# ONTO MINUS EDGE DOMINATION NUMBERS IN GRAPHS

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## Abstract

Let $G = (V, E)$ be a graph. *The onto minus edge dominating function* is a function $f: E \rightarrow \{-1, 0, 1\}$ such that $f$ is onto and $f(N[e]) \geq 1$ for all $e \in E(G)$. The *onto minus edge domination number* of a graph $G$ is a minimum weight of a set of onto minus edge dominating functions on $G$ and it is denoted by $\gamma' om\ (G)$.

In this paper we discuss about the onto minus edge domination number of Paths, Cycles and Bipartite graph.

**Keywords and Phrases:** Dominating function, onto minus edge dominating function, Paths, Cycles, Bipartite graph.

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## 1. Introduction and Preliminaries

Mitchell and Hedetniemi were introduced the concept of edge domination. The minus dominating function was introduced by Dunbar et al [2]. Further the concept was extended to define other edge parameter like minus edge domination number which was introduced by B. Xu and S. Zhou. Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. The closed neighborhood $N_G[e]$ of an edge $e$ in a graph $G$ is the set consisting of $e$ and all the edges having a common vertex with $e$. A function $f: E \rightarrow \{0, 1\}$ is called an edge dominating function of $G$ if $f(N[e]) \geq 1$ for every $e \in E(G)$. A function $f: E \rightarrow \{-1, 0, 1\}$ is called a minus edge dominating function of $G$ if $f(N[e]) \geq 1$ for every $e \in E(G)$. The minus edge domination number for a graph $G$ is $\gamma' m\ (G) = min\ \{w(f): f \text{ is minus edge dominating function of } G\}$

[4]. We denote *f (N [e]) by f[e]*. A function *f: A → B* is said to be onto if every element in *B* has a pre-image in *A*. The onto minus edge dominating function is a function *O: E → {-1, 0, 1}* such that *O* is onto and *O (N [e]) ≥ 1* for every *e ∈ E(G)*. The onto minus edge domination number of a graph *G* denoted by $\gamma' om\ (G)$ is the minimum weight of a set of all onto minus edge dominating functions on *G*. That is $\gamma' om\ (G)$ = *min {w (f): f is onto minus edge dominating function of G}*. Minus edge dominating function exists for all graphs, but the onto minus edge dominating function do not exists for all graphs. For example the graph $P_4$ has no onto minus edge dominating function for such graphs we define the onto minus edge domination number is equal to ∞.

## 2. Main Results

### Theorem 2.1

For n > 6,

$$\gamma' om\ (P_n) = \{\lceil\frac{n}{3}\rceil + 1 \quad if\ \ n\ =\ 3k, k\ > 2\ \lceil\frac{n}{3}\rceil \qquad otherwise$$

**Proof :** We prove the result by considering three cases. Let $e_1, e_2, e_3, \ldots\ e_{n-1}$ be the edges of $P_n$.

***Case ( i ) :*** *n ≡ 0 (mod3)*

In this case *n = 3k*, where *k > 2* is a positive integer.

Define a function f*: E → {-1, 0, 1}* by

$$f(e_i) = \{0\ \ if\ i = 1, n-2 \qquad\qquad 1\ \ if\ i \equiv 2\ (mod 3)\ and\ i \equiv 0 (mod3)\ -1\ if\ i \equiv 1\ (mod3)\ except\ i = 1,\ \ n-1$$

Then $f(N[e_2]) = f(N[e_{n-3}]) = f(N[e_{n-2}]) = 2$ and $f(N[e_i]) = 1$ for other edges. Thus f is an onto minus edge dominating function of $P_n$ if *n ≡ 0 (mod3)*. Also, the weight of the function *f* is

$$f(E) = (k - 2)(-1) + (2k - 1)(1) + 2(0)$$
$$= k + 1$$
$$= \lceil 3k/3 \rceil + 1$$
$$= \lceil n/3 \rceil + 1$$

***Case ( ii ) :*** *n ≡ 1 (mod3)*

In this case *n = 3k + 1*, where *k > 1* is a positive integer.

Define a function f*: E → {-1, 0, 1}* by

$f(e_i) = \{0 \;\; if\; i = 1 \qquad 1\; if\; i \equiv 2\;(mod 3)\; and\; i \equiv 0 (mod 3) \;\; -1\; if\;\; i \equiv 1\;(mod 3)\; except\; i = 1$

Then $f(N[e_2]) = f(N[e_{n-1}]) = 2$ and $f(N[e_i]) = 1$ for other edges. Thus f is an onto minus edge dominating function of $P_n$ if $n \equiv 1\ (mod3)$. Also, the weight of the function *f* is

$$f(E) = (k - 1)(-1) + (2k)(1) + 1(0)$$

$$= k + 1$$

$$= \lceil (3k + 1)/3 \rceil$$

$$= \lceil n/3 \rceil$$

***Case ( iii ) :*** $n \equiv 2\ (mod3)$

In this case $n = 3k + 2$, where $k > 1$ is a positive integer.

Define a function f: $E \rightarrow \{-1, 0, 1\}$ by

$f(e_i) = \{\; 0 \;\; if\; i = 1, n-1 \qquad 1\;\; if\; i \equiv 2\;(mod 3)\; and\; i \equiv 0 (mod 3) \;\; -1\; if\; i \equiv 1\;(mod 3)\; except\; i = 1,\;\; n-1$

Then $f(N[e_2]) = f(N[e_{n-2}]) = 2$ and $f(N[e_i]) = 1$ for other edges. Thus f is an onto minus edge dominating function of $P_n$ if $n \equiv 2\ (mod3)$. Also, the weight of the function *f* is

$$f(E) = (k - 1)(-1) + (2k)(1) + 2(0)$$

$$= k + 1$$

$$= \lceil (3k + 2)/3 \rceil$$

$$= \lceil n/3 \rceil$$

Hence,

$\gamma' om\ (P_n) = \{\lceil \frac{n}{3} \rceil + 1\; if\;\; n \; = \; 3k, k \; > 2 \quad \lceil \frac{n}{3} \rceil \qquad otherwise$ for $n > 6$.

**Theorem 2.2**

For $n > 5$,

$$\gamma' om\ (C_n) = \{\lceil \frac{n}{3} \rceil \qquad if \;\; n \; = \; 3k + 1, k > 1 \;\; \lceil \frac{n}{3} \rceil + 1 \quad otherwise$$

**Proof :** We prove the result by considering three cases. Let $e_1, e_2, e_3, \ldots e_n$ be the edges of $C_n$.

***Case ( i ) :*** $n \equiv 0 \ (mod3)$

In this case $n = 3k$, where $k > 1$ is a positive integer. Define a function f$: E \rightarrow \{-1, 0, 1\}$ by

$$f(e_i) = \{0 \ \ if \ i = n \qquad\qquad 1 \ \ if \ i \equiv 1 \ (mod3) \ and \ i \equiv 2(mod3) \ -1 \ if \ i \equiv 0 \ (mod3) \ except \ i = n$$

Then $f(N[e_1]) = f(N[e_{n-1}]) = f(N[e_n]) = 2$ and $f(N[e_i]) = 1$ for other edges. Thus f is an onto minus edge dominating function of $C_n$ if $n \equiv 0 \ (mod3)$. Also, the weight of the function $f$ is

$$f(E) = (k - 1)(-1) + (2k)(1) + 1(0)$$

$$= k + 1$$

$$= \lceil 3k/3 \rceil + 1$$

$$= \lceil n/3 \rceil + 1$$

***Case ( ii ) :*** $n \equiv 1 \ (mod3)$

In this case $n = 3k + 1$, where $k > 1$ is a positive integer.

Define a function f$: E \rightarrow \{-1, 0, 1\}$ by

$$f(e_i) = \{0 \ \ if \ i \equiv 1 \ (mod3) \ and \ i \ \equiv 2(mod3) except \ i = 2, 4, 5 \ \ 1 \ \ if \ i \equiv 0 \ (mod3) \ and \ i = 2, 5 \qquad -1 \ if \ i = 4$$

Then $f(N[e_i]) = 1$ for all edges $i$ except $i = 2, 6$.

For $i = 2, 6$ $f(N[e_i]) = 2$. Thus f is an onto minus edge dominating function of $C_n$ if $n \equiv 1 \ (mod3)$. Also, the weight of the function $f$ is

$$f(E) = (k - 1)(-1) + (2k)(1) + 2(0)$$

$$= k + 1$$

$$= \lceil (3k + 1)/3 \rceil$$

$$= \lceil n/3 \rceil$$

***Case ( iii ) :*** $n \equiv 2 \ (mod3)$

In this case $n = 3k + 2$, where $k > 1$ is a positive integer.

Define a function f*: E* → *{-1, 0, 1}* by

$$f(e_i) = \{0 \;\; if\; i = 3k, 3k+2 \qquad 1 \;\; if\; i \equiv 1\ (mod3)\ and\ i \equiv 2(mod3)\ except\ i = 3k+2 \qquad -1\ if\ i \equiv 0\ (mod3)\ except\ i = 3k$$

Then $f(N[e_1]) = f(N[e_{n-2}]) = f(N[e_n]) = 2$ and $f(N[e_i]) = 1$ for other edges. Thus f is an onto minus edge dominating function of $C_n$ if $n \equiv 2\ (mod3)$. Also, the weight of the function *f* is

$$f(E) = (k - 1)(-1) + (2k + 1)(1) + 2(0)$$

$$= k + 1 + 1$$

$$= \lceil(3k + 2)/3\rceil + 1$$

$$= \lceil n/3\rceil + 1$$

Hence,

$$\gamma' om\ (C_n) = \{\lceil \tfrac{n}{3}\rceil \quad if\ n = 3k+1, k > 1 \quad \lceil \tfrac{n}{3}\rceil + 1 \quad otherwise$$ for $n > 5$.

**Theorem 2.3**

For $m \le n$, $\gamma' om\ (K_{m,n}) = \{m\ if m,n \ge 3 \qquad 3\ if\ m = 2\ and\ n > 2$

**Proof :** Let $K_{m,n}$ be the bipartite graph with $m + n$ vertices and $mn$ edges. Let $e_1$, $e_2$, $e_3$,…, $e_{mn}$ be the edges of $K_{m,n}$.

***Case ( i ):*** If $m, n \ge 3$

Define a function f*: E* → *{-1, 0, 1}* by

$$f(e_i) = \{1\ \ if\ i = 1, n+2, n+3, nk+1\ where\ k = 2, 3, \dots, m-1 \quad -1\ if\ i = n+1 \qquad 0\ \ otherwise$$

Then $f(N[e_1]) = m - 2$, $f(N[e_{n+1}]) = m$

For $i = 2, 3$ $f(N[e_i]) = 2$

For $4 \le i \le n$, $f(N[e_i]) = 1$

For $n + 2 \le i \le 2n$, $f(N[e_i]) = 1$

For $i = nk + 1$ where $k = 2, 3, \dots, m - 1$, $f(N[e_i]) = m - 2$

For $i = nk + 2$ , $nk + 3$ where $k = 2, 3, \dots, m - 1$, $f(N[e_i]) = 2$

For $nk + 4 \leq i \leq n(k + 1)$ where $k = 2, 3, \ldots, m - 1$, $f(N[e_i]) = 1$

Thus f is an onto minus edge dominating function of $K_{m,n}$.

Also, the weight of the function $f$ is

$$f(E) = (1)(-1) + [(m + 1)(1) + mn - (m + 2)](0)$$

$$= m$$

***Case (i i ):*** If $m = 2$ and $n > 2$

Define a function f: $E \rightarrow \{-1, 0, 1\}$ by

$$f(e_i) = \{1 \text{ if } i = 1, 2, n + 2, n + 3 \quad -1 \text{ if } i = n + 1 \quad 0 \text{ otherwise}$$

Then $f(N[e_1]) = 1$

For i = 2, 3 $f(N[e_i]) = 3$

For $4 \leq i \leq n + 2$, $f(N[e_i]) = 2$

For $n + 3 \leq i \leq 2n$, $f(N[e_i]) = 1$

Thus f is an onto minus edge dominating function of $K_{m,n}$. Also, the weight of the function $f$ is

$$f(E) = (1)(-1) + (4)(1) + (mn - 5)(0)$$

$$= -1 + 4 = 3$$

Thus $\gamma' om\ (K_{m,n}) = \{m \text{ if } m, n \geq 3 \quad 3 \text{ if } m = 2 \text{ and } n > 2$ for $m \leq n$.

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