# Connectedness and Compactness via $\mathcal{J}i$-Open Sets

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**Abstract**
The aim of this paper is to define the notion of $\mathcal{J}i$-compactness in Intuitionistic topological spaces. Besides, we define the $\mathcal{J}i$-$C_s$-connected sets, $\mathcal{J}i$-$C_m$-connected sets and discuss the relationship between $\mathcal{J}i$-$C_i$-connected ($i$ = 1,2) sets and $\mathcal{J}i$-$C_s$-connected sets. Further, we obtain several characterizations of $\mathcal{J}i$-compactness in Intuitionistic Topological Spaces.

**Keywords** *$\mathcal{J}i$-$C_1$-connected, $\mathcal{J}i$-$C_2$-connected, $\mathcal{J}i$-$C_s$-connected, $\mathcal{J}i$-compact*
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**Introduction**
After the introduction of the concept of fuzzy set by Zadeh [9], Atanassov[1] proposed the concept of intuitionistic fuzzy sets. The concept of intuitionistic sets and intuitionistic topological spaces(also named as intuitionistic fuzzy special topological spaces) was first introduced by Coker [2,3]. He studied some properties of compactness, continuity and separation axioms in intuitionistic topological spaces. Selma ozcag and Dogan Coker [4] also examined connectedness in intuitionistic topological spaces. Suganya etal.[6] defined intuitionistic $i$-open sets in Intuitionistic topological spaces and determine their properties. Also, they explained continuous function[7], irresolute function[7], $\mathcal{J}i$-connectedness[8] associated with $\mathcal{J}i$-open sets in ITS. In this paper, we introduce the concepts of $\mathcal{J}i$-$C_s$-connected set, $\mathcal{J}i$-$C_m$-connected set and $\mathcal{J}i$-compactness s in Intuitionistic Topological Spaces. Besides, we study the properties of $\mathcal{J}i$-compactness in Intuitionistic Topological Spaces. Also we discuss the relationship between $\mathcal{J}i$-$C_i$-connected ($i$ = 1,2) sets and $\mathcal{J}i$-$C_s$-connected sets.

**Preliminaries**

**Definition 2.1. [3]** An intuitionistic topology (for short IT) on a non-empty set $\mathcal{K}$ is a family $\tau$ of intuitionistic sets in $A$ satisfying following axioms.
1) $\widetilde{\emptyset}, \tilde{A} \in \tau_I$
2) $G_1 \cap G_2 \in \tau_I$, for any $G_1$ , $G_2 \in \tau_I$
3) $\cup G_\alpha \in \tau_I$ for any arbitrary family $\{G_\alpha/\alpha \epsilon \mathrm{J}\}$ where $(A, \tau_I)$ is called an intuitionistic topological space and any intuitionistic set is called an intuitionistic open set (for short $\mathcal{J}OS$) in $A$. The complement of an $\mathcal{J}OS$ is called an intuitionistic closed set (for short $\mathcal{J}CS$) in $A$.

**Definition 2.2.[6]** An intuitionistic set $D$ of an ITS $(A, \tau_I)$ is named as intuitionistic $i$-open set (shortly $\mathcal{J}iOS$) if there exist an intuitionistic open set $H \neq \widetilde{\emptyset}$ and $\tilde{A}$ such that $D \subseteq \mathcal{J}cl(D \cap H)$. The complement of $\mathcal{J}i$-open set is called $\mathcal{J}i$-closed set. The set of all intuitionistic $i$-open sets of $(A, \tau_I)$ is denoted by $\mathcal{J}iO$.

**Definition 2.3.[6]** Let $(A, \tau_I)$ be an ITS and let $H \subseteq A$. The intuitionistic $i$-closure of $H$ is defined as the intersection of all intuitionistic $i$-closed sets in $A$ containing $H$, and is denoted by $\mathcal{J}cl_i(H)$.

**Definition 2.4.[9]** Let $D$ be an intuitionistic set in the ITS $(A,\tau_I)$ . If there exists $\mathcal{J}i$-open sets $P$ and $Q$ in $A$ satisfying the following properties, then $D$ is called $\mathcal{J}i$- $C_k$-disconnected $(k = 1,2)$.
$\mathcal{J}i$- $C_1$ : $D \subseteq P \cup Q$, $P \cap Q \subseteq \overline{D}$ , $D \cap P \neq \widetilde{\emptyset}$ , $D \cap Q \neq \widetilde{\emptyset}$
$\mathcal{J}i$- $C_2$ : $D \subseteq P \cup Q$ , $P \cap Q \cap D = \widetilde{\emptyset}$ , $D \cap P \neq \widetilde{\emptyset}$ , $D \cap Q \neq \widetilde{\emptyset}$ .
**Theorem 2.5.[9]** If the $\mathcal{J}i$-closure of the subsets of $(A,\tau_I)$ are $\mathcal{J}i$-closed, then the nonempty sets $P$ and $Q$ are $\mathcal{J}i$ weakly separated if and only if there exists $C,D \in \mathcal{J}iO$ such that $P \subseteq C, Q \subseteq D, P \subseteq \overline{D}$ and $Q \subseteq \bar{C}$ .
**Definition 2.6. [2]** Let $A$ be a non empty set and $p \in A$ a fixed element in $A$. Then the intutionistic set $\tilde{p} = < A,\{p\},\{p\}^c >$ is called intutionistic point and $\tilde{\tilde{p}} =< A,\emptyset,\{p\}^c >$ is called intutionistic vanishing point.
**Definition 2.7. [2]** Let $p \in A$ and $H = < A,H_1,H_2 >$ be an intutionistic set. Then
(i) $\tilde{p} \subseteq H$ iff $\tilde{p} \in H_1$
$(ii)$ $\tilde{\tilde{p}} \subseteq H$ iff $\tilde{\tilde{p}} \in H_2$
**Proposition 2.8. [3]** Let $(A,\tau_I)$ be an ITS and $H = < A,H_1,H_2 >$ be an IS in $A$. Then the several intuitionistic topologies (a),(b) and general topologies (c),(d) generated by $(A,\tau_I)$ are (a) $\tau_{I_{0,1}} = \{[]H : H \in \tau_I\}$ (b) $\tau_{I_{0,2}} = \{<> H : H \in \tau_I\}$ (c) $\tau_{I1} = \{ H_1 : < A,H_1,H_2 > \in \tau_I\}$ (d) $\tau_{I2} = \{ (H_2)^c < A,H_1,H_2 > \in \tau_I\}$
**Definition 2.9.[3]** Let $(A,\tau_I)$ be an intuitionistic topological space. If a family $\{< A,H_{1_k},H_{2_k} >; k \in J\}$ of intuitionistic open sets in $A$ satisfies the condition $\cup \{< A,H_{1_k},H_{2_k} >; k \in J\} = \tilde{A}$ , then it is called an intuitionistic open cover of $A$. A finite subfamily of an intuitionistic open cover $\{< A,H_{1_k},H_{2_k} >; k \in J\}$ of $A$ ,which is also an intuitionistic open cover of $A$ is called a finite intuitionistic subcover of $\{< A,H_{1_k},H_{2_k} > ; k \in J\}$
**Definition 2.10.[3]** Let $(A,\tau_I)$ be an intuitionistic topological space . A family $\{< A,H_{1_k},H_{2_k} >; k \in J\}$ of intuitionistic closed sets in $A$ satisfies the finite intersection property iff every finite subfamily $\{H_1,H_2,H_3,....,H_n\}$ of $H$ satisfies the condition $\bigcap_{k=1}^{n} < A,H_{1_k},H_{2_k} > \neq \widetilde{\emptyset}$
**Definition 2.11.[3]** An ITS $(A,\tau_I)$ is said to be intuitionistic compact iff each intuitionistic open cover has a finite intuitionistic subcover.
**Definition 2.12.[7]** Let $(A,\tau_{I_1})$ and $(B,\tau_{I_2})$ be two intuitionistic topological spaces. A mapping $s$: $(A,\tau_{I_1}) \to (B,\tau_{I_2})$ is intuitionistic i-continuous function if the inverse image of every intuitionistic open set in $(B,\tau_{I_2})$ is intuitionistic i-open in $(A,\tau_{I_1})$ .
**Definition 2.13.[7]** A function $s$: $(A,\tau_{I_1}) \to (B,\tau_{I_2})$ is said to be $\mathcal{J}$i-irresolute if $s^{-1}(O)$ is a $\mathcal{J}i$-open in $(A,\tau_{I_1})$ for every $\mathcal{J}i$-open set $O$ in $(B,\tau_{I_2})$.

**3. $\mathcal{J}i$-connected sets in Intuitionistic Topological Spaces**

**Definition 3.1.** An intuitionistic set $H$ in the ITS $(A, \tau_I)$ is said to be $\mathcal{J}i$-$C_s$-disconnected if and only if there are two non-empty $\mathcal{J}i$-weakly separated sets $P$ and $Q$ in $(A, \tau_I)$ such that $H = P \cup Q$. $H$ is called $\mathcal{J}i$-$C_s$-connected if $H$ is not $\mathcal{J}i$-$C_s$-disconnected.
**Theorem 3.2.** If $\mathcal{J}cl_i(H)$ is $\mathcal{J}i$-closed for every intuitionistic set $H$ in $(A, \tau_I)$ , then $G$ is $\mathcal{J}i$-$C_s$-connected if $G$ is $\mathcal{J}i$-$C_1$-connected.
**Proof:** Let $G$ be $\mathcal{J}i$-$C_s$-disconnected. Then there exists intuitionistic nonempty sets $M$ and $N$ such that $G = M \cup N$, $M$ and $N$ are $\mathcal{J}i$-weakly separated. So $\mathcal{J}cl_i(M) \subseteq N^c$ and $\mathcal{J}cl_i(N) \subseteq M^c$. Let $P = \left(\mathcal{J}cl_i(M)\right)^c$ and $Q = \left(\mathcal{J}cl_i(N)\right)^c$ . Then $P$ and $Q$ are $\mathcal{J}i$-open sets. Since $M$ and $N$ are $\mathcal{J}i$-weakly separated, $\mathcal{J}cl_i(M) \cap \mathcal{J}cl_i(N) \subseteq N^c \cap M^c = (M \cup N)^c = G^c$ which implies $G \subseteq \left(\mathcal{J}cl_i(M) \cap \mathcal{J}cl_i(N)\right)^c = ((\mathcal{J}cl_i(M))^c \cup ((\mathcal{J}cl_i(N) )^c = P \cup Q$ which implies $G \subseteq$

$P \cup Q$. Now $P \cap Q = ((\mathcal{J}cl_i(M))^c \cap ((\mathcal{J}cl_i(N))^c = (\mathcal{J}cl_i(M) \cup \mathcal{J}cl_i(N))^c \subseteq (M \cup N)^c = G^c$. If $P \cap G = \widetilde{\emptyset}$, then $P \subseteq G^c$ which implies $G \subseteq P^c$ which implies $G \subseteq \mathcal{J}cl_i(M) \subseteq N^c$ that is $M \cup N \subseteq N^c$ which is a contradiction. Hence $P \cap G \neq \widetilde{\emptyset}$. Similarly $Q \cap G \neq \widetilde{\emptyset}$. So, $G$ is $\mathcal{J}i$-$C_1$-disconnected which is a contradiction. Therefore, $G$ is $\mathcal{J}i$-$C_s$-connected.

**Remark 3.3.** Every $\mathcal{J}i$-$C_s$-connected is not $\mathcal{J}i$-$C_1$-connected.

**Example 3.4.** Let $A = \{x, y\}$ with $\tau_I = \{\tilde{A}, \widetilde{\emptyset}, < A, \{y\}, \{x\} >, < A, \emptyset, \{x\} >\}$. The $\mathcal{J}i$-open sets are $\{\tilde{A}, \widetilde{\emptyset}, < A, \emptyset, \emptyset >, < A, \emptyset, \{x\} >, < A, \{x\}, \emptyset >, < A, \{y\}, \emptyset >, < A, \{y\}, \{x\} >\}$. Let $H = < A, \emptyset, \{y\} >$. Then there does not exist $\mathcal{J}i$-weakly separated sets $P$ and $Q$ in $(A, \tau_I)$ such that $H = P \cup Q$. Therefore, $H$ is $\mathcal{J}i$-$C_s$-connected. Also, for $H = < A, \emptyset, \{y\} >$ there exist $\mathcal{J}i$-open sets $P = < A, \emptyset, \emptyset >$ and $Q = < A, \{x\}, \emptyset >$ such that $H \subseteq P \cup Q$, $P \cap Q \subseteq \overline{H}$, $H \cap P \neq \widetilde{\emptyset}$, $H \cap Q \neq \widetilde{\emptyset}$. Hence, $H = < A, \emptyset, \{y\} >$ is not $\mathcal{J}i$-$C_1$-connected.

**Theorem 3.5.** If $\mathcal{J}cl_i(H)$ is $\mathcal{J}i$-closed for every intuitionistic set $H$ in $(A, \tau_I)$, then $G$ is $\mathcal{J}i$-$C_2$-connected if $G$ is $\mathcal{J}i$-$C_s$-connected.

**Proof :** Let $G$ be $\mathcal{J}i$-$C_s$-connected. Suppose $G$ is $\mathcal{J}i$-$C_2$-disconnected. Then by definition there exists $\mathcal{J}i$-open sets $M$ and $N$ such that $G \subseteq M \cup N, M \cap N \cap G = \widetilde{\emptyset}$, $G \cap M \neq \widetilde{\emptyset}$, $G \cap N \neq \widetilde{\emptyset}$. Let $P = G \cap M$ and $Q = G \cap N$. Since $G \subseteq M \cup N$, $G = G \cap (M \cup N) = (G \cap M) \cup (G \cap N) = P \cup Q$. Let $P \subseteq M, Q \subseteq N$. Suppose $P \nsubseteq N^c$, then $P \cap N \neq \widetilde{\emptyset}$ which implies $G \cap M \cap N \neq \widetilde{\emptyset}$ which is a contradiction. Hence $P \subseteq N^c$. Similarly we can prove $Q \subseteq M^c$. By Theorem 2.5, $P$ and $Q$ are $\mathcal{J}i$-weakly separated. Hence $G$ is $\mathcal{J}i$-$C_s$-disconnected which is a contradiction. Hence, $G$ is $\mathcal{J}i$-$C_2$-connected.

**Remark 3.6.** Every $\mathcal{J}i$-$C_2$-connected is not $\mathcal{J}i$-$C_s$-connected.

**Example 3.7.** Consider example 3.4. Let $H = < A, \emptyset, \emptyset >$.Then there does not exist $\mathcal{J}i$-open sets such that $P \cap Q \cap H = \widetilde{\emptyset}$. Therefore, $H$ is $\mathcal{J}i$-$C_2$-connected. But, for $H = < A, \emptyset, \emptyset >$ there exists $\mathcal{J}i$-weakly separated sets $P = < A, \emptyset, \{x\} >$ and $Q = < A, \emptyset, \{y\} >$ in $(A, \tau_I)$ such that $H = P \cup Q$. Therefore, $H$ is $\mathcal{J}i$-$C_s$-disconnected.

**Theorem 3.8.** If $G_x$ and $G_y$ are intersecting $\mathcal{J}i$-$C_1$-connected sets, then $G_x \cup G_y$ is also $\mathcal{J}i$-$C_1$-connected **.**

**Proof :** Let $G_x \cup G_y$ be $\mathcal{J}i$-$C_1$-disconnected. Then there exists $\mathcal{J}i$-open sets $M$ and $N$ such that $G_x \cup G_y \subseteq M \cup N$, $M \cap N \subseteq (G_x \cup G_y)^c$and $(G_1 \cup G_y) \cap M \neq \widetilde{\emptyset}$, $(G_x \cup G_y) \cap N \neq \widetilde{\emptyset}$. Suppose $G_x$ and $G_y$ are $\mathcal{J}i$-$C_1$-connected then ($G_x \cap M = \widetilde{\emptyset}$ or $G_x \cap N = \widetilde{\emptyset}$) and ($G_y \cap M = \widetilde{\emptyset}$ or $G_y \cap N = \widetilde{\emptyset}$). Since $G_x \cap G_y \neq \widetilde{\emptyset}$, there exists $\tilde{\tilde{p}} \in G_x \cap G_y$ of the following cases. Case (i): Let $G_x \cap M = \widetilde{\emptyset}$ and $G_y \cap M = \widetilde{\emptyset}$. Then $(G_x \cap M) \cup (G_y \cap M)$ = $(G_x \cup G_y) \cap M = \widetilde{\emptyset}$ which is a contradiction. Case (ii): Let $G_x \cap M = \widetilde{\emptyset}$ and $G_y \cap N = \widetilde{\emptyset}$. Then there exists $\tilde{\tilde{p}} \notin M$, $\tilde{\tilde{p}} \notin N$ which is impossible since $\tilde{\tilde{p}} \in G_x \cup G_y \subseteq$ A ∪ B. Case (iii): Let $G_x \cap N = \widetilde{\emptyset}$ and $G_y \cap M = \widetilde{\emptyset}$. Then there exists $\tilde{\tilde{p}} \notin M$, $\tilde{\tilde{p}} \notin N$ which is impossible as above. Case (iv): Let $G_x \cap N = \widetilde{\emptyset}$ and $G_y \cap N = \widetilde{\emptyset}$. Then $(G_x \cap N) \cup (G_y \cap N) = (G_x \cup G_y) \cap N = \widetilde{\emptyset}$ which is a contradiction. Hence $G_x$ and $G_y$ are $\mathcal{J}i$-$C_1$-disconnected.

**Theorem 3.9.** If $G_x$ and $G_y$ are intersecting $\mathcal{J}i$-$C_2$-connected sets, then $G_x \cup G_y$ is also $\mathcal{J}i$-$C_2$-connected **.**

**Proof :** Similar to Theorem 3.8.

**Theorem 3.10.** Let $(G_k)$: $k \in J$ be a family of $\mathcal{J}i$-$C_1$-connected sets such that $\cap G_k \neq \widetilde{\emptyset}$. Then $\cup G_k$ is also $\mathcal{J}i$-$C_1$-connected.

**Proof :** Let $G = \cup G_k$ be $\mathcal{J}i$-$C_1$-disconnected. Then there exists $\mathcal{J}i$-open sets $M$ and $N$ such that $G \subseteq M \cup N$, $M \cap N \subseteq G^c$, $G \cap M \neq \widetilde{\emptyset}$, $G \cap N \neq \widetilde{\emptyset}$. Consider any index $k_0 \in J$.

Since $G_{k_0}$ is $\mathcal{J}i$-$C_1$-connected, we have $G_{k_0} \cap M = \widetilde{\emptyset}$, or $G_{k_0} \cap N = \widetilde{\emptyset}$, . So we have three cases. Case (i): If $G_k \cap M = \widetilde{\emptyset}$ for each $k \in J_1$ and $G_k \cap M = (\cup G_k) \cap M = \cup (G_k \cap M) = \widetilde{\emptyset}$ which is a contradiction. Case (ii): If $G_k \cap N = \widetilde{\emptyset}$ for each $k \in J_1$ and $G_k \cap N = (\cup G_k) \cap N = \cup (G_k \cap N) = \widetilde{\emptyset}$ which is a contradiction. Case (iii): If $G_k \cap M = \widetilde{\emptyset}$ for each $k \in J_1$ and $G_k \cap N = \widetilde{\emptyset}$ for each $k \in J_2$ where $J = J_1 \cup J_2$ and $J_1 \neq \widetilde{\emptyset}$ , $J_2 \neq \widetilde{\emptyset}$. Since $\cap G_k \neq \widetilde{\emptyset}$ , $\tilde{\tilde{p}} \in \cap G_k$. In this case $\tilde{\tilde{p}} \notin M$ and $\tilde{\tilde{p}} \notin N$ which is a contradiction $\tilde{\tilde{p}} \in G \subseteq M \cup N$ . Hence $G$ is also $\mathcal{J}i$-$C_1$-disconnected.

**Theorem 3.11.** Let $(G_k): k \in J$ be a family of $\mathcal{J}i$-$C_2$-connected sets such that $\cap G_k \neq \widetilde{\emptyset}$. Then $\cup G_k$ is also $\mathcal{J}i$-$C_2$-connected .

**Proof :** Similar to Theorem 3.6.

**Theorem 3.12.** Let $(A, \tau_I$ )) be an intuitionistic topological space. Then

(1) $\tilde{a}$ is $\mathcal{J}i$-$C_1$-connected

(2) $\tilde{a}$ is $\mathcal{J}i$-$C_2$-connected.

**Proof.** (1) Suppose $\tilde{a}$ be $\mathcal{J}i$-$C_1$-disconnected. Then there exist $\mathcal{J}i$-open sets $M$ and $N$ such that $\tilde{a} \subseteq M \cup N$, $M \cap N \subseteq \tilde{a}^c$, $\tilde{a} \cap M \neq \widetilde{\emptyset}$ , $\tilde{a} \cap N \neq \widetilde{\emptyset}$ where $\tilde{a}^c = < A, \{a\}^c, \{a\} >$. Since $\tilde{a} \cap M \neq \widetilde{\emptyset}$ and $\tilde{a} \cap N \neq \widetilde{\emptyset}$ , we get $\tilde{\tilde{a}} \in M$ and $\tilde{\tilde{a}} \in N$. But $M \cap N \subseteq \tilde{a}^c$ implies $M_1 \cap N_1 \subseteq \tilde{a}^c$ and $M_2 \cup N_2 \supseteq \tilde{a}^c$ which is impossible. Hence $\tilde{a}$ is $\mathcal{J}i$-$C_1$-connected. (ii) Let $\tilde{a}$ be $\mathcal{J}i$-$C_2$-disconnected. Then there exist $\mathcal{J}i$-open sets $M$ and $N$ such that $\tilde{a} \subseteq M \cup N$, $M \cap N \cap \tilde{a} = \widetilde{\emptyset}$ , $\tilde{a} \cap M \neq \widetilde{\emptyset}$ , $\tilde{a} \cap N \neq \widetilde{\emptyset}$ . Since $\tilde{a} \cap M \neq \widetilde{\emptyset}$ and $\tilde{a} \cap N \neq \widetilde{\emptyset}$ , we get $\tilde{\tilde{a}} \in M$ and $\tilde{\tilde{a}} \in N$ which implies $a \notin M_2$ and $a \notin N_2$. But $M \cap N \cap \tilde{a} = \widetilde{\emptyset}$ which implies $M_2 \cup N_2 \cup \{a\}^c = \tilde{A}$ which is impossible. Hence $\tilde{a}$ is $\mathcal{J}i$-$C_2$-connected.

**Definition 3.13.** An intuitionistic set $H$ in the ITS $(A, \tau_I$ ) is said to be $\mathcal{J}i$-$C_m$-disconnected if there exists an $\mathcal{J}i$-$q$-separated non-empty sets $P$ and $Q$ in $(K, \tau)$ such that $H = P \cup Q$ . $H$ is called $\mathcal{J}i$-$C_m$-connected set if it is not $\mathcal{J}i$-$C_m$-disconnected set.

## 4. $\mathcal{J}i$- compactness in Intuitionistic Topological Spaces

**Definition 4.1.** Let $(A, \tau_I$ ) be an intuitionistic topological space. If a family $\{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ of $\mathcal{J}i$-open sets in $A$ satisfies the condition $\cup \{< A, H_{1_k}, H_{2_k} > ; k \in J\} = \tilde{A}$ , then it is called an $\mathcal{J}i$-open cover of $A$. A finite subfamily of an $\mathcal{J}i$-open cover $\{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ of $A$ ,which is also an $\mathcal{J}i$-open cover of $A$ is called a finite $\mathcal{J}i$-subcover of $\{< A, H_{1_k}, H_{2_k} > ; k \in J\}$

**Definition 4.2.** Let $(A, \tau_I$ ) be an intuitionistic topological space . A family $\{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ of $\mathcal{J}i$-closed sets in $A$ satisfies the finite intersection property iff every finite subfamily $\{H_1, H_2, H_3, ...., H_n\}$ of $H$ satisfies the condition $\bigcap_{k=1}^{n} < A, H_{1_k}, H_{2_k} > \neq \widetilde{\emptyset}$

**Definition 4.3.** An ITS $(A, \tau_I$ ) is said to be $\mathcal{J}i$-compact iff each $\mathcal{J}i$-open cover has a finite subcover.

**Definition 4.4.** Let $(A, \tau_I$ ) be an intuitionistic topological space and $G$ be an IS in $A$. The family $\{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ of $\mathcal{J}i$-open sets in $A$ is called a $\mathcal{J}i$-open cover of $G$ if $G \subseteq \cup \{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ .

**Definition 4.5.** An IS $G = < A, G_1, G_2 >$ in an ITS $(A, \tau_I$ ) is called $\mathcal{J}i$-compact iff every $\mathcal{J}i$-open cover of $G$ has a finite sub cover. Also we can define an IS $G = < A, G_1, G_2 >$ in $(A, \tau_I$ ) is $\mathcal{J}i$-compact iff for each family $H = \{H_k : k \in J\}$ where $H_k = \{< A, H_{1_k}, H_{2_k} > ; k \in J\}$ of $\mathcal{J}i$-open sets in $A$, $G_1 \subseteq \bigcup_{k \in J} H_{1_k}$ and $G_2 \supseteq \bigcup_{k \in J} H_{2_k}$, there exists a finite subfamily $\{H_1, H_2, H_3, ...., H_n\}$ of $H$ such that $G_1 \subseteq \bigcup_{k=1}^{n} H_{1_k}$and $G_2 \supseteq \bigcup_{k=1}^{n} H_{2_k}$ .

**Proposition 4.6.** Let $(A, \tau_I$ ) be an intuitionistic topological space. Then $(A, \tau_I$ ) is $\mathcal{J}i$-compact iff the ITS $(A, \tau_{I_{0,1}}$ ) is $\mathcal{J}i$-compact.

**Proof. Necessity:** Let $(A,\tau_I)$ be $\mathcal{J}i$-compact and consider an $\mathcal{J}i$-open cover $\{[\,]H_k : k \in J\}$ of $A$ in $(A,\tau_{I_{0,1}})$. Since $\cup([\,]H_k) = \tilde{A}$ , we obtain $\cup H_{1_k} = A$ and hence $H_{2_k} \subseteq (H_{1_k})^c$ which implies $\cap H_{2_k} \subseteq (\cup H_{1_k})^c = \emptyset$ which implies $\cap H_{2_k} = \emptyset$ and hence $\cup H_k = \tilde{A}$. Since $(A,\tau_I)$ is $\mathcal{J}i$-compact, there exists $H_1, H_2, H_3, \ldots, H_n$ such that $\bigcup_{k=1}^{n} H_k = \tilde{A}$ which implies $\bigcup_{k=1}^{n} H_{1_k} = A$ and $\bigcap_{k=1}^{n} H_{2_k} = \emptyset$. Hence $(A,\tau_{I_{0,1}})$ is $\mathcal{J}i$-compact.

**Sufficiency:** Suppose $(A,\tau_{I_{0,1}})$ is $\mathcal{J}i$-compact. Consider an $\mathcal{J}i$-open cover $\{H_k : k \in J\}$ of $A$ in $(A,\tau_I)$. Since $\cup H_k = \tilde{A}$, we obtain $\cup H_{1_k} = A$ and hence $\cap (H_{1_k})^c = \emptyset$ which implies $\cup([\,]H_k) = \tilde{A}$. Since $(A,\tau_{I_{0,1}})$ is $\mathcal{J}i$-compact, there exists $H_1, H_2, H_3, \ldots, H_n$ such that $\bigcup_{k=1}^{n}([\,]H_k) = \tilde{A}$ which implies $\bigcup_{k=1}^{n} H_{1_k} = A$ and $\bigcap_{k=1}^{n}(H_{1_k})^c = \emptyset$. Hence $H_{1_k} \subseteq (H_{2_k})^c$ which implies $A = \bigcup_{k=1}^{n} H_{1_k} \subseteq \left(\bigcap_{k=1}^{n} H_{2_k}\right)^c$ which implies $\bigcap_{k=1}^{n} H_{2_k} = \emptyset$. Thus $\bigcup_{k=1}^{n} H_k = \tilde{A}$. So $(A,\tau_I)$ is $\mathcal{J}i$-compact.

**Proposition 4.7.** The ITS $(A,\tau_I)$ is $\mathcal{J}i$-compact iff $(A,\tau_{I1})$ is $\mathcal{J}i$-compact.

**Proof.** Similar to Proposition 4.6.

**Proposition 4.8.** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be a surjective $\mathcal{J}i$-continuous mapping. If $(A,\tau_{I_1})$ is $\mathcal{J}i$-compact then $(B,\tau_{I_2})$ is intuitionistic compact.

**Proof:** Let $\{H_k : k \in J\}$ be any intuitionistic open cover of $B$. Since $s$ is $\mathcal{J}i$-continuous, $\{s^{-1}(H_k) : k \in J\}$ is an $\mathcal{J}i$-open cover of $A$. Since $(A,\tau_{I_1})$ is $\mathcal{J}i$-compact, it has a finite subcover $\{s^{-1}(H_1), s^{-1}(H_2), s^{-1}(H_3), \ldots, s^{-1}(H_n)\}$ such that $\bigcup_{k=1}^{n} s^{-1}(H_{1_k}) = \tilde{A}$ and $\bigcap_{k=1}^{n} s^{-1}(H_{2_k}) = \tilde{\emptyset}$ that is $s^{-1}(\bigcup_{k=1}^{n}(H_{1_k})) = A$ and $s^{-1}(\bigcap_{k=1}^{n}(H_{2_k})) = \emptyset$ which implies $\bigcup_{k=1}^{n}(H_{1_k}) = s(A)$ and $\bigcap_{k=1}^{n}(H_{2_k}) = s(\emptyset)$. Since $s$ is surjective $\{H_1, H_2, H_3, \ldots, H_n\}$ is an open cover of $B$ and hence $(B,\tau_{I_2})$ is intuitionistic compact.

**Corollary 4.9.** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be $\mathcal{J}i$-continuous. If $N$ is $\mathcal{J}i$-compact in $(A,\tau_{I_1})$, then $s(N)$ is intuitionistic compact in $(B,\tau_{I_2})$.

**Proof:** Let $\{G_k : k \in J\}$ be an $\mathcal{J}$-open set of $B$ such that $s(N) \subseteq \cup\{G_k : k \in J\}$. Then $N \subset \cup\{s^{-1}(G_k) : k \in J\}$ where $s^{-1}(G_k)$ is $\mathcal{J}i$-open in $A$ for each $k$. Since $N$ is $\mathcal{J}i$-compact relative to $A$, there exists a finite sub collection $\{G_1, G_2, \ldots, G_n\}$ such that $N \subset \cup\{s^{-1}(G_k) : k = 1,2,\ldots,n\}$ . Hence $s(N) \subset s(\cup\{s^{-1}(G_k) : k = 1,2,\ldots,n\}) = \cup\{s(s^{-1}(G_k)) : k = 1,2,\ldots,n\}) \subset \cup\{G_k : k = 1,2,\ldots,n\}$. Hence $s(N)$ is $\mathcal{J}i$-compact relative 12to $B$.

**Proposition 4.10.** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be an $\mathcal{J}i$-irresolute mapping and if $M$ is $\mathcal{J}i$-compact relative to $A$, then $s(M)$ is $\mathcal{J}i$-compact relative to $B$.

**Proof.** Let $\{H_k : k \in J\}$ be an $\mathcal{J}i$-open set of $B$ such that $s(M) \subseteq \cup\{H_k : k \in J\}$. Then $M \subset \cup\{s^{-1}(H_k) : k \in J\}$ where $s^{-1}(H_k)$ is $\mathcal{J}i$-open in $A$ for each $k$. Since $M$ is $\mathcal{J}i$-compact relative to $A$, there exists a finite sub collection $\{H_1, H_2, \ldots, H_n\}$ such that $M \subset \cup\{s^{-1}(H_k) : k = 1,2,\ldots,n\}$ which implies $s(M) \subset \cup\{H_k : k = 1,2,\ldots,n\}$. Hence $s(M)$ is $\mathcal{J}i$-compact relative to $B$.

**Proposition 4.11.** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be an $\mathcal{J}i$-irresolute mapping. If $A$ is $\mathcal{J}i$-compact, then $B$ is also an $\mathcal{J}i$-compact space .

**Proof.** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be an $\mathcal{J}i$-irresolute mapping from $\mathcal{J}i$-compact space $(A,\tau_{I_1})$ onto an intuitionistic topological space $(B,\tau_{I_2})$. Let $\{G_k : k \in J\}$ be an $\mathcal{J}i$-open cover of $B$. Then $\{s^{-1}(G_k) : k \in J\}$ is an $\mathcal{J}i$-open cover of $A$. Since $A$ is $\mathcal{J}i$-compact, there is a finite subfamily $\{s^{-1}(G_{k_1}), s^{-1}(G_{k_2}), s^{-1}(G_{k_3}), \ldots, s^{-1}(G_{k_n})\}$ of $\{s^{-1}(G_k) : k \in J\}$ such that $\bigcup_{j=1}^{n} s^{-1}(G_{k_j}) = \tilde{A}$ . Since $s$ is onto, $s(\tilde{A}) = \tilde{B}$ and $s(\bigcup_{j=1}^{n} s^{-1}(G_{k_j})) = \bigcup_{j=1}^{n} s(s^{-1}(G_{k_j})) = \bigcup_{j=1}^{n} G_{k_j}$. It follows that $\bigcup_{j=1}^{n} G_{k_j} = \tilde{B}$ and the family $\{G_{k_1}, G_{k_2}, G_{k_3}, \ldots, G_{k_n}\}$ is an intuitionistic finite subcover of $\{G_k : k \in J\}$. Hence $(B,\tau_{I_2})$ is an $\mathcal{J}i$- compact.

**Theorem 4.12.** An ITS $(A, \tau_I)$ is $\mathcal{I}i$-compact iff every family $\{< A, H_{1_k}, H_{2_k} >; k \in J\}$ of $\mathcal{I}i$-closed sets in $A$ having the FIP has a nonempty intersection.

**Proof:** Assume that $A$ is $\mathcal{I}i$-compact that is every $\mathcal{I}i$-open cover of $A$ has a finite $\mathcal{I}i$-subcover. Let $H_k = \{< A, H_{1_k}, H_{2_k} >; k \in J\}$ be a family of $\mathcal{I}i$-closed sets of $A$. Also assume that this family has finite intersection property. We have to show that $\bigcap_{k \in J} H_k \neq \tilde{\emptyset}$. Suppose on the contrary , $\bigcap_{k \in J} H_k = \tilde{\emptyset}$ which implies $\overline{\bigcap_{k \in J} H_k} = \overline{\tilde{\emptyset}}$ which implies $\bigcup_{k \in J} \overline{H_k} = \tilde{A}$ that is $\bigcup_{k \in J} < A, H_{2_k}, H_{1_k} > = \tilde{A}$. Since for every $k \in J$, $H_k$ is an $\mathcal{I}i$-closed set of $A$, therefore $\overline{H_k}$ will be an $\mathcal{I}i$-open set of $A$. Thus, $\{ \overline{H_k} = < A, H_{2_k}, H_{1_k} > : k \in J\}$ is an $\mathcal{I}i$-open cover for $A$. Since $A$ is $\mathcal{I}i$-compact, this $\mathcal{I}i$-cover has a finite $\mathcal{I}i$-subcover, say, $\bigcup_{k=1}^{n} \overline{H_k} = \bigcup_{k=1}^{n} < A, H_{2_k}, H_{1_k} > = \tilde{A}$ .Then, $\overline{\bigcup_{k=1}^{n} \overline{H_k}} = \overline{\tilde{A}}$ which implies $\bigcap_{k=1}^{n} H_k = \tilde{\emptyset}$ Thus, the above considered family does not satisfy the FIP which is a contradiction. Therefore, $\bigcap_{k \in J} H_k \neq \tilde{\emptyset}$. Conversely, assume that the family of $\mathcal{I}i$-closed sets of $A$ having FIP has nonempty intersection. To show that $A$ is $\mathcal{I}i$-compact. Let $\{H_k = \{< A, H_{1_k}, H_{2_k} >; k \in J\}$ be an $\mathcal{I}i$-open cover of $A$. Suppose this $\mathcal{I}i$-open cover has no finite $\mathcal{I}i$-subcover, that is for every finite subcollection of the given cover, say, $\bigcup_{k=1}^{n} H_k \neq \tilde{A}$ which implies $\overline{\bigcup_{k=1}^{n} H_k} \neq \overline{\tilde{A}}$ which implies $\bigcap_{k=1}^{n} \overline{H_k} \neq \tilde{\emptyset}$ . As each $H_k$ is an $\mathcal{I}i$-open set of $A$ therefore, each $\overline{H_k}$ is an $\mathcal{I}i$-closed set of $A$. Thus, $\{ \overline{H_k} = < A, H_{2_k}, H_{1_k} > : k \in J\}$ is a family of $\mathcal{I}i$-closed set of $A$ having FIP. So by the hypothesis it has nonempty intersection, that is $\bigcap_{k \in J} \overline{H_k} \neq \tilde{\emptyset}$ which implies $\overline{\bigcap_{k \in J} \overline{H_k}} \neq \overline{\tilde{\emptyset}}$ which implies $\bigcup_{k \in J} H_k \neq \tilde{A}$. This shows that the family $\{ H_k = \{< A, H_{1_k}, H_{2_k} >; k \in J\}$ is not an $\mathcal{I}i$-open cover for $A$, which is a contradiction. Therefore, the given family must have a finite $\mathcal{I}i$-subcover and hence $A$ is $\mathcal{I}i$-compact.

**References**

[1] K Atanassov, 1986 Intuitionistic fuzzy sets , Fuzzy Sets and Systems 20 pp 87-66

[2] D. Coker. A note on intuitionistic sets and intuitionistic points. Turkish J. Math., 20(3)(1996), 343-351.

[3] D. Coker. An introduction to intuitionistic topological spaces. BUSEFAL, 81(2000), 51-56.

[4] Selma Ozcag and D.Coker. A Note on Connectedness in Intuitionistic Fuzzy Special Topological Spaces. Int. J. Math. Math. Sci., 23(1)(2000), 45-54.

[5] S.Selvanayaki and Gnanambal Ilango, IGPR Continuity and Compactness in Intuitionistic Topological Spaces, British Journal of Mathematics and Computer Science, 11(2): 1-8, 2015

[6] P.Suganya And J. Arul Jesti, On intuitionistic i-open sets in Intuitionistic Topological Spaces, Journal of Algebraic Statistics, May 2022, Vol. 13, Issue 2, 3182-3187

[7] P.Suganya and J. Arul Jesti, $\mathcal{I}i$-continuous functions in Intuitionistic Topological Spaces, GIS Science Journal, Volume 9 , Issue 6(2022),824-831.

[8] P.Suganya and J. Arul Jesti, Some New Connectedness in Intuitionistic Topological Spaces, Purakala, Volume 32 , Issue 1(2023),849-854.

[9] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338-353.