Split Domination in Lict Subdivision Graph of a Graph

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Abstract

For any graph G, the Lict subdivision dominating set $D \subseteq V\{n[S(G)]\}$ is a Split lict subdivision dominating set, in the event that the sub graph $\langle V\{n[S(G)]\} - D\rangle$ is disconnected. The least cardinality of vertices in such a set denotes the Split lict sub division domination number in $S(G)$ or Split domination in lict subdivision graph of a graph and is represented by $\gamma_{sns}(G)$. We study the graph theoretic properties of $\gamma_{sns}(G)$ and many bounds were obtained in terms of the various components of G and it was also discovered how it related to other domination parameters.

Keywords:

Split domination number in lict subdivision graph, Roman domination number in line graph, total domination number , co total domination number and connected domination number.

Introduction

We refer to a graph as a finite, undirected graph with numerous edges and no loops. Any term that has not been defined and the notations in this paper may be found in [5] .

A graph G comprises a finite non empty set $V = V(G)$ of p vertices together with prescribed set E of q unoredered pairs of distinct vertices of V. Each pair $e = \{u, v\}$ of vertices in E is called an edge and e is said to join u and .

The removal of a vertex v from a graph G results the more components of graph G, than the vertex v is said to be cut vertex. The degree of a vertex v in a graph G is denoted by $deg(v)$ is the number of edges incident to v. The minimum / greatest degree between the vertices of a graph G is denoted by $\delta(G)/\Delta(G)$. If a vertex and an edge are incident, they are said to cover each other. A cover of G is a set of vertices that covers all of the edges of a graph G. The minimum number of vertices in every G cover is known as the covering number, which is indicated by $\alpha_0(G)$. An edge cover of G is a set of edges that cover all vertices of G. The edge covering number of a graph G is denoted by $\alpha_1(G)$ and is the smallest number of edges in any edge cover of the graph.

For some real number $x, \lceil x \rceil$ represents the least integer not less that x and $\lfloor x \rfloor$ represents the greatest integer not greater than x. A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge by a path uvw. The subdivision of G is the graph formed by subdividing each edge of G exactly once and is denoted by $S(G)$.

A graph G without a cycle is called a tree. A line graph $L(G)$ is the graph whose vertices corresponds to the edges of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges of G are adjacent.

We'll start by recalling some common domination theory definitions.

If every vertex not in D is adjacent to at least one vertex in D, then set $D \subseteq V$ of a graph $G = (V, E)$ is a dominating set. The lowest cardinality of a dominating set in G is the domination number $\gamma(G)$. A dominating set D is a connected dominating set, if an induced subgraph $\langle D \rangle$ is connected and the minimal cardinality of the set D is connected domination number $\gamma_c(G)$ of G .

If the induced subgraph $\langle V - D \rangle$ is not connected, the dominating set D of a graph G is a split dominating set. The minimal cardinality of a split dominating set is the split domination number $\gamma_s(G)$ of a graph . Kulli and Janakiram [8] were the ones who first offered this notion.

Analogously , we define domination number in split lict subdivision graph of a graph as fallows.

A lict subdivision dominating set $D \subseteq V\{n[S(G)]\}$ is a split lict subdivision dominating set , if the subgraph $\langle V\{n[S(G)]\} - D \rangle$ is not connected. The least number of vertices in such a set is called Split lict subdivision domination number in G or split domination number in lict subdivision graph of a graph G and is characterised by $\gamma_{sns}(G)$.

The idea of total domination in graph was introduced by Cockayne , Dawas and Hedetniemi [3]. A set D of vertices of a graph G is a total dominating set if each vertex v is adjacent to some vertex in D. The total domination number $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set. A dominating set D of G is a cototal dominating set if the induced subgraph $\langle V - D \rangle$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ of G is the lowest cardinality of a cototal dominating set. This concept was introduced by Kulli, Janakiram & Iyer in [9] . This concept was also studied as restrained domination in graphs[4] .

On a graph $G = (V, E)$, a Roman dominating function is $f: v \to \{0,1,2\}$ that meets the criterion that every vertex $u \in v$ for which $f(u) = 0$ is adjacent to atleast one vertex $u \in v$ for which $f(v) = 2$. The value of $f(v) = \sum_{u\in v} f(v)$ represents the weight of Roman dominating function. The minimum weight of a Roman dominating function of G is the Roman domination number $\gamma_R(G)$.

With preference from the above definition we express Roman domination number in line graph as shown below.

The Roman domination number of line graph $\gamma_{RL}(G)$ of $L(G)$ is the minimal weight of a Roman dominating function of $L(G)$. The weight of Roman dominating function of $L(G)$ is the value $(v) = \sum_{u\in V[L(G)]} f(v)$.

In this paprer many constraints on $\gamma_{sns}(G)$ were obtained in terms of vertices, edges of . Also we establish split domination number of lict subdivision graph $n[S(G)]$ and express the result with other different domination parameter of G .

Results: The definition of the split domination graph of a graph inspired us to introduce the split domination in lict subdivision graph of a graph in domination theory.

Theorem 1: For any connected (p, q) graph G ,

$$\gamma_{sns}(G) + \gamma_{RL}(G) \geq p + \Delta(G) .$$

Proof: Let $f: V[n(G)] \to \{0,1,2\}$ and partition, the vertex set of $n(G)$ into (V_0, V_1, V_2) induced by f with $|V_i| = p$ for $= 0,1,2$. Suppose the set V_2 dominates V_0, then $S = V_1 \cup V_2$ forms a minimal dominating set of $n(G)$. Further let $F = \{v_1, v_2, \dots\dots\dots\dots, v_k\} \subseteq V[n(G)]$ be the vertex set with $deg(v_j) \geq 2$; $1 \leq j \leq k$ and let $F' = \{v_1', v_2', v_3', \dots\dots\dots, v_{k-1}'\}$ be the vertex of subdivision graph of $n(G)$ with $\deg(v_{k-1}') = 2$, where as $F \subseteq F'$. Now assume there is a vertex set $D \subseteq F'$ with $N[D] = V\{n[S(G)]\}$ and if $\langle V\{n[S(G)]\} - D\rangle$ is disconnected. Then D forms a split lict dominating set in $S(G)$. Otherwise, there exists atleast one vertex $z \in F$ and $z \notin D$ such that $D \cup \{z\}$ forms a minimal split dominating set of subdivision of $n(G)$. Since for any graph G, there exists atleast one vertex $v \in V(G)$ with maximum degree $\Delta(G)$,

Thus we can conclude that $|D \cup \{Z\}| \cup |S| \geq |V(G)| \cup \max[\deg(v)]$

Hence $\gamma_{sns}(G) + \gamma_{RL}(G) \geq p + \Delta(G)$.

Hence the proof.

In the following theorem we obtain the relation for a tree $\gamma_{sns}(T)$ in terms of cutvertices of .

Theorem 2: For any Tree T with $C > 1$, where C is the number of cutvertices then

$$\gamma_{sns}(T) \geq C - 1.$$

Proof: Let $V(T)=\{v_1,v_2,v_3,\dots\dots\dots\dots\dots,v_n\}$ and $H=\{v_1,v_2,v_3,\dots\dots\dots\dots\dots\dots,v_i\}$; $1\le i\le n$ such that $H\subset V(T)$ is the set of cutvertices. Further $E(T)=\{e_1,e_2,e_{3},\dots\dots,e_m\}$ and $J=\{e_1,e_2,e_{3},\dots\dots,e_j\}$; $1\le j\le m$ be the set of all non end edges in T, such that $J\subset E(T)$. Let $V'(T)=\{v_1{}',v_2{}',v_3{}',\dots\dots\dots,v_{n-1}{}'\}$ be the vertex set of $S(T)$, formed by inserting a vertex v_j ,$\forall\ 1\le j\le n-1$ between every edges of $n(T)$ with degree two for each v_j , where as $v_i\subset v_j{}'$. Now since $V\{n[S(T)]\}=\{E(T)-J(T)\}\cup\{J\}\cup\{H\}\cup\{v_j\}$, let $D=\{u_1,u_2,u_3,\dots\dots\dots\dots\dots,u_m\}\subseteq J(T)\subseteq V\{n[S(T)]\}\cup\{v_j\}$; $\forall\ u_i,v_j,1\le i\le j\le m$ have maximum degree in $n[S(T)]$ and $\langle V\{n[S(T)]\}-D\rangle$ is disconnected such that $N\{v_j\}=V\{n[S(T)]\}$;$\forall v_j\in n$ then D forms a minimal split dominating set of $n[S(T)]$.

Clearly it follows that $\quad |D|\ge|H|-1$

And hence $\quad \gamma_{sns}(T)\ge C-1.$

Theorem 3: For any connected (p,q) graph G,

$$\gamma_{sns}(G)\ge\gamma_{nc}(G).$$

Proof: Let $E(G)=\{e_1,e_2,e_{3},\dots\dots,e_n\}$ and $C(G)=\{c_1,c_2,c_{3},\dots\dots,c_j\}$ be the set of edges and cutvertices in G. In (G), $V[n(G)]=E(G)\cup\ C(G)$, and let $E'=\{e_1{}',e_2{}',e_3{}',\dots\dots\dots,e_n{}'\}$ be the set of edge set of $n[S(G)]$ such that $E\subseteq E'$. Now let $E_1{}'=\{e_1{}',e_2{}',e_3{}',\dots\dots\dots,e_k{}'\}\subseteq E'$ be the set of non end edges which are the cutvertices in $n[S(G)]$. Suppose $E_1{}'=\emptyset$. Then $n[S(G)]$ is non-separable. Let $V\{n[S(G)]\}=\{v_1,v_2,v_3,\dots\dots\dots\dots\dots,v_n\}$; $V_1\subseteq V\{n[S(G)]\}$ and $\forall\ v_i\in V_1,\deg(v_i)=\Delta\{n[S(G)]\}$. Now there exists $V_2\subseteq V_1$such that $D=V\{n[S(G)]\}-V_2$ and $\forall\ v_j\in D$ are adjacent to at least one vertex $V_m\in V\{n[S(G)]\}-V_2$ also $\langle D\rangle$ is disconnected then D forms a split lict dominating set of $S[G]$. Suppose $\langle D\rangle$ has an isolates then consider $N(D)-\{w_i\}$, such that $\langle D\cup\{w_i\}\rangle$ is connected. Otherwise D is both split lict dominating and connected lict dominating set of G .

In both cases we have $|D|\le\langle D\cup\{w_i\}\rangle$ which gives

$$\gamma_{sns}(G)\ge\gamma_{nc}(G).$$

Hence the result.

The next theorem relates $\gamma_{sns}(G)$ in terms of $\gamma_t(G)$ and vertices of G.

Theorem 4: For any connected (p,q) graph G,

$$\gamma_{sns}(G)+\gamma_t(G)\ge p\ .$$

Proof: Let $V_1=\{v_1,v_2,v_3,\dots\dots\dots\dots\dots,v_k\}\subseteq V[G]$;$\forall\ i\le k\le n$ is the set of all non-end vertices in G. Suppose $V_2\subseteq V_1$be the minimum set of vertices in G and if $\deg(v_i)\ge1$;$\forall\ v_i\in V_2,1\le i\le m$ in the subgraph $\langle V_2\rangle$ then V_2 forms a total dominating set of G. Otherwise, if $\deg(v_i)<1$,then attach the vertices

$w_i \in N(v_i)$ to make $\deg(v_i) \geq 2$ such that $\langle V_2 \cup \{w_i\} \rangle$ does not contain any isolated vertex. Clearly $V_2 \cup \{w_i\}$ forms minimal total dominating set of . Now let $E' = \{e_1', e_2', e_3', \dots \dots \dots \dots \dots .., e_{k-1}'\}$ be the edge set formed by adding a vertex of degree two between every edge of $n[S(G)]$ such that $E' \subset E\{n[S(G)]\}$ and also $V[n(G)] \subseteq E(G) \cup C(G) \subseteq E'[S(G)]$ where $C(G)$ is a set of all cutvertices in G. Now let $D = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots, v_{k-1}\} \subseteq V_1'\{n[S(G)]\} = E_1'[S(G)] \cup C_1'[S(G)]$ when E_1' be the set of edges which are incident with the vertices of $V_2 \cup \{w_1\}$, clearly D be the $\gamma_{sn} - set$ of $S(G)$. Hence

$$|D| \cup |V_2 \cup \{w_1\}| \geq |V(G)| \quad \text{gives}$$

$$\gamma_{sns}(G) + \gamma_t(G) \geq p .$$

Thus the result.

In the following theorem we establish the result on upper bound for $\gamma_{sns}(G)$.

Theorem 5: For any tree T ,

$$\gamma_{sns}(T) \leq \left\lceil p + \frac{m}{2} \right\rceil, \text{ where } m \text{ be the number of end vertices in } T.$$

Proof: The result forms an equality if $diam(T) \leq 3$. If $diam(T) > 3$ then let $M = \{v_1, v_2, v_{3\,,\dots\dots\dots\dots\dots\dots,} v_m\}$ be the set of all end vertices in T such that $|M| = m$. Since by the definition of lict graph we can observe that $V[n(T)] = E_i \cup C_i$ where E_i and C_i for all $1 \leq i \leq m$ is the edge set and cutvertex set of T respectively. Let $n[S(T)]$ be the subdivision graph of lict graph of T where as vertices of $n(T)$ are adjacernt if the corresponding edges E_i and cutvertices C_i are incident and edges in T .. Now let $F = \{u_1, u_2, u_{3\,,\dots\dots\dots\dots\dots,} u_k\} \subseteq V\{n[S(T)]\}$ be the set of vertices such that $\{u_i\} = \{e_j\} \in E[S(T)]$; $1 \leq i \leq k$ where $\{e_i\}$ are incident with the vertices of F. Further let $D \subseteq F$ be the set of vertices with $N[D] = V\{n[S(T)]\}$ and if the subgraph $\langle V\{n[S(T)]\} - \mathrm{D} \rangle$ is disconnected then D forms a split dominating set of $[S(T)]$. Otherwise there exists at least one vertex $\{u_j\} \in V\{n[S(T)]\} - D$ for $i \leq j$ such that $\langle V\{n[S(T)]\} - \mathrm{D} - \{\mathrm{u_j}\} \rangle$ forms more than one component. Thus $D \cup \{u_j\}$ forms a minimal γ_s set of $n[S(T)]$.

Thus in all case we obtain $\quad D \cup \{u_j\} \leq \left\lceil p + \frac{|M|}{2} \right\rceil.$

$$\gamma_{sns}(T) \leq \left\lceil p + \frac{m}{2} \right\rceil .$$

Thus the result .

Theorem 6: For any graph $G \cong T$,

$$\gamma_{sns}(T) \leq e + \left\lceil \frac{\alpha_1(T)}{2} \right\rceil + 2, \text{ where } e \text{ be the number of end edges.}$$

Proof: Suppose $E' = \{e_1, e_2, e_{3,\ldots\ldots\ldots\ldots}, e_m\}$ be a set of all end edges in T, then $E' \cup I$ where $I \subseteq E(T) - E'$ be the least set of edges which covers the vertices of T and is not covered by E' , such that $|E' \cup I| = \alpha_1(T)$. Now without loss of generality in $[S(T)]$, let $U = \{u_1, u_2, u_{3,\ldots\ldots\ldots\ldots}, u_i\} \subseteq V\{n[S(T)]\}$ is the set of all vertices in $n[S(T)]$ formed by the $E(T) \cup C(T) \subseteq V\{n[S(T)]\}$. Let D be the minimal dominating set of $n[S(T)]$ such that subgraph $\langle \{U\} \cup D \rangle$ is disconnected and D forms a minimal split lict dominating set of $S(T)$.

Clearly it follows that $|D| \leq |E(T)| + \left\lceil \frac{|E' \cup T|}{2} \right\rceil + 2.$

$$\gamma_{sns}(T) \leq e + \left\lceil \frac{\alpha_1(T)}{2} \right\rceil + 2 \ .$$

Hence the result.

In next result we obtain the relation of $\gamma_{sns}(G)$ and $\gamma_{cot}(G)$.

Theorem 7: For any (p, q) graph G ,

$$\gamma_{sns}(G) \geq \gamma_{cot}(G) + 2 \ .$$

Proof: Suppose $S = \{v_1, v_2, v_{3,\ldots\ldots\ldots\ldots}, v_n\} \subseteq V(G)$ be the smallest set of vertices that covers all the vertices in G such that $\langle V(G) - S \rangle$ has no isolates. Than $|S| = \gamma_{cot}(G)$. Now without loss of generality, let $D = \{u_1, u_2, u_{3,\ldots\ldots\ldots\ldots}, u_k\} \subseteq V\{n[S(G)]\}$, where as $V\{n[S(G)]\} = E(G) \cup C(G) \subseteq \{v_i, u_i\}$ that belongs to components of $n[S(G)]$ such that $\{u_i\} = \{e_j\} \subset E[S(G)]\,; 1 \leq j \leq k$. Where $\{e_j\}$ are incident with the vertices of (G) . Further since $D \subseteq V\{n[S(G)]\}$ with $N(D) = u_j \subseteq V\{n[S(G)]\}$; for $1 \leq j \leq i$ and if the induced subgraph $\{u_j - D\}$ contains multiple component . Then D forms a split dominating set of subdivision of $n(G)$ .Otherwise there exists at least one vertex $\{u_k\} \in V\{n[S(G)]\} - D$ for $k \leq j$ such that $\langle V\{n[S(G)]\} - D - \{u_k\} \rangle$ yields more than one component.

Clearly $D \cup \{u\}$ forms a minimal $\gamma_{sn} - set$ of $S(G)$. Therefore it follows that

$$|D \cup \{u\}| \geq |S| + 2 \ .$$

Hence $\gamma_{sns}(G) \geq \gamma_{cot}(G) + 2$.

Theorem 8: For any (p, q) graph G,

$$\gamma_{sns}(G) \geq \left\lceil \frac{p}{2} \right\rceil \text{ provided } G \neq K_{1,p} \text{ with } p \geq 2 \text{ vertices .}$$

Proof: Case 1: If $G \cong K_{1,p}$ it is obvious that for any star $K_{1,p}$, $n(K_{1,p}) = K_{p+1}$. Thus $S(K_{p+1})$ is formed by inserting a vertex v_j for all $j \leq n$, with $\deg(v_j) = 2$ in between every edge of K_{p+1} . Thus number of vertices $v_j \geq v_n$ in $S(K_{p+1})$ and it is clear that $V[K_{1,p}] \subseteq V[S(K_{p+1})]$ and if $V' = \{v_1', v_2', v_3', \dots, v_m'\}$ is $V\{n[S(G)]\}$ and let $D = \{v_i'\}; \forall\, 1 \leq i \leq m$ be the minimal γ_s −set of $n[S(G)]$, then there exists a vertex $\{u_j\}$; $\forall\, i \leq j$ such that $\langle \{u_j\} - D \rangle$ is disconnected where as $\langle \{u_j\} - D \rangle \supset V(K_{1,p})$ hence a contradiction.

Case 2: If $G \ncong K_{1,p}$, then by the definition of $n(G)$. $V[n(G)] = E(G) \cup C(G)$, let $F = \{e_1, e_2, e_{3,\dots\dots}, e_{n-1}, e_n\} = E(G)$ and $C = \{c_1, c_2, c_{3,\dots\dots,\dots\dots}, c_{n-1}\}$ be the set of cutvertices of G, where as $F \subseteq e_i \cup c_j$; for all $1 \leq i \leq n$ and $1 \leq j \leq n-1$. Let $F' = \{u_1', u_2', u_3', \dots\dots\dots\dots\dots, u_{n-1}'\}$ be the vertex set of $n[S(G)]$ with $F' \subset e_i \cup c_j$ and $F \subseteq F'$ be the minimal dominating set of $n[S(G)]$, such that $|F| = D$. Further if there exists a vertex $\{u_j'\}$; $\forall\, 1 \leq j \leq n-1$ such that subgraph $\langle \{u_j'\} - D \rangle$ is disconnected and hence D forms a minimal split dominating set of $n[S(G)]$. Also $|\{u_j'\} - D| \geq 2$ that contains at least two vertices such that $p \leq 2n$.

Hence it follows that

$$|\{u_j'\} - F| \geq 2n > p$$

$$|\{u_j'\} - F| \geq n > \left\lceil \frac{p}{2} \right\rceil.$$

Hence $\gamma_{sns}(G) \geq \left\lceil \frac{p}{2} \right\rceil$.

Thus the result follows.

Conclusion:

Here we discuss and establish the results on Split domination in Lict subdivision graph of a graph. Also we derive few relations between Split Domination in Lict subdivision domination number and some other standard parameters. Also we extend this results in future.

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