers are extended to fuzzy numbers. The inclusion of uncertainty regarding parameters, characteristics, geometry, initial circumstances, etc. is possible when using fuzzy numbers in calculations. Fuzzy arithmetic procedures can be carried out using either the interval arithmetic technique or the extension principle approach, to accomplish the arithmetic calculations on fuzzy integers. A fuzzy interval is the same as a fuzzy number.
Uncertainty presents a difficult aspect of daily living for humans. Because it is impossible to anticipate the future. Lack of information is the primary source of uncertainty. Information may be lacking in some other way, incomplete, unreliable, hazy, or conflicting. There may be fuzziness or ambiguity as a result of these numerous information adequacies. Professor Zadeh's introduction of fuzzy set theory [7] in 1965 serves as a qualitative computational strategy for describing uncertainty.

In the area of fuzzy medical diagnostic models, many papers have been published. A few of them include: Using fuzzy matrices that capture the medical information between the characteristic of symptoms and illness of diseases, Sanchez [5] created diagnostic models. Kennedy Shriver, Eunice [3]. Sanchez's method for diagnosis of illness has been expanded by Meenakshi and Kaliraja [4] utilizing an interval value fuzzy matrix representation. Elizabeth and Sujatha [2] have expanded Sanchez's technique to medical diagnosis [6] by assigning and utilizing the representation of the TFM matrix, membership grades are allotted to linguistic concepts in fuzzy occurrence relations and conformability relations.

The definitions of the terms (TFN) and (TFMN) are briefly discussed. The terms union and intersection of (TFMNs) are defined. Some basic characteristics of set operations and propositions are confirmed using the definition. A procedure for the fuzzy medical diagnostic model is suggested in section 4. The simulation result is used to show an example.

1.1. Defuzzification:

After the fuzzy reasoning is finished, it is typically required to communicate the results of the reasoning in a way that is understandable to humans. Arithmetic defuzzification and linguistic approximation are the two main forms of defuzzification.

1.2. Triangular Fuzzy Number (TFN):

This is the most widely used type of fuzzy number, and it is a number with three points that are expressed as membership functions. $E = (e_1, e_2, e_3)$

1.3. Poly Cystic Ovarian Syndrome (PCOS):

The ovaries, uterus, fallopian tubes and vagina are all parts of the female reproductive system. It is formatted to carry out a different function such as the development and movement of gametes and the generation of sex hormones. Ovarian health is affected by PCOS. The most noticeable signs of PCOD include hirsutism, severe skin acne, no irregular periods and no ovulation. In addition, if the lady doesn't receive prompt medical attention, she may also experience other problems like insulin-resistant diabetes, obesity, and high cholesterol. Heart problems are the direct result of this. Despite its great frequency, PCOS's actual aetiology is still unknown and there is no known treatment for it.

2. Pre-Requisites

2.1. Definition: Triangular fuzzy number (TFN):

This is the most widely used type of fuzzy number, and it is a number with three points that are expressed as membership functions. $E = (e_1, e_2, e_3)$

TFN is denoted as $\tilde{E} = (e_1, e_2, e_3),\ e_1, e_2, e_3 \in R,\ e_1 < e_2 < e_3$.

Triangular Membership function is $\mu\tilde{E}(x) = (x; e_1, e_2, e_3) = \{0,\ x \le e_1\ \frac{x-e_1}{e_2-e_1}\ ,\ e_1 \le x \le e_2\ \frac{e_3-x}{e_3-e_2}\ ,\ e_2 \le x \le e_3\ 0,\ e_3 \le x$

2.2. Definition: Triangular fuzzy membership number (TFMN) [1] is found by converting a triangular fuzzy number (TFN):

Let $\tilde{E} = (e_1, e_2, e_3)$ be a triangular fuzzy number. Then $TFMN = \left(\frac{e_1}{a}, \frac{e_2}{a}, \frac{e_3}{a}\right)$, where $0 \le e_1 \le e_2 \le e_3 \le a$. Thus $0 \le \frac{e_1}{a} \le \frac{e_2}{a} \le \frac{e_3}{a} \le 1$.

2.3. Definition: Triangular fuzzy membership number (TFMN)/ Arithmetic mean (AM):

Let $\mu_{\tilde{E}} = \left(\frac{e_1}{a}, \frac{e_2}{a}, \frac{e_3}{a}\right)$ be a TFMN then AM $(\mu_{\tilde{E}}) = (e_1 + e_2 + e_3)\ /\ 3a$.

2.4. Definition: Triangular fuzzy number matrix:

A TFN matrix of order $a \times b$ is denoted as $\tilde{E} = (e_{ij})_{a\times b}$ where $e_{ij}\ is\ equal\ to\ (e_{ijL}, e_{ijM}, e_{ijU}) is\ the\ ij^{th}\ element\ of\ \tilde{E}. e_{ijL}\ is\ the\ left\ and\ e_{ijU}\ right\ exposes\ of\ e_{ij} respectively\ and\ e_{ijM} is\ the\ mean\ value.$

2.5. Definition: Addition operation on TFN matrix:

Assume $\tilde{E} = (e_{ij})_{b\times b}\ and\ \tilde{F} = (f_{ij})_{b\times b}$ be TFN of same order matrices. Then $\tilde{E}(+)\tilde{F} = (e_{ij} + f_{ij})_{b\times b}$ where $e_{ij} + f_{ij} = (e_{ijL} + f_{ijU}, e_{ijM} + f_{ijM}, e_{ijU} + f_{ijL})$ is the ij^{th} element of $\tilde{E}\ (+)\tilde{F}$. The same condition holds for TFMN matrix.

2.6. Definition: Subtraction operation on TFN matrix:

Assume $\tilde{E} = (e_{ij})_{b\times b}\ and\ \tilde{F} = (f_{ij})_{b\times b}$ be TFN of same order matrices. Then $\tilde{E}(-)\tilde{F} = (e_{ij} - f_{ij})_{b\times b}$ where $e_{ij} - f_{ij} = (e_{ijL} - f_{ijU}, e_{ijM} - f_{ijM}, e_{ijU} - f_{ijL})$ is the ij^{th} element of $\tilde{E}\ (-)\tilde{F}$. The same condition holds for TFMN matrix.

2.7. Definition: Max-Min Composition on fuzzy membership value matrices:

Assume C_{ab} is the collection of all $a \times b$ matrices over C. Elements of C_{ab} are called fuzzy membership value matrices. For $\tilde{E} = (e_{ij}) \in C_{ad}\ and\ \tilde{F} = (f_{ij}) \in C_{db}$ the max min product is $\tilde{E}(\cdot)\tilde{F} = (sup_k[(e_{ik}, f_{bk})\ ]) \in C_{ab}$.

3. Union and Intersection of TFMNs:

Sarala and Prabhavathi [6] provide the mentioned definition for the fuzzy soft matrix. The concept of union and intersection for TFMNs is based on this.

3.1. Definition: Union and Intersection of TFMNs:

Assume $\tilde{E} = (\mu_{EL}, \mu_{EM}, \mu_{EU}), \tilde{F} = (\mu_{FL}, \mu_{FM}, \mu_{FU})$ be two TFMN, then the union and intersection of $\tilde{E}\ and\ \tilde{F}$ is given by

a)$\tilde{E} \cup \tilde{F} = (\mu_{EL}, \mu_{EM}, \mu_{EU}) \cup (\mu_{FL}, \mu_{FM}, \mu_{FU}) = (\mu_{EL} + \mu_{FL} - \mu_{EL} \times \mu_{FL}, \mu_{EM} + \mu_{FM} - \mu_{EM} \times \mu_{FM}, \mu_{EU} + \mu_{FU} - \mu_{EU} \times \mu_{FU})$.

b)$\tilde{E} \cap \tilde{F} = (\mu_{EL}, \mu_{EM}, \mu_{EU}) \cap (\mu_{FL}, \mu_{FM}, \mu_{FU}) = (\mu_{EL} \times \mu_{FL}, \mu_{EM} \times \mu_{FM}, \mu_{EU} \times \mu_{FU})$.

Example:

Let $\tilde{E} = (0.5, 0.6, 0.8), \tilde{F} = (0.6, 0.7, and\ 0.9)$ be two TFMNs.

$Then\ \tilde{E} \cup \tilde{F} = (0.5 + 0.6 - 0.5 \times 0.6, 0.6 + 0.7 - 0.6 \times 0.7, 0.8 + 0.9 - 0.8 \times 0.9) = (0.8, 0.88, 0.98)$.

$\tilde{E} \cap \tilde{F} = (0.5 \times 0.6, 0.6 \times 0.7, 0.8 \times 0.9) = (0.3, 0.4, 0.7)$.

3.2. Fundamental properties of set operations:

Assume $\tilde{E}, \tilde{F}, \tilde{g}$ be TFMNs. Using the previous definition following properties are satisfied.

1.Involution $\widetilde{\overline{E}} = \tilde{E}, \underline{\tilde{E}} = \tilde{U} - \tilde{E}$, where $\tilde{U} = (1,1,1)$.

2.Commutativity: a) $\tilde{E} \cup \tilde{F} = \tilde{F} \cup \widetilde{E}$; b) $\tilde{E} \cap \tilde{F} = \tilde{F} \cap \tilde{E}$.

3.Associativity: a) $(\widetilde{E \cup} \tilde{F}) \cup \tilde{g} = \tilde{E} \cup (\tilde{F} \cup \tilde{g})$; b) $(\tilde{E} \cap \tilde{F}) \cap \tilde{g} = \tilde{E} \cap (\tilde{F} \cap \tilde{g})$.

4. Distributive laws: a) $\tilde{E} \cap (\tilde{F} \cup \tilde{g}) = (\tilde{E} \cap \tilde{F}) \cup (\tilde{E} \cap \tilde{g})$; b) $\tilde{E} \cup (\tilde{F} \cap \tilde{g}) = (\tilde{E} \cup \tilde{F}) \cap (\tilde{E} \cap \tilde{g})$.

3.3. Proposition:

Let $\tilde{E}$ be TFMN and $\tilde{U} = (1,1,1), \tilde{Z} = (0,0,0)$.

1. a) $\tilde{E} \cup \tilde{U} = \tilde{U}$ b) $\tilde{E} \cap \tilde{U} = \tilde{E}$.
2. a) $\tilde{E} \cup \tilde{Z} = \tilde{E}$ b) $\tilde{E} \cap \tilde{Z} = \tilde{Z}$.
3. a) $\tilde{E} \cup \underline{\tilde{E}} \neq \tilde{U}$ b) $\tilde{E} \cap \underline{\tilde{E}} = \tilde{Z}$.

4. An Application of TFM Matrix in Medical Diagnostic Mode:

Let P be the group of patients, S be the body of symptoms and D be the series of diseases. The elements of TFN matrix are designated as $\tilde{E} = (e_{ij})\, a \times b$ where $e_{ij} = (e_{ijL}, e_{ijM}, e_{ijU})$ is the ij^{th} element of $\tilde{E}$. Here $0 \leq e_{ijL} \leq e_{ijM} \leq e_{ijU} \leq 10$, where e_{ijL} is minimum bound, e_{ijM} is absolute value and e_{ijU} is maximum bound.

4.1. Medical Diagnostic Model using fuzzy follow these steps:

Step1: Construct a TFN matrix (T, D) over S, where T is a shaping given by $T: D \rightarrow \tilde{T}(S)$, $\widetilde{T}(S)$ is the collection of all triangular fuzzy sets of S. This matrix signified by $\widetilde{X_O}$ is the symptom-disease TFN matrix or fuzzy occurrence matrix.

Step 2: Construct another TFN matrix (T_1, S) over P, where T1 is a shaping given by $T_1: S \rightarrow \tilde{T}(P)$. This matrix signified by $\widetilde{X_S}$ is the patient-symptom TFN matrix.

Step 3: The matrices $\widetilde{X_O}$ and $\widetilde{X_S}$ were given by TFM matrices namely $(X_O)_{mem}$ and $(X_S)_{mem}$ using the 2.2 definition.

Step 4: Convert $(X_O)_{mem}$ and $(X_S)_{mem}$ into fuzzy matrices X_O and X_S by taking the AM for TFMN using the 2.3 definitions.

Step5: Enumerate the following relation matrices to diagnose the disease

i)$X_1 = (X_S)(\cdot)(X_O)$, ii)$X_2 = (1 - X_S)(\cdot)X_O$, iii)$X_3 = (X_S)(\cdot)(1 - X_O)$,

iv)$X_4 = (1 - X_S)(\cdot)(1 - X_O)$, v)$X_5 = max(X_1, X_2)$, vi)$X_6 = min(X_3, X_4)$,

vii) $X_7 = X_5 - X_6$.

Finding the top value in each row of $the\ X_7$ matrix, gives the assured diagnosis of the disease to the patient.

4.2. Illustrative example:

Let the group of patients $is\ P = (P_1,$) P2, P3, P4.

Let the body of symptoms $is\ S = (S_1,$) S2, S3, S4, S5, S6.

S_1= Acne; S_2 = Excess hair growth (hirsutism – hair growth in face, chest and back); S_3=Irregular or infrequent periods; S_4= Weight gain; S_5= Difficulties with fertility; S_6= Depression.

Let D = (D_1, $D2, D_3, D_4$) denote the diseases Insulin Resistance, Obesity, Cardiovascular Disease (CVD) and Excess androgen respectively.

Step 1: Given the universal set as the collection of S and the series D = $(D_1, \quad D2, D_3, D_4)$ representation of multiple PCOS kinds. The TFN matrix (T, D) is derived from professional medical documentation and is a parameterized family $\big(T(D_1),\ T(D_2),\ T(D_3),\ T(D_4)\big)\ of$ all TFN matrices over the set S. Consequently, the TFN matrix (T, D) depicts a relation matrix $\widetilde{X_O}$ that provides medical information about four different types of sickness or diseases and associated symptoms.

$\widetilde{X_O} = \quad D_1 \quad D_2 \quad D_3 \quad D_4$

$S_1\ S_2\ S_3\ S_4\ S_5\ S_6$ [((9,9,9) (4,6,8) (4,5,6) (2,4,6) (9,9,9) (4,5,6) (2,4,6) (4,5,6) (1,1,1) (10,10,10) (1,1,1) (1,1,1) (1,1,

Step2: Let $P = (P_1, P_2, P_3, P_4)$ being patient's representation, with PCOS and S = $(s_1, s_2, s_3, s_4, s_5, s_6)$ selected collection of parameters. Another parameterized family of TFN matrices $(T_1,)$ S, the TFN matrix represents a relation matrix known as $\tilde{X}_S$ is the patient-symptom matrix provided here

$\widetilde{X_S} = \quad s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6$

$P1\ P2\ P3\ P4$ [(10,10,10) (10,10,10) (1,2,3) (1,2,3) (1,1,1) (1,1,1) (4,6,8) (4,5,6) (9,9,9) (10,10,10) (0,0,0) (1,1,1) (

Following the steps in 4.1, we reach to arrive at the following result D_3.

Step 3: The matrices $\widetilde{X_O}\ and\ \widetilde{X_S}$ are altered into TFM matrices were $(X_O)_{mem}\ and\ (X_S)_{mem}$ using the 2.2 definition.

$(X_O)_{mem}$= $[9\ 6\ 5\ 4\ 9\ 5\ 4\ 5\ 1\ 10\ 1\ 1\ 1\ 9\ 0\ 1\ 1\ 0\ 10\ 0\ 2\ 1\ 1\ 10\ ]$ $(X_S)_{mem}$= $[10\ 10\ 2\ 1\ 1\ 1\ 6\ 5\ 9\ 10\ 0\ 1\ 4\ 4\ 2\ 2\ 10\ 0\ 4\ 6\ 1\ 1\ 1\ 10\ ]$

Step 4: Converted $(X_O)_{mem}\ and\ (X_S)_{mem}$ into fuzzy matrices $X_O\ and\ X_S$ by taking the AM for TFMN using the 2.3 definitions.

X_O= $[0.9\ 0.6\ 0.5\ 0.4\ 0.9\ 0.5\ 0.4\ 0.5\ 0.1\ 1\ 0.1\ 0.1\ 0.1\ 0.9\ 0\ 0.1\ 0.1\ 0\ 1\ 0\ 0.2\ 0.1\ 0.1\ 1\ ]$ and X_S = $[1\ 1\ 0.2\ 0.1\ 0.1\ 0.1\ 0.6\ 0.5\ 0.9\ 1\ 0\ 0.1\ 0.4\ 0.4\ 0.2\ 0.2\ 1\ 0\ 0.4\ 0.6\ 0.1\ 0.1\ 0.1\ 1\ ]$

Step 5: Computed the relation matrices to diagnose the illness or disease.

$X_1 = [0.9\ 0.6\ 0.5\ 0.5\ 0.6\ 0.9\ 0.5\ 0.5\ 0.4\ 0.4\ 1\ 0.4\ 0.6\ 0.5\ 0.4\ 1]$ $X_2 = [0.2\ 0.9\ 0.9\ 0.9\ 0.5\ 0.5\ 1\ 0.9\ 0.6\ 0.8\ 0.5\ 1\ 0.6\ 0.9\ 0.9\ 0.4]$ $X_3 = [0.2\ 0.5\ 0.6\ 0.6\ 1\ 0.5\ 1\ 0.9\ 0.9\ 1\ 0.4\ 1\ 0.8\ 0.9\ 0.9\ 0.5]$

$X_4 = [0.9\ 0.9\ 0.9\ 0.9\ 0.9\ 1\ 0.9\ 1\ 0.8\ 0.9\ 0.9\ 0.8\ 0.9\ 0.9\ 0.9\ 0.9]$ $X_5 = [0.9\ 0.9\ 0.9\ 0.9\ 0.6\ 0.9\ 1\ 0.9\ 0.6\ 0.8\ 1\ 1\ 0.6\ 0.9\ 0.9\ 1]$ $X_6 = [0.2\ 0.5\ 0.6\ 0.6\ 0.9\ 0.5\ 0.9\ 0.9\ 0.8\ 0.9\ 0.4\ 0.8\ 0.8\ 0.9\ 0.9\ 0.5]$

$X_7 =$ D1 D2 D3 D4 Max

$$P_1\ P_2\ P_3\ P_4\ [\ 0.7\ 0.4\ 0.3\ 0.3\ -0.3\ 0.4\ 0.1\ 0\ -0.2\ -0.1\ 0.6\ 0.2\ -0.2\ 0\ 0\ 0.5\ ]0.7\ 0.4\ 0.6\ 0.5$$

FromX_7, we can final that the patient (P_1) is affected by the disease D_1, P_2 *by* D_2, P_3*by* D_3 andP_4 *by* D_4.

4.3. Verification using the method proposed in [2]:

The relation matrices are i) $(X_S)_{mem}(\cdot)(X_O)_{mem}$. ii) $(X_S)_{mem}(\cdot)(1 - X_O)_{mem}$
iii) $AM\ ((X_S)_{mem}(\cdot)(X_O)_{mem}) - AM(X_S)_{mem}(\cdot)(1 - X_O)_{mem}$

The top value in each row finals the illness or disease of the patient.

D1 D2 D3 D4 Max

$$P1\ P2\ P3\ P4\ [\ 0.7\ \ 0.1\ -0.1\ -0.1\ -0.4\ \ 0.4\ -0.5\ -0.4\ -0.5\ -0.6\ \ 0.6\ -0.6\ -0.2\ -0.4\ -0.5\ \ 0.5\ ]0.7\ 0.4\ 0.6\ 0.5$$

5. Conclusion

In this research paper, we have used the idea of a TFM matrix in a medical diagnostic model. The benefit of this work is that if the patient's symptoms and the symptoms disease matrices are known; it would be used to finalize which patients have which diseases. This offers an alternative approach to dealing with the challenge of medical diagnosis in an uncertain setting.

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